Mark Scheme for June 2010
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1 Direction of \( l_1 = k[7, 0, -10] \)  
Direction of \( l_2 = k[1, 3, -1] \)  

**EITHER**  \( \mathbf{n} = [7, 0, -10] \times [1, 3, -1] \)  
**OR**  \([x, y, z], [7, 0, -10] = 0 \Rightarrow 7x - 10z = 0 \)  
\([x, y, z], [1, 3, -1] = 0 \Rightarrow x + 3y - z = 0 \)  
\[ \Rightarrow \mathbf{n} = k[10, -1, 7] \]  

**METHOD 1**  
Vector \((\mathbf{a} - \mathbf{b})\) from \( l_1 \) to \( l_2 \) is \( \pm[4, 6, -10] \)  
**OR**  \( \pm[-4, 3, 1] \)  
\( \pm[3, 3, -9] \)  
\( \pm[-3, 6, 0] \)  
\[ d = \frac{|(\mathbf{a} - \mathbf{b}) \cdot \mathbf{n}|}{|\mathbf{n}|} = \frac{36}{\sqrt{150}} \]  
\[ d = \frac{6 \sqrt{6}}{5} = 2.94 \]  

**METHOD 2**  
Planes containing \( l_1 \) and \( l_2 \) perpendicular to \( \mathbf{n} \) are  
\( \mathbf{r} \cdot [10, -1, 7] = p_1, 70 \)  
\( \mathbf{r} \cdot [10, -1, 7] = p_2, 34 \)  
\[ d = \frac{|70 - 34|}{\sqrt{150}} = \frac{36}{\sqrt{150}} = \frac{6 \sqrt{6}}{5} = 2.94 \]  

**METHOD 3**  
\( \mathbf{r}_1 = [7\lambda, 0, 10 - 10\lambda] \)  
**OR**  \([7 + 7\lambda, 0, -10\lambda] \)  
\( \mathbf{r}_2 = [4 + \mu, 6 + 3\mu, -\mu] \)  
**OR**  \([3 + \mu, 3 + 3\mu, 1 - \mu] \)  
\[ 7\lambda + 10\alpha - \mu = 4 \]  
\[ -\alpha - 3\mu = 6 \]  
\[ -10\lambda + 7\alpha + \mu = -10 \]  
\[ \Rightarrow \alpha = -\frac{6}{25} \]  
\( |\mathbf{n}| = \sqrt{150} \)  
\[ \Rightarrow d = \frac{6 \sqrt{6}}{25} = 2.94 \]  

**For both directions**  
**For finding vector product of directions of**  
**\( l_1 \) and \( l_2 \)**  
**OR**  for using 2 scalar products and obtaining equations  
**For correct**  

**For a correct vector**  
**For finding**  \((\mathbf{a} - \mathbf{b}) \cdot \mathbf{n}\)  
**For**  \(|\mathbf{n}|\)  **in denominator**  
**For**  \( p_1 - p_2 \) **seen**  
**For**  \( p_1 = 70k \) **and**  \( p_2 = 34k \)  
**For correct distance**  

**For finding planes and**  
**\( p_1 - p_2 \) **seen**  
**For**  \( p_1 = 70k \) **and**  \( p_2 = 34k \)  
**For**  \(|\mathbf{n}|\)  **in denominator**  
**For**  \( \mathbf{n} \)  **in denominator**  
**For correct distance**  

**For**  **correct points on**  
**\( l_1 \) and \( l_2 \)**  
**using different parameters**  
**For setting up 3 linear equations from**  
\( \mathbf{r}_1 + \alpha \mathbf{n} = \mathbf{r}_2 \) **and solving for**  \( \alpha \)  
**For**  \(|\mathbf{n}|\)  **seen multiplying**  \( \alpha \)  
**For correct distance**  

