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Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk
(i) Using $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$,  
\[ 1020 = 80 \times 15 + \frac{1}{2} \alpha \times 15^2 \]
$\alpha = -1.6$  
Angular deceleration is $1.6 \text{ rad s}^{-2}$

(ii) Using $\theta = \omega_0 t - \frac{1}{2} \alpha t^2$,  
\[ \theta = 0 - \frac{1}{2} x (-1.6) \times 5^2 \]
\[ \theta = 20 \text{ rad} \]
Angle is $20 \text{ rad}$

(iii) Using $\omega_2^2 = \omega_1^2 + 2\alpha \theta$,  
\[ 0 = 80^2 + 2 \times (-1.6) \theta \]
\[ \theta = 2000 \]
Number of revolutions is $318$ (3 sf)

| 2 | Area is $\int_0^{\ln 3} e^{-x} \, dx$ | M1 | Limits not required |
|   | $= \left[ -e^{-x} \right]_0^{\ln 3} \quad (= \frac{2}{3})$ | A1 | For $-e^{-x}$ |
|   | $\int x \, e^{-x} \, dx = \int_0^{\ln 3} x \, e^{-x} \, dx$ | M1 | Limits not required |
|   | $= \left[ -xe^{-x} - e^{-x} \right]_0^{\ln 3} \quad (= \frac{2}{3} - \frac{1}{3} \ln 3)$ | M1 | Integration by parts |
|   | $x = \frac{3}{2} - \frac{1}{2} \ln 3 = 1 - \frac{1}{2} \ln 3$ | A1 | For $-xe^{-x} - e^{-x}$ |
|   | $\int \frac{1}{2} y^2 \, dx = \int_0^{\ln 3} \frac{1}{2} (e^{-x})^2 \, dx$ | M1 | Limits not required |
|   | $= \left[ -\frac{1}{4} e^{-2x} \right]_0^{\ln 3} \quad (= \frac{2}{9})$ | A1 | $\int (e^{-x})^2 \, dx$ or $\int (-\ln y) y \, dy + (\frac{1}{3} \ln 3) \times \frac{1}{5}$ |
|   | $\frac{y}{3} = \frac{9}{3} = \frac{1}{3}$ | A1 | $-\frac{1}{4} e^{-2x}$ or $-\frac{1}{2} y^2 \ln y + \frac{1}{4} y^2 \quad (\text{dep on M1})$ |
|   | Max penalty of 1 mark for correct answers in an unacceptable form (eg decimals) | |

3 (i) By conservation of angular momentum  
\[ I_2 \times 15 = 0.9 \times 16 \]
\[ I_2 = 0.96 \]
Mass is $0.375 \text{ kg}$

(ii) KE before is $\frac{1}{2} \times 0.9 \times 16^2$  
KE after is $\frac{1}{2} \times 0.96 \times 15^2$  
Loss of KE is $115.2 - 108 = 7.2 \text{ J}$
### Question 4

#### Part (i)

\[
\cos \alpha = \frac{12}{15}
\]

\[
\alpha = 36.87^\circ \text{ (4 sf)}
\]

**Bearing of** \(v_B\) **is**

\[
110 - 36.87 = 073.13 = 073^\circ \text{ (nearest degree)}
\]

- **M1** Velocity triangle with 90° opposite \(v_C\)
- **A1** Correct velocity triangle
- **M1** Finding a relevant angle

#### Part (ii)

**Magnitude is** \(\sqrt{15^2 - 12^2} = 9 \text{ ms}^{-1}\)

**Direction is** 90° from \(v_B\)

**Bearing is**

\[
73.13 + 90 = 163^\circ \text{ (nearest degree)}
\]

**B1** Accept 8.95 to 9.05

**M1**

**A1**

#### Alternative for (ii) (using given answer in (i))

\[
v^2 = 12^2 + 15^2 - 2 \times 12 \times 15 \cos 37^\circ
\]

\[
v = 9
\]

\[
\sin \beta = \frac{\sin 37^\circ}{v}
\]

\[
\beta = 53^\circ
\]

