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MATHEMATICS
HIGHER LEVEL
PAPER 3 – SETS, RELATIONS AND GROUPS

Thursday 15 May 2014 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics HL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 12]*

The binary operation Δ is defined on the set $S = \{1, 2, 3, 4, 5\}$ by the following Cayley table.

Δ	**1**	**2**	**3**	**4**	**5**
1	1	1	2	3	4
2	1	2	1	2	3
3	2	1	3	1	2
4	3	2	1	4	1
5	4	3	2	1	5

(a) State whether S is closed under the operation Δ and justify your answer. *[2]*

(b) State whether Δ is commutative and justify your answer. *[2]*

(c) State whether there is an identity element and justify your answer. *[2]*

(d) Determine whether Δ is associative and justify your answer. *[3]*

(e) Find the solutions of the equation $a \Delta b = 4 \Delta b$, for $a \neq 4$. *[3]*

2. *[Maximum mark: 19]*

Consider the set S defined by $S = \{s \in \mathbb{Q} : 2s \in \mathbb{Z}\}$.

You may assume that + (addition) and × (multiplication) are associative binary operations on $\mathbb{Q}$.

(a) (i) Write down the six smallest non-negative elements of S.

(ii) Show that $\{S, +\}$ is a group.

(iii) Give a reason why $\{S, \times\}$ is not a group. Justify your answer. *[9]*

(b) The relation R is defined on S by $s_1 R s_2$ if $3s_1 + 5s_2 \in \mathbb{Z}$.

(i) Show that R is an equivalence relation.

(ii) Determine the equivalence classes. *[10]*

3. *[Maximum mark: 15]*

Sets X and Y are defined by $X =]0, 1[$; $Y = \{0, 1, 2, 3, 4, 5\}$.

(a) (i) Sketch the set $X \times Y$ in the Cartesian plane.

(ii) Sketch the set $Y \times X$ in the Cartesian plane.

(iii) State $(X \times Y) \cap (Y \times X)$. *[5]*

Consider the function $f: X \times Y \to \mathbb{R}$ defined by $f(x, y) = x + y$ and the function $g: X \times Y \to \mathbb{R}$ defined by $g(x, y) = xy$.

(b) (i) Find the range of the function f.

(ii) Find the range of the function g.

(iii) Show that f is an injection.

(iv) Find $f^{-1}(\pi)$, expressing your answer in exact form.

(v) Find all solutions to $g(x, y) = \frac{1}{2}$. *[10]*

4. *[Maximum mark: 14]*

Let $f: G \to H$ be a homomorphism of finite groups.

(a) Prove that $f(e_G) = e_H$, where e_G is the identity element in G and e_H is the identity element in H. *[2]*

(b) (i) Prove that the kernel of f, $K = \text{Ker}(f)$, is closed under the group operation.

(ii) Deduce that K is a subgroup of G. *[6]*

(c) (i) Prove that $gkg^{-1} \in K$ for all $g \in G$, $k \in K$.

(ii) Deduce that each left coset of K in G is also a right coset. *[6]*