**For correct distance**  

**For correct distance**  

**For correct distance**  

**For correct distance**  

**For correct distance**  

7
2 (i) \[ ar = r^5a \Rightarrow rar = r^6a \] 
\[ r^6 = e \Rightarrow rar = a \]  
M1 Pre-multiply \( ar = r^5a \) by \( r \)  
A1 2 Use \( r^6 = e \) and obtain answer \( AG \)

(ii) METHOD 1

For \( n = 1, rar = a \)  OR  For \( n = 0, r^0ar^0 = a \)  
Assume \( r^k ar^k = a \)

EITHER  Assumption \( \Rightarrow r^{k+1}ar^{k+1} = rar = a \)  
OR  \( r^{k+1}ar^{k+1} = r.r^k ar^k .r = rar = a \)  
OR  \( r^{k+1}ar^{k+1} = r^k .rar , r^k = r^k ar^k = a \)  
A1 4 For stating true for \( n = 1 \) OR for \( n = 0 \)

Hence true for all \( n \in \mathbb{Z}^+ \)

METHOD 2

\[ r^2ar^2 = r.rar , r = rar = a , \text{ similarly for } r^3ar^3 = a \]
\[ r^4ar^4 = r.r^3ar^3 .r = rar = a , \text{ similarly for } r^5ar^5 = a \]
\[ r^6ar^6 = eae = a \]

For \( n > 6, r^n = r^{n \text{mod6}}, \text{ hence true for all } n \in \mathbb{Z}^+ \)  
B1 4 For showing true for \( n = 6 \)

METHOD 3

\[ r^n ar^n = r^{n-1}ar , r^{n-1} \]  
OR  \( r^n ar^n = r^n , r^5a , r^{n-1} = r^{n+5}ar^{n-1} \)
\[ = r^{n-1}ar^{n-1} \]
\[ = r^{n-2}ar^{n-2} = \ldots \]
\[ = rar = a \]  
A1 For proving true for \( n-1 \)

A1 For continuation from \( n-2 \) downwards  
B1 For final use of \( rar = a \)

SR can be done in reverse

METHOD 4

\[ ar = r^5a \Rightarrow ar^2 = r^5ar = r^{10}a \text{ etc.} \]
\[ \Rightarrow ar^n = r^{5n}a \]
\[ \Rightarrow r^n ar^n = r^{6n}a \]
\[ = eaa = a \]  
M1 For attempt to derive \( ar^n = r^{5n}a \)
A1 For correct equation  
SR may be stated without proof  
B1 For pre-multiplication by \( r^n \)
A1 For obtaining \( a \) (\( r^6 = e \) may be implied)
3

(i) \( w^2 = \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \)
\( w^3 = \cos \frac{\pi}{5} + i \sin \frac{\pi}{5} \)
\( w^4 = \cos \frac{\pi}{5} - i \sin \frac{\pi}{5} \)
\( w^5 = \cos \frac{\pi}{5} + i \sin \frac{\pi}{5} \)

For correct value

(ii) \( 1 + w + w^2 + w^3 + w^4 = 0 \)

For 1 + \( w \) in approximately correct position

B1

For correct value

SR For exponential form with \( i \) missing, award B0 first time, allow others

(iii) \( z^5 - 1 = 0 \) OR \( z^5 + z^4 + z^3 + z^2 + z = 0 \)

For correct equation AEF (in any variable)

B1

Allow factorised forms using \( w \), exp or trig

4

(i) \( y = xz \Rightarrow \frac{dy}{dx} = z + x \frac{dz}{dx} \)
\( \Rightarrow xz + x^2 \frac{dz}{dx} = x \cos z \Rightarrow x \frac{dz}{dx} = \cos z \)
\( \Rightarrow \int \sec z \ dz = \int \frac{1}{x} \ dx \)
\( \Rightarrow \ln (\sec z + \tan z) = \ln kx \)

For correct differentiation of substitution

B1

For substituting into DE

A1

For DE in variables separable form

M1

For attempt at integration to ln form on LHS

A1

For correct integration (\( k \) not required here)