**B1**

**M1** Finding a relevant angle

**A1**

#### Part (iii)

**As viewed from** \(B\)

\[
d = 3500 \sin 56.87^\circ
\]

**Shortest distance is** 2930 m (3 sf)

**M1** Diagram indicating initial displacement and relative velocity  *May be implied*

**M1**

**A1**

#### Alternative for (iii)

\[
d^2 = (3500 \sin 40^\circ + 2.6...t)^2
\]

\[
+ (3500 \cos 40^\circ - 8.6...t)^2
\]

Minimum when \(-34432 + 162t = 0\)

\[
t = 213
\]

**M1**

**M1**

**M1** Differentiating or completing the square

**A1**

**Accept 2910 to 2950**

**M1**

**A1**

**Accept 2910 to 2950**
<table>
<thead>
<tr>
<th>5 (i)</th>
<th></th>
<th>5 (ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I = \int_{-a}^{a} \frac{m}{6a} x^2 , dx ) or ( \int_{-a}^{a} \rho x^2 , dx )</td>
<td>( \delta m = \frac{m \delta x}{6a} ) or ( \rho = \frac{m}{6a} )</td>
<td>( \text{WD by couple is } \frac{6mga}{\pi} \times 3\pi \quad (= 18mga) )</td>
</tr>
<tr>
<td>[ \int_{-a}^{a} ] ( \frac{m}{18a} x^3 , dx = \frac{m}{18a} (125a^3 + a^3) ) or ( 42 \rho a^3 )</td>
<td>Correct integral expression for ( I )</td>
<td>( \text{Gain of PE is } mg(4a) )</td>
</tr>
<tr>
<td>[ = \frac{126ma^3}{18a} = 7ma^2 ]</td>
<td>( I = \int_{0}^{5a} ... + \int_{0}^{a} ... )</td>
<td>( 18mga = 4mga + \frac{1}{2}(7ma^2) \omega^2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Angular speed is ( \sqrt{\frac{4g}{a}} )</td>
</tr>
<tr>
<td></td>
<td>( \text{Evaluating definite integral} )</td>
<td>( \text{Equation involving WD, PE and } \frac{1}{2} I \omega^2 )</td>
</tr>
<tr>
<td></td>
<td>( \text{Dependent on integrating } x^2 )</td>
<td>Using ( C\theta )</td>
</tr>
</tbody>
</table>
\[
\frac{dV}{d\theta} = mga(3\cos \theta + 4\sin \theta - 3)
\]

When \( \theta = 0 \), \( \frac{dV}{d\theta} = mga(3 + 0 - 3) = 0 \)

so \( \theta = 0 \) is a position of equilibrium

\[
\frac{d^2V}{d\theta^2} = mga(-3\sin \theta + 4\cos \theta)
\]

When \( \theta = 0 \), \( \frac{d^2V}{d\theta^2} = 4mga > 0 \)

hence the equilibrium is stable

\[\text{B1} \quad \text{M1} \quad \text{A1} \quad \text{ag} \]

Considering \( \frac{dV}{d\theta} = 0 \)

Correctly shown

\[\text{Considering } \frac{d^2V}{d\theta^2} \text{ (or other method)}\]

\[V^* = 4mga \implies \text{Stable M1A0}\]

\[V^* = 4mga \implies \text{Minimum } \implies \text{Stable M1A1}\]

(ii) Speed of \( P \) and \( Q \) is \( a\dot{\theta} \)

KE is \( \frac{1}{2}(5m)(a\dot{\theta})^2 + \frac{1}{2}(3m)(a\dot{\theta})^2 \) or

\[\frac{1}{2}(8m)(a\dot{\theta})^2 = \frac{5}{2}ma^2\dot{\theta}^2 + \frac{3}{2}ma^2\dot{\theta}^2 = 4ma^2\dot{\theta}^2\]

\[\text{M1} \quad \text{ag} \quad [5]\]

