Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Thursday 24 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer all the questions.
- Do not write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- This document consists of 4 pages. Any blank pages are indicated.
1 For the variables $A$ and $B$, it is given that $\text{Var}(A) = 9$, $\text{Var}(B) = 6$ and $\text{Var}(2A - 3B) = 18$.

(i) Find $\text{Cov}(A, B)$. [3]

(ii) State with a reason whether $A$ and $B$ are independent. [1]

2 The probability generating function of the discrete random variable $X$ is $e^{4t}$. Find

(i) $E(X)$. [3]

(ii) $P(X = 2)$. [3]

3 $X_1$ and $X_2$ are continuous random variables. Random samples of 5 observations of $X_1$ and 6 observations of $X_2$ are taken. No two observations are equal. The 11 observations are ranked, lowest first, and the sum of the ranks of the observations of $X_1$ is denoted by $R$.

(i) Assuming that all rankings are equally likely, show that $P(R \leq 17) = \frac{2}{231}$. [5]

The marks of 5 randomly chosen students from School A and 6 randomly chosen students from School B, who took the same examination, achieving different marks, were ranked. The rankings are shown in the table.

<table>
<thead>
<tr>
<th>Rank</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>School</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>A</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
</tbody>
</table>

(ii) For a Wilcoxon rank-sum test, obtain the exact smallest significance level for which there is evidence of a difference in performance at the two schools. [2]

4 The moment generating function of a continuous random variable $Y$, which has a $\chi^2$ distribution with $n$ degrees of freedom, is $(1 - 2t)^{-\frac{n}{2}}$, where $0 \leq t < \frac{1}{2}$.

(i) Find $E(Y)$ and $\text{Var}(Y)$. [5]

For the case $n = 1$, the sum of 60 independent observations of $Y$ is denoted by $S$.

(ii) Write down the moment generating function of $S$ and hence identify the distribution of $S$. [2]

(iii) Use a normal approximation to estimate $P(S \geq 70)$. [3]

5 In order to test whether the median salary of employees in a certain industry who had worked for three years was £19 500, the salaries $x$, in thousands of pounds, of 50 randomly chosen employees were obtained.

(i) The values $|x - 19.5|$ were calculated and ranked. No two values of $x$ were identical and none was equal to 19.5. The sum of the ranks corresponding to positive values of $(x - 19.5)$ was 867. Stating a required assumption, carry out a suitable test at the 5% significance level. [10]

(ii) If the assumption you stated in part (i) does not hold, what test could have been used? [1]
Nuts and raisins occur in randomly chosen squares of a particular brand of chocolate. The numbers of nuts and raisins are denoted by \( N \) and \( R \) respectively and the joint probability distribution of \( N \) and \( R \) is given by

\[
f(n, r) = \begin{cases} 
  c(n + 2r) & n = 0, 1, 2 \text{ and } r = 0, 1, 2, \\
  0 & \text{otherwise},
\end{cases}
\]

where \( c \) is a constant.

(i) Find the value of \( c \). [3]

(ii) Find the probability that there is exactly one nut in a randomly chosen square. [2]

(iii) Find the probability that the total number of nuts and raisins in a randomly chosen square is more than 2. [2]

(iv) For squares in which there are 2 raisins, find the mean number of nuts. [4]

(v) Determine whether \( N \) and \( R \) are independent. [2]

The continuous random variable \( X \) has probability density function given by

\[
f(x) = \begin{cases} 
  \frac{x}{2\theta^2} & 0 \leq x \leq 2\theta, \\
  0 & \text{otherwise},
\end{cases}
\]

where \( \theta \) is an unknown positive constant.

(i) Find \( E(X^n) \), where \( n \neq -2 \), and hence write down the value of \( E(X) \). [3]

(ii) Find

(a) \( \text{Var}(X) \),

(b) \( \text{Var}(X^2) \). [4]

(iii) Find \( E(X_1 + X_2 + X_3) \) and \( E(X_1^2 + X_2^2 + X_3^2) \), where \( X_1, X_2 \) and \( X_3 \) are independent observations of \( X \). Hence construct unbiased estimators, \( T_1 \) and \( T_2 \), of \( \theta \) and \( \text{Var}(X) \) respectively, which are based on \( X_1, X_2 \) and \( X_3 \). [6]

(iv) Find \( \text{Var}(T_2) \). [2]

For the events \( L \) and \( M \), \( P(L \mid M) = 0.2 \), \( P(M \mid L) = 0.4 \) and \( P(M) = 0.6 \).

(i) Find \( P(L) \) and \( P(L' \cup M') \). [3]

(ii) Given that, for the event \( N \), \( P(N \mid (L \cap M)) = 0.3 \), find \( P(L' \cup M' \cup N') \). [3]
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