M1

For correct solution AEF including RHS = \( e^{(\ln x) + c} \)

A1

(ii) \( (4, \pi) \Rightarrow \sec \frac{\pi}{3} + \tan \frac{\pi}{3} = 4k \)
\( \Rightarrow \sec \left(\frac{y}{x}\right) + \tan \left(\frac{y}{x}\right) = 4k \)
\( \Rightarrow \sec \left(\frac{y}{2x}\right) + \tan \left(\frac{y}{2x}\right) = \frac{1}{4} (1 + \sqrt{2}) x \)

For substituting \((4, \pi)\) into their solution (with \( k \))

M1

For correct solution AEF

A1

Allow decimal equivalent 0.60355

Allow \( e^{\ln x} \) for \( x \)
5 (i) \[ C + iS = 1 + \frac{1}{2} e^{i\theta} + \frac{1}{4} e^{2i\theta} + \frac{1}{8} e^{3i\theta} + \ldots \]
\[= \frac{1}{1 - \frac{1}{2} e^{i\theta}} = \frac{2}{2 - e^{i\theta}} \]

M1 For using \( \cos n\theta + i\sin n\theta = e^{in\theta} \)
at least once for \( n \geq 2 \)
A1 For correct series

M1 For using sum of infinite GP
A1 4 For correct expression AG
SR For omission of 1st stage award up to M0 A0 M1 A1 OEW

(ii) \[ C + iS = \frac{2(2 - e^{-i\theta})}{(2 - e^{i\theta})(2 - e^{-i\theta})} \]
\[= \frac{4 - 2e^{-i\theta}}{4 - 2(e^{i\theta} + e^{-i\theta}) + 1} = \frac{4 - 2\cos \theta + 2i \sin \theta}{4 - 4\cos \theta + 1} \]
\[\Rightarrow C = \frac{4 - 2\cos \theta}{5 - 4\cos \theta}, \quad S = \frac{2\sin \theta}{5 - 4\cos \theta} \]

M1 For multiplying top and bottom by complex conjugate
M1 For reverting to \( \cos \theta \) and \( \sin \theta \)
and equating \( \text{Re OR Im parts} \)
A1 For correct expression for \( C \ AG \)
A1 4 For correct expression for \( S \)

6 (i) Aux. equation \( m^2 + 2m + 17 = 0 \)
\[\Rightarrow m = -1 \pm 4i \]

CF \( y = e^{-x}(A\cos 4x + B\sin 4x) \)

A1√ For correct CF (allow \( A\cos(4x + C) \))
(trig terms required, not \( e^{\pm 4ix} \))

f.t. from their \( m \) with 2 arbitrary constants

M1 For attempting to solve correct auxiliary equation
A1 For correct roots
A1 For correct value of \( p \)
A1 For correct value of \( q \)

PI \( y = px + q \Rightarrow 2p + 17(px + q) = 17x + 36 \)

\[\Rightarrow p = 1 \]
and \( q = 2 \)

GS \( y = e^{-x}(A\cos 4x + B\sin 4x) + x + 2 \)

B1√ 7 For GS. f.t. from their CF+PI with 2 arbitrary constants in CF and none in PI.
Requires \[ y \]

(ii) \( x \gg 0 \Rightarrow e^{-x} \rightarrow 0 \ OR \ very \ small \)
\[\Rightarrow y = x + 2 \ approximately \]

B1 For correct statement. Allow graph
B1√ 2 For correct equation
Allow \( = \rightarrow \) and in words
Allow relevant f.t. from linear part of GS
7 (i) \((1, 3, 5)\) and \((5, 2, 5) \Rightarrow \pm [4, -1, 0] \text{ in } II\)

\[ n = [2, -2, 3] \times [4, -1, 0] = k[1, 4, 2] \]

\[ r = [1, 4, 2] + 23 \]

