INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer all the questions.
- Do not write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- This document consists of 4 pages. Any blank pages are indicated.
1 (i) The number of inhabitants of a village who are selected for jury service in the course of a 10-year period is a random variable with the distribution \( \text{Po}(4.2) \).

(a) Find the probability that in the course of a 10-year period, at least 7 inhabitants are selected for jury service. [2]

(b) Find the probability that in 1 year, exactly 2 inhabitants are selected for jury service. [3]

(ii) Explain why the number of inhabitants of the village who contract influenza in 1 year can probably not be well modelled by a Poisson distribution. [2]

2 A university has a large number of students, of whom 35% are studying science subjects. A sample of 10 students is obtained by listing all the students, giving each a serial number and selecting by using random numbers.

(i) Find the probability that fewer than 3 of the sample are studying science subjects. [3]

(ii) It is required that, in selecting the sample, the same student is not selected twice. Explain whether this requirement invalidates your calculation in part (i). [2]

3 Tennis balls are dropped from a standard height, and the height of bounce, \( H \) cm, is measured. \( H \) is a random variable with the distribution \( \text{N}(40, \sigma^2) \). It is given that \( P(H < 32) = 0.2 \).

(i) Find the value of \( \sigma \). [3]

(ii) 90 tennis balls are selected at random. Use an appropriate approximation to find the probability that more than 19 have \( H < 32 \). [6]

4 The proportion of commuters in a town who travel to work by train is 0.4. Following the opening of a new station car park, a random sample of 16 commuters is obtained, and 11 of these travel to work by train. Test at the 1% significance level whether there is evidence of an increase in the proportion of commuters in this town who travel to work by train. [7]

5 The time \( T \) seconds needed for a computer to be ready to use, from the moment it is switched on, is a normally distributed random variable with standard deviation 5 seconds. The specification of the computer says that the population mean time should be not more than 30 seconds.

(i) A test is carried out, at the 5% significance level, of whether the specification is being met, using the mean \( \bar{T} \) of a random sample of 10 times.

(a) Find the critical region for the test, in terms of \( \bar{T} \). [4]

(b) Given that the population mean time is in fact 35 seconds, find the probability that the test results in a Type II error. [3]

(ii) Because of system degradation and memory load, the population mean time \( \mu \) seconds increases with the number of months of use, \( m \). A formula for \( \mu \) in terms of \( m \) is \( \mu = 20 + 0.6m \). Use this formula to find the value of \( m \) for which the probability that the test results in rejection of the null hypothesis is 0.5. [4]
6  (a) The random variable $D$ has the distribution Po(24). Use a suitable approximation to find $P(D > 30)$.  

(b) An experiment consists of 200 trials. For each trial, the probability that the result is a success is 0.98, independent of all other trials. The total number of successes is denoted by $E$.

(i) Explain why the distribution of $E$ cannot be well approximated by a Poisson distribution.  

(ii) By considering the number of failures, use an appropriate Poisson approximation to find $P(E \leq 194)$.  

7  A machine is designed to make paper with mean thickness 56.80 micrometres. The thicknesses, $x$ micrometres, of a random sample of 300 sheets are summarised by

$$n = 300, \quad \Sigma x = 17\,085.0, \quad \Sigma x^2 = 973\,847.0.$$ 

Test, at the 10% significance level, whether the machine is producing paper of the designed thickness.  

8  The continuous random variable $X$ has probability density function given by

$$f(x) = \begin{cases} kx^{-a} & x \geq 1, \\ 0 & \text{otherwise}, \end{cases}$$

where $k$ and $a$ are constants and $a$ is greater than 1.

(i) Show that $k = a - 1$.  

(ii) Find the variance of $X$ in the case $a = 4$.  

(iii) It is given that $P(X < 2) = 0.9$. Find the value of $a$, correct to 3 significant figures.