Rewarding Learning

ADVANCED
General Certificate of Education
2015

Mathematics

Assessment Unit C4
assessing
Module C4: Core Mathematics 4

AMC41

[AMC41]
TUESDAY 26 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find the angle between the two lines whose vector equations are

$$\mathbf{r}_1 = \mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j})$$

$$\mathbf{r}_2 = \mathbf{i} - \mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} - \mathbf{k})$$ [5]

2 Using the substitution $u = x - 1$ or otherwise find

$$\int_2^5 \frac{x-2}{\sqrt{x-1}}\,\mathrm{d}x$$ [6]

3 **(a)** A curve is given by the parametric equations

$$x = \mathrm{e}^{2t} \qquad y = 1 + \mathrm{e}^{t}$$

Find the gradient of the curve when $t = 0$ [5]

(b) Find the equation of the normal to the curve

$$2x^2 + y^2 - 3y = 0$$

at the point $(1, 2)$. [9]

4 The number of yeast cells increases at a rate proportional to the number of yeast cells, N, present at any time t.

This can be modelled by the differential equation

$$\frac{\mathrm{d}N}{\mathrm{d}t} = 0.02N$$

If the number of yeast cells present at time $t = 0$ minutes is N_0, find the length of time it will take for the number of yeast cells to be $2N_0$ [8]

5 **(a)** Sketch the graph of

$$y = \tan^{-1}x$$

where $-\frac{\pi}{2} < y < \frac{\pi}{2}$ [2]

(b) Solve the equation

$$\tan 2\theta = 3\cot\theta$$

where $0^\circ \leqslant \theta \leqslant 360^\circ$ [8]

6 The maximum volume of liquid that a chinese wok can hold can be modelled by the volume generated by the rotation of the curve

$$x^2 + y^2 = a^2$$

through 2π radians about the x-axis between $x = \frac{a}{2}$ and $x = a$ as shown in **Fig. 1** below.

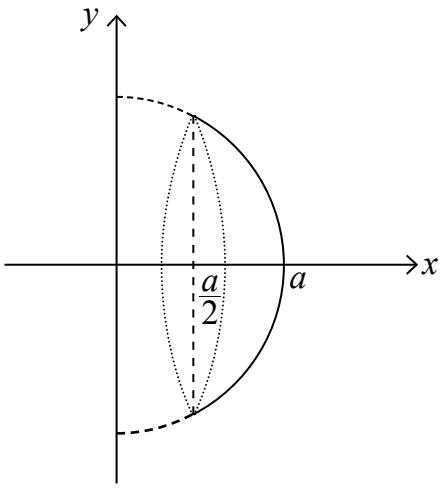

Fig. 1

(i) Find the volume generated by the rotation. [7]

(ii) Given that when the wok is 'full' it holds $2880\pi\,\text{cm}^3$, find a. [2]

7 A function f is defined by

$$\mathrm{f} : x \to \frac{1}{x+2} \qquad x \in \mathbb{R}\ x > -2$$

(i) Sketch the graph of $y = \mathrm{f}(x)$. [1]

(ii) State the equations of the asymptotes to this graph. [2]

A function g is defined by

$$\mathrm{g} : x \to \frac{1}{x-1} \qquad x \in \mathbb{R}\ x > 3$$

(iii) Find the range of g. [1]

(iv) Find the composite function fg. [2]

(v) Find the inverse function $(\mathrm{fg})^{-1}$ [4]

8 **(a)** Find

$$\int x^2 \ln x^2 \, \mathrm{d}x$$ [6]

(b) Find the exact value of

$$\int_0^{\frac{\pi}{4}} \cos^2 x \sin^3 x \, \mathrm{d}x$$ [7]

THIS IS THE END OF THE QUESTION PAPER