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Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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### 1(i)

**a**
- 31 75 87 42 43 70 56 61 95 28
  - (may be shown vertically or as separate swaps)
  - 9 comparisons and 8 swaps
  - The smallest (final) mark, 28
  - 28 moved to the end of the list, no other values moved
  - Correct list at end of first pass (cao)
  - 9 and 8 (written, not tallies) (cao) - if not specified, assume the larger value is comparisons (their) 28 or smallest/least or final/last/end
  - If sorted into increasing order: 28 31 75 42 43 70 56 61 87 95
  - M0 A0, then 9 and 6 = B1 and (their) 95 or largest/greatest/biggest or final/last/end = B1

### 1(b)
- 75 87 42 43 70 56 61 95 31 28
  - Correct list at end of second pass
  - If sorted into increasing order and already penalised in (i)(a) then condone here: 28 31 42 43 70 56 61 75 87 95

### 1(c)
- 7 more passes
  - 7 (cao)

### 1(ii)
- 31 28 75 87 42 43 70 56 61 95
  - 75 31 28 87 42 43 70 56 61 95
  - 1 comparison and 0 swaps in first pass
  - 2 comparisons and 2 swaps in second pass
  - 31 28 75 or 31 28 75 …
  - Correct list, in full, at end of second pass
  - Lists must be easily found, not picked out from working, if the candidate has labelled passes use them as labelled 1 and 0 (written)(cao) may appear next to list
  - 1 and 0 (written)(cao) may appear next to list
  - If sorted into increasing order: 28 31 75 …
  - M0, A0, then 1 and 1 = B1; 1 and 0 = B1

### 1(iii)
- Bubble sort does not terminate early, since it takes 9 passes to get 95 to the front of the list, so it uses 9+8+…+1 or 45 comparisons
  - Shuttle sort takes fewer than 1+2+…+9 comparisons, since, for example, in the fourth pass 42 will be compared with 28, 31 and 75 but not with 87.
  - Identifying that bubble sort does not terminate early (Just stating 9+8+…+1 or 45 = B0)
  - Allow ‘the largest number is at the end of the list’ or ’95 at end’
  - A good explanation of why shuttle sort requires fewer comparisons in this particular case
  - Do not accept ‘because the list is not in reverse order’

### 1(iv)
- \[20 \times \left(\frac{50}{10}\right)^2\]
- = 500 seconds
  - Correct method
  - 500 seconds or 8 mins 20 sec (without wrong working)
### Question 2

<table>
<thead>
<tr>
<th>Part</th>
<th>Description</th>
<th>Marks</th>
<th>Notes</th>
</tr>
</thead>
</table>
| (i)  | Cannot have an odd number of odd nodes  
Odd vertices come in pairs | B1 [1] | Sum of orders must be even  
Sum of orders is 9 so 4.5 arcs (which is impossible) |
| (ii) | eg  
[Diagram of a graph with 4 vertices, not connected and not simple] | M1  
A1 [2] | A diagram showing a graph with four vertices that is  
not connected and not simple  
Vertices have orders 1, 2, 3, 4 |
| (iii) | The vertex of order 4 needs to connect to four  
other vertices, but there are only three other  
vertices available, so one vertex must be joined  
twice or the vertex of order 4 is connected to  
itself. Hence the graph cannot be simple | M1  
A1 [2] | Specifically identifying that the problem is with the vertex of order 4  
Explaining why the graph cannot be simple (either reason)  
and stating that simple cannot be achieved  
Ignore any claims about whether or not the graph is connected |
| (iv) | Each vertex of order 4 connects to each of the  
others, since graph is simple. Hence the other two  
vertices must have order (at least) 3.  
But Eulerian, so all must have order 4. | B1 [1] | Any reasonable explanation, but not just a diagram of a  
specific case  
‘the other two must be odd but they can’t because  
Eulerian’ is not enough  
Note: the graph has five vertices |
| (a)  | Graph is Eulerian - so each vertex order is even;  
simple - so no vertex has order more than 4; and  
connected - so no vertex has order 0. Hence each  
vertex has order either 2 or 4. But cannot have 3  
or 4 vertices of order 4. So must have 0, 1, 2 or 5  
vertices of order 4.  
[Diagram of a graph with 5 vertices, not connected and not simple] | B1  
M1  
A1 [3] | Explaining why there are only four such graphs  
Or list all the possibilities (eg 22222 42222 44222 44444)  
Any two correct (note: must be simply connected and  
Eulerian)  
All four correct and no extras (apart from topologically  
equivalent variations) |
### Question 3