Or moment of inertia of \( P \) is \( 5ma^2 \)

\[\frac{5}{2}ma^2\dot{\theta}^2 + \frac{3}{2}ma^2\dot{\theta}^2 \quad \text{M1A1}\]

\[\frac{1}{2}(5ma^2)\dot{\theta}^2 + \frac{1}{2}(3ma^2)\dot{\theta}^2 \quad \text{M1A0}\]

\[\frac{1}{2}(8ma^2)\dot{\theta}^2 \quad \text{M1A0}\]

(iii) \[V + 4ma^2\dot{\theta}^2 = K\]

\[\frac{dV}{d\theta} + 8ma^2\dot{\theta}^2 = 0\]

\[mga(3\cos \theta + 4\sin \theta - 3)\dot{\theta} + 8ma^2\dot{\theta}^2 = 0\]

For small \( \theta \), \( \sin \theta \approx \theta \), \( \cos \theta \approx 1 \)

\[mga(3 + 4\theta - 3) + 8ma^2\dot{\theta} = 0\]

\[\dot{\theta} = -\frac{g}{2a}\theta\]

Approximate period is \[2\pi \sqrt{\frac{2a}{g}}\]

\[\text{M1} \quad \text{A1} \quad \text{M1} \quad \text{A1} \quad \text{ft} \quad \text{A1} \quad [5]\]

\[= 0 \text{ is required for A1 (may be implied by later work)}\]

Linear approximation (ft is dep on M1M1)
\[
I = \frac{1}{2} m((3a)^2 + (4a)^2) + m(5a)^2 \\
= \frac{100ma^2}{3}
\]

Using parallel (or perpendicular) axes rule
or \( I = \frac{4}{7} m(3a)^2 + \frac{4}{7} m(4a)^2 \)

(ii)

By conservation of energy,
\[
\frac{1}{2} \left( \frac{100}{3} ma^2 \right) \omega^2 = mg(4a - 3a)
\]
\[
\frac{50}{3} ma^2 \omega^2 = mga
\]
Angular speed is \( \sqrt{\frac{3g}{50a}} \)
\[
-mg(3a) = \left( \frac{100}{3} ma^2 \right) \alpha
\]
Angular acceleration is \( (-) \frac{9g}{100a} \)

(iii)

\[
P = mg \cos \theta = m(5a)\omega^2
\]
\[
P = \frac{1}{10} mg
\]
\[
Q = mg \sin \theta = m(5a)\alpha
\]
\[
Q = \frac{3}{20} mg
\]
\[
F = \sqrt{P^2 + Q^2} = \frac{1}{20} mg\sqrt{22^2 + 3^2}
\]
\[
= \frac{\sqrt{493}}{20} mg
\]

Alternative for (iii)

\[
H = m(5a)\omega^2 \sin \theta - m(5a)\alpha \cos \theta
\]
\[
V = m(5a)\left( \frac{3g}{50a} \right) \left( \frac{2}{3} \right) + m(5a)\left( \frac{9g}{100a} \right) \left( \frac{4}{5} \right)
\]
\[
H = \frac{27}{2} \frac{mg}{2} , \quad V = \frac{97}{100} mg
\]

Equation involving KE and PE

Equation involving \( P \) and \( r\omega^2 \)

Give A1 if correct apart from sign(s)
(Allow \( \frac{1}{2} H + \frac{1}{2} V \) in place of \( P \))

Equation involving \( Q \) and \( r\alpha \)

Give A1 if correct apart from sign(s)
ft for wrong value of \( \alpha \)
ft for wrong value of \( r \) in second equation
(Allow \( \frac{1}{3} H - \frac{1}{3} V \) in place of \( Q \))

Dependent on previous M1M1

Equation involving \( H, r\omega^2 \) and \( r\alpha \)

Give A1 if correct apart from sign(s)

Equation involving \( V, r\omega^2 \) and \( r\alpha \)

Give A1 if correct apart from sign(s)
\[ F = \sqrt{H^2 + V^2} = \frac{1}{100} \, mg \sqrt{54^2 + 97^2} \]
\[ = \frac{\sqrt{12325}}{100} \, mg = \frac{\sqrt{493}}{20} \, mg \]

- M1
- A1
- ag

| Dependent on previous M1M1 |