Rewarding Learning

ADVANCED
General Certificate of Education
2015

Mathematics

Assessment Unit C4
assessing
Module C4: Core Mathematics 4

MV18

[AMC41]
TUESDAY 26 MAY, MORNING

TIME

1 hour 30 minutes, plus your additional time allowance.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find the angle between the two lines whose vector equations are

$\mathbf{r}_1 = \mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j})$

$\mathbf{r}_2 = \mathbf{i} - \mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} - \mathbf{k})$ [5 marks]

2 Using the substitution $u = x - 1$ or otherwise find

$$\int_2^5 \frac{x-2}{\sqrt{x-1}}\,\mathrm{d}x$$ [6 marks]

3 **(a)** A curve is given by the parametric equations

$x = e^{2t}$ $y = 1 + e^t$

Find the gradient of the curve when $t = 0$ [5 marks]

(b) Find the equation of the normal to the curve

$2x^2 + y^2 - 3y = 0$

at the point (1, 2). [9 marks]

4 The number of yeast cells increases at a rate proportional to the number of yeast cells, N, present at any time t.

This can be modelled by the differential equation

$$\frac{dN}{dt} = 0.02N$$

If the number of yeast cells present at time $t = 0$ minutes is N_0, find the length of time it will take for the number of yeast cells to be $2N_0$ [8 marks]

5 **(a)** Sketch the graph of

$$y = \tan^{-1} x$$

where $-\frac{\pi}{2} < y < \frac{\pi}{2}$ [2 marks]

(b) Solve the equation

$$\tan 2\theta = 3\cot\theta$$

where $0° \leqslant \theta \leqslant 360°$ [8 marks]

6 The maximum volume of liquid that a chinese wok can hold can be modelled by the volume generated by the rotation of the curve

$$x^2 + y^2 = a^2$$

through 2π radians about the x-axis between $x = \frac{a}{2}$ and $x = a$ as shown in **Fig. 1** below.

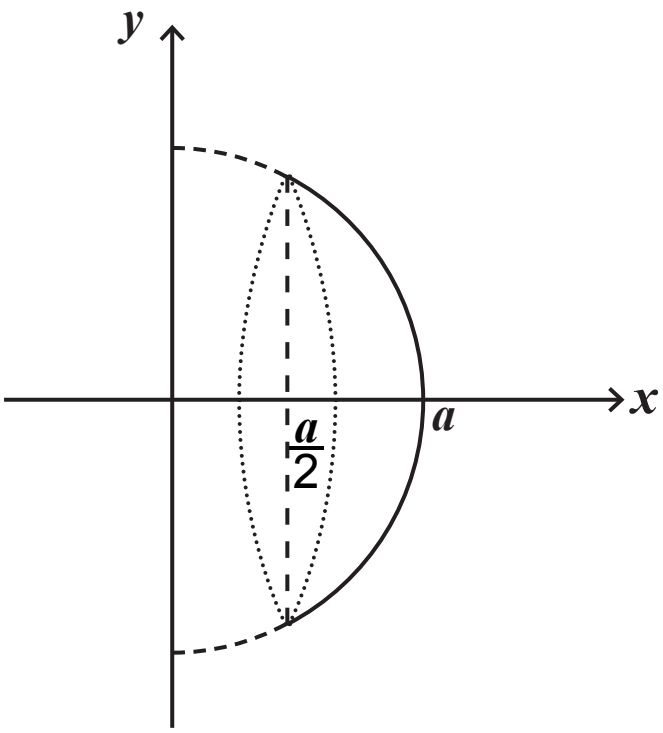

Fig. 1

(i) Find the volume generated by the rotation. [7 marks]

(ii) Given that when the wok is 'full' it holds $2880\pi\text{cm}^3$, find a. [2 marks]

7 A function f is defined by

$$f: x \to \frac{1}{x+2} \qquad x \in \mathbb{R}\ x > -2$$

(i) Sketch the graph of $y = f(x)$. [1 mark]

(ii) State the equations of the asymptotes to this graph. [2 marks]

A function g is defined by

$$g: x \to \frac{1}{x-1} \qquad x \in \mathbb{R}\ x > 3$$

(iii) Find the range of g. [1 mark]

(iv) Find the composite function fg. [2 marks]

(v) Find the inverse function $(fg)^{-1}$ [4 marks]

8 **(a)** Find

$$\int x^2 \ln x^2 \, dx$$ [6 marks]

(b) Find the exact value of

$$\int_0^{\frac{\pi}{4}} \cos^2 x \sin^3 x \, dx$$ [7 marks]

THIS IS THE END OF THE QUESTION PAPER