#### (i)

| y ≥ x  
| x ≥ 0  
| y ≤ 7 − \(\frac{2}{3}x\) | M1 | Boundaries \(y = x\) and \(x = 0\) in any form (may be shown as an equality or an inequality with inequality sign wrong)  
| Boundary \(2x + 3y = 21\) in any form  
| All inequalities correct (and any extras do not affect the feasible region) | M1 A1 [3] |

#### (ii)

| (0, 7) \(\Rightarrow\) 42  
| (4.2, 4.2) \(\Rightarrow\) 29.4 or \((\frac{21}{3}, \frac{21}{3})\) \(\Rightarrow\) \(\frac{147}{3}\) | M1 A1 [3] | Substantially correct attempt at testing vertices (at least one vertex apart from (0, 0)) or using a line of constant profit (may be implied)  
| Accept (0, 7) identified (cao)  
| 42 (stated) (cao) NOT deduced from earlier working, unless identified | |

#### (iii)

| (4.2, 4.2)  
| \(P_k = 4.2(k + 6)\) or \(4.2k + 25.2\) | B1 | cao |

#### (iv)

| Compare \(kx + 6y\) with boundary \(2x + 3y\) or algebraically, \(4.2(k + 6)\) with 42  
| or \(\frac{k}{6}\) with \(-\frac{2}{3}\)  
| \(\Rightarrow k ≤ 4\)  
| \(k ≤ 4\) or \(k < 4\) implies M1, A1 | M1 A1 [2] | Algebraically or using line, or implied (allow = here)  
| Accept \(k < 4\)  
| No need to say that \(k > 0\), but candidates may also say \(k > 0\) or \(k ≥ 0\)  
| Note: \(k\) is continuous, so answers such as ‘\(k = 1, 2, 3, 4\)’ or ‘\(k = 1, 2, 3\)’, with no other working, would get M1, A0 |
### 4(i)

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>B</td>
<td>0.5</td>
<td>3</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>0.6</td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>G</td>
<td>1.1</td>
<td></td>
<td>1.1</td>
<td></td>
</tr>
</tbody>
</table>

**Route:** A → B → D → F → G

- M1: 1.7 shown as a temporary label at G
- A1: All temporary labels correct with no extras (may not have written temporary label when it becomes permanent)
- B1: All permanent labels correct (cao)
- B1: Order of labelling correct (cao)
- B1: This route written down (not reversed) (cao)

#### 4(ii)

**Route Inspection problem**

- B1: [1] Accept Chinese postman
  - Allow ‘postman’, ‘postman route’, but not just ‘inspection’

#### 4(iii)

- **CD (CBD) = 0.3, DG (DFG) = 0.65,**
- **CG (CBDFG) = 0.95**
- **CD (CBD) and FG = 0.75**
  - or **CD (CBD) and EG (EFG) = 1.05**

**Length:**

\[
\text{Length} = 3.7 + 0.5 + 0.3 + 0.75 = 5.25 \text{ km}
\]

- M1: Any one of these seen (explicitly or as part of a calculation)
- A1: All three of these seen (explicitly or as parts of calculations)
- M1: Or either of these with AB to give 1.25 or 1.55 respectively
- A1: Adding their 0.75 to 3.7 or their 0.75 to 3.7 + 0.5 + 0.3 (cao) units not needed
  - 5.25 implies M1, M1 A1, irrespective of working

#### 4(iv)

- **B → D → F → G → C → B**
- **1.9 km**

- B1: cao
- B1: [2] 1.9 (cao) irrespective of method

#### 4(v)

**[TREE]**

- Vertices added in order BDCF or BDFC
- Arcs added in order BD, BC, DF or BD, DF, BC
- Two shortest arcs from G total 0.45 + 0.65 = 1.1
- Lower bound = 0.5 + 1.1 = 1.6 km

- B1: Correct tree drawn
- B1: A valid order of adding vertices or a valid order of adding arcs
- M1: 0.45 and 0.65, or total 1.1 (may be implied from 1.6)
- A1: 1.6 (cao) units not needed
  - 1.6 implies M1, A1

- [4]
5(i) \[ 600x + 800y + 500z \leq 5000 \]
\[ \Rightarrow 6x + 8y + 5z \leq 50 \]

\[ 120x + 80y + 120z \leq 800 \]
\[ \Rightarrow 3x + 2y + 3z \leq 20 \]

May use slack variables, provided they also specify slack variables non-negative
eg \[ 6x + 8y + 5z + t = 50, \ t \geq 0 \ = M1, \ A1 \]

<table>
<thead>
<tr>
<th>( s )</th>
<th>( t )</th>
<th>( u )</th>
<th>( \text{RHS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>60</td>
</tr>
</tbody>
</table>

Correct inequality, allow < for M mark only
Correct fully simplified form (cao)