M1 For finding a vector in II

M1 For finding vector product of direction vectors of \(l\) and a line in II

A1 For correct \(n\)

A1 4 For correct equation. Allow multiples

\[ \Rightarrow (1, 3, 5) \text{ and } (5, 2, 5) \Rightarrow \pm [4, -1, 0] \text{ in } II\]

(ii) \[ (7, 3, 0) \text{ and a line in } II\]

METHOD 1

Perpendicular to \(II\) through \((-7, -3, 0)\) meets \(II\)

where \((-7 + k) + 4(3 + 4k) + 2(2k) = 23\)

\[ k = 2 \Rightarrow d = 2\sqrt{1^2 + 4^2 + 2^2} = 2\sqrt{21} = 9.165 \]

M1 For finding vector product of direction vectors of \(l\) and a line in II

A1 For correct \(n\)

A1 4 For correct distance \(AEF\)

METHOD 2

\(II\) is \(x + 4y + 2z = 23\)

\[ d = \frac{|(-7) + 4(3) + 2(0) - 23|}{\sqrt{1^2 + 4^2 + 2^2}} = 2\sqrt{21} = 9.165 \]

M1 For normalising the \(n\) used in this part

A1 For correct distance \(AEF\)

METHOD 3

\(m = [1, 3, 5] - [-7, -3, 0] = (\pm)[8, 6, 5]\)

\(OR = [5, 2, 5] - [-7, -3, 0] = (\pm)[12, 5, 5]\)

\[ d = \frac{m \cdot [1, 4, 2]}{\sqrt{1^2 + 4^2 + 2^2}} = \frac{42}{\sqrt{21}} = 2\sqrt{21} = 9.165 \]

M1 For finding \(m \cdot n\)

A1 For normalising the \(n\) used in this part

A1 For correct distance \(AEF\)

METHOD 4

\((-7, -3, 0) + k[1, 4, 2] = [1, 3, 5] + s[2, -2, 3] + t[4, -1, 0]\)

\[ k - 2s - 4t = 8 \]

\[ 4k + 2s + t = 6 \]

\[ 2k - 3s = 5 \]

\[ k = 2 \left( s = -\frac{1}{3}, t = -\frac{4}{3} \right) \]

\[ d = 2\sqrt{1^2 + 4^2 + 2^2} = 2\sqrt{21} = 9.165 \]

M1 For normalising the \(n\) used in this part

A1 For correct distance \(AEF\)

METHOD 5

\[ d_1 = \frac{23}{\sqrt{1^2 + 4^2 + 2^2}} = \frac{23}{\sqrt{21}} \]

\[ d_2 = \frac{|-7, -3, 0| \cdot [1, 4, 2]}{\sqrt{1^2 + 4^2 + 2^2}} = \frac{-19}{\sqrt{21}} \]

\[ d_1 - d_2 = \frac{23 - (-19)}{\sqrt{21}} = 2\sqrt{21} = 9.165 \]

M1 For finding \(d_1 - d_2\)

A1 For correct distance \(AEF\)

(iii) \((-7, -3, 0) + k (1, 4, 2)\)

Use \(k = 4\)

\[ b = [2, -2, 3] \]

\[ a = [-3, 13, 8] \]

\[ r = [-3, 13, 8] + t[2, -2, 3] \]

M1 State or imply coordinates of a point on the reflected line

M1 State or imply \(2 \times\) distance from (ii)

Allow \(k = \pm 4\) OR \(\pm 4\sqrt{21}\) ft. from (ii)

B1 For stating correct direction

A1 4 For correct point seen in equation \(r = a + tb\)

AEF in this form
8 (i) \( \{A, D\} \) OR \( \{A, E\} \) OR \( \{A, F\} \)  
\[ \text{B1 1 For stating any one subgroup} \]

(ii) \( A \) is the identity  
5 is not a factor of 6  
OR elements can only be of order 1, 2, 3, 6  
\[ \text{B1 1 For identifying } A \text{ as the identity} \]
\[ \text{B1 2 For reference to factors of 6} \]

