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General Certificate of Education

2015

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# Mathematics

Assessment Unit F2

*assessing*

Module FP2: Further Pure Mathematics 2

[AMF21]

TUESDAY 9 JUNE, MORNING

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## TIME

1 hour 30 minutes.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all eight** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$

**Answer all eight questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

**1 (i)** Show that

$$\frac{1}{r^2} - \frac{1}{(r+1)^2} \equiv \frac{2r+1}{r^2(r+1)^2} \quad [2]$$

**(ii)** Hence or otherwise find

$$\sum_{r=1}^n \frac{2r+1}{r^2(r+1)^2} \quad [3]$$

**2** Find, in terms of  $\pi$ , the general solution of the equation

$$\tan 2x + \tan 4x = 0 \quad [7]$$

**3 (i)** Using partial fractions show that

$$\frac{x-4x^2}{(2-x)^2(3+x^2)} \equiv \frac{1}{2-x} - \frac{2}{(2-x)^2} + \frac{x}{3+x^2} \quad x \neq 2 \quad [6]$$

**(ii)** Hence or otherwise find the exact value of

$$\int_0^1 \frac{x-4x^2}{(2-x)^2(3+x^2)} dx \quad [5]$$

- 4 (i) Using Maclaurin's theorem, find the first three terms of the series expansion for

$$f(x) = e^{\tan x} \quad [5]$$

- (ii) Hence write down the first three terms of the expansion for  $e^{-\tan x}$  [1]

- 5 Using the principle of mathematical induction, prove that

$$3^{2n+1} + 2^{n+2}$$

- is divisible by 7 for any positive integer  $n$ . [6]

- 6 (a) (i) Show that the equation of the tangent to the ellipse

$$\frac{x^2}{16} + y^2 = 1$$

at the point P ( $4\cos t$ ,  $\sin t$ ) is

$$4y \sin t = -x \cos t + 4 \quad [4]$$

The tangent at P cuts the coordinate axes at Q and R. The midpoint of QR is M.

- (ii) As  $t$  varies find the Cartesian equation of the locus of the point M. [5]

- (b) Show that if the line with equation  $y = mx + c$  is to be a tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

then

$$a^2m^2 + b^2 = c^2 \quad [4]$$

- 7 A particle P is constrained to move on a fixed line so that at time  $t$  seconds its displacement  $x$  metres from an origin O is given by the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 40 \cos 3t$$

Given that  $x = 2$  and  $\frac{dx}{dt} = 13$  when  $t = 0$ , find the displacement  $x$  as a function of  $t$ . [13]

- 8 (a) (i) Write down the modulus and argument of the complex number  $4 + 4i$  [2]

(ii) Solve the equation

$$z^5 = 4 + 4i$$

leaving your answers in exponential form. [6]

- (b) Find the smallest positive integer values of  $p$  and  $q$  so that

$$\frac{\left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}\right)^p}{\left(\cos \frac{\pi}{6} - i \sin \frac{\pi}{6}\right)^q} = -1$$
 [6]

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**THIS IS THE END OF THE QUESTION PAPER**

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