(ii)

\[ 60 + 15 = 4, \ 50 + 5 = 10, \ 20 + 3 = 6 \frac{2}{3} \]

Pivot on the 15 in the \( z \) column

New row 2 = row 2 + 15
New row 1 = row 1 + 120 \times \text{new row 2}
New row 3 = row 3 - 5 \times \text{new row 2}
New row 4 = row 4 - 3 \times \text{new row 2}

<table>
<thead>
<tr>
<th>( P )</th>
<th>( x )</th>
<th>( y )</th>
<th>( z )</th>
<th>( s )</th>
<th>( t )</th>
<th>( u )</th>
<th>( \text{RHS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>480</td>
</tr>
<tr>
<td>0</td>
<td>\frac{4}{3}</td>
<td>1 \frac{1}{3}</td>
<td>1</td>
<td>\frac{1}{15}</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>1 \frac{1}{3}</td>
<td>0</td>
<td>\frac{1}{3}</td>
<td>1</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>0</td>
<td>\frac{1}{3}</td>
<td>\frac{2}{3}</td>
<td>0</td>
<td>\frac{1}{3}</td>
<td>0</td>
<td>1</td>
<td>8</td>
</tr>
</tbody>
</table>

Correct pivot choice from their \( z \) column
Correct method for their pivot row seen (or implied from correct row in tableau if no attempt seen)
Correct method for their three other rows seen as a formula

Iterate to get a tableau with exactly four basis columns and non-negative entries in final column, in which the value of the objective has not decreased
Values in final column correct (follow through)

\[ 4 + \frac{4}{3} = 5, \ 30 + 2 = 15, \ 8 + \frac{2}{3} = 13 \frac{1}{3} \]

Pivot on the \( \frac{4}{3} \) in the \( x \) column

New row 2 = row 2 + \frac{4}{3}
New row 1 = row 1 + 4 \times \text{new row 2}
New row 3 = row 3 - 2 \times \text{new row 2}
New row 4 = row 4 - \frac{4}{3} \times \text{new row 2}

<table>
<thead>
<tr>
<th>( P )</th>
<th>( x )</th>
<th>( y )</th>
<th>( z )</th>
<th>( s )</th>
<th>( t )</th>
<th>( u )</th>
<th>( \text{RHS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>126 \frac{2}{3}</td>
<td>5</td>
<td>8 \frac{1}{3}</td>
<td>0</td>
<td>0</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1 \frac{1}{3}</td>
<td>1 \frac{1}{3}</td>
<td>\frac{1}{12}</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>\frac{2}{3}</td>
<td>\frac{1}{4}</td>
<td>1</td>
<td>0</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>\frac{3}{3}</td>
<td>\frac{2}{3}</td>
<td>\frac{1}{3}</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

Correct pivot choice for their second iteration
Correct method for their pivot row seen (or implied from correct row in tableau if no attempt seen)
Correct method for their three other rows seen as a formula

Iterate to get a tableau with exactly four basis columns and non-negative entries in final column, in which the value of the objective has not decreased
Values in final column correct (follow through)
<table>
<thead>
<tr>
<th>Expression</th>
<th>Marks</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make 5 litres of <em>fruit salad</em> only</td>
<td>B1</td>
<td>Interpretation of <em>their</em> final (non-negative) (x, y \text{ and } z), in context (need ‘only’ or equivalent; ‘5 fruit salads’ is not enough)</td>
</tr>
<tr>
<td>(x = 5, y = 0, z = 0) gives B0</td>
<td>[13]</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(60 \div 12 = 5, 50 \div 6 = 8 \frac{1}{2}, 20 \div 3 = 6 \frac{2}{3})</td>
<td>B1</td>
<td>Correct pivot choice from <em>their</em> (x) column</td>
</tr>
<tr>
<td>Pivot on the 12 in the (x) column</td>
<td>M1</td>
<td>Correct method for <em>their</em> pivot row (seen or implied from correct row in tableau)</td>
</tr>
<tr>
<td>New row 2 = row 2 ÷ 12</td>
<td>A1</td>
<td>Correct method for <em>their</em> objective row seen as a formula</td>
</tr>
<tr>
<td>New row 1 = row 1 + 100 \times \text{new row 2}</td>
<td></td>
<td>Showing that there are no negative entries in objective row</td>
</tr>
<tr>
<td>Showing that there are no negative entries in objective row</td>
<td>M1</td>
<td>Or achieving a final tableau, in one iteration, with exactly four basis columns and non-negative entries in final column, in which the value of the objective has not decreased</td>
</tr>
<tr>
<td>Saying that optimum has been achieved (‘no negatives in top row’)</td>
<td>A1</td>
<td></td>
</tr>
</tbody>
</table>