(iii)  
\[ BE = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = D, \quad EB = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = F \]  
\[ D \text{ or } \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad F \text{ or } \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} \in M \]  
\[ \Rightarrow \text{ closure property satisfied} \]  
\[ \text{M1 For finding } BE \text{ and } EB \text{ AND using } \omega^3 = 1 \]
\[ \text{A1 For correct } BE \text{ (D or matrix)} \]
\[ \text{A1 For correct } EB \text{ (F or matrix)} \]

(iv)  
\[ B^{-1} = \begin{pmatrix} \omega^2 & 0 \\ 0 & 1 \end{pmatrix} = C \]  
\[ E^{-1} = \begin{pmatrix} 1 & -\omega \\ -\omega & 0 \end{pmatrix} = E \]  
\[ \text{A1 3 For correct } E^{-1} = E \text{ Allow } \begin{pmatrix} \omega^2 & 0 \\ 0 & 1 \end{pmatrix} \]

(v) METHOD 1  
\[ M \text{ is not commutative} \]  
e.g. from \( BE \neq EB \) in part (iii)  
\[ N \text{ is commutative (as } \times \text{ mod 9 is commutative)} \]  
\[ \Rightarrow M \text{ and } N \text{ not isomorphic} \]  
\[ \text{B1 For justification of } M \text{ being not commutative} \]
\[ \text{B1 For statement that } N \text{ is commutative} \]
\[ \text{B1# 3 For correct conclusion} \]

METHOD 2  
Elements of \( M \) have orders 1, 3, 3, 2, 2, 2  
Elements of \( N \) have orders 1, 6, 3, 2, 3, 6  
Different orders OR self-inverse elements  
\[ \Rightarrow M \text{ and } N \text{ not isomorphic} \]  
\[ \text{B1* For all orders of one group correct} \]
\[ \text{B1 (dep) For sufficient orders of the other group correct} \]
\[ \text{B1# For correct conclusion} \]
\[ \text{SR Award up to B1 B1 B1 if the self-inverse elements are sufficiently well identified for the groups to be non-isomorphic} \]

METHOD 3  
\[ M \text{ has no generator} \]  
since there is no element of order 6  
\[ N \text{ has 2 OR 5 as a generator} \]  
\[ \Rightarrow M \text{ and } N \text{ not isomorphic} \]  
\[ \text{B1 For all orders of } M \text{ shown correctly} \]
\[ \text{B1 For stating that } N \text{ has generator 2 OR 5} \]
\[ \text{B1# For correct conclusion} \]

METHOD 4  
\[ M = \begin{array}{llllll} \{A, B, C, D, E, F\} \\
A & A & B & C & D & E & F \\
B & B & C & A & F & D & E \\
C & C & A & B & E & F & D \\
D & D & E & F & A & B & C \\
E & E & F & D & C & A & B \\
F & F & D & E & B & C & A \\
\end{array} \]  
\[ N = \begin{array}{llllll} \{1, 2, 4, 8, 7, 5\} \\
1 & 1 & 2 & 4 & 8 & 7 & 5 \\
2 & 2 & 4 & 8 & 7 & 5 & 1 \\
4 & 4 & 8 & 7 & 5 & 1 & 2 \\
8 & 8 & 7 & 5 & 1 & 2 & 4 \\
7 & 7 & 5 & 1 & 2 & 4 & 8 \\
5 & 5 & 1 & 2 & 4 & 8 & 7 \\
\end{array} \]
\[ \Rightarrow M \text{ and } N \text{ not isomorphic} \]  
\[ \text{B1 For stating correctly all 6 squared elements of one group} \]
\[ \text{B1 (dep) For stating correctly sufficient squared elements of the other group} \]
\[ \text{B1# For correct conclusion} \]

\[ \text{# In all Methods, the last B1 is dependent on at least one preceding B1} \]