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General Certificate of Education
2014

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

THURSDAY 22 MAY, MORNING

AMC41

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(i)** Write

$$\frac{2x-7}{(5-x)(1+x)}$$

in partial fractions. [6]

(ii) Hence find

$$\int \frac{2x-7}{(5-x)(1+x)}\,dx$$ [4]

2 The points A and B have position vectors:

$$\overrightarrow{OA} = 3\mathbf{i} - 4\mathbf{j}$$

and $$\overrightarrow{OB} = 7\mathbf{i} + 5\mathbf{j}$$

relative to a fixed origin O.

(i) Find $\overrightarrow{AB}$. [2]

The point C has position vector

$$\overrightarrow{OC} = 3\mathbf{i} - 2\mathbf{j}$$

(ii) Find a vector equation of the line through C, parallel to AB. [3]

(iii) Show that the point with position vector $(11\mathbf{i} + 16\mathbf{j})$ lies on this line. [4]

3 Use the substitution $u = 3x - 5$ to evaluate

$$\int_2^3 6x\sqrt{3x-5}\,\mathrm{d}x$$ [9]

4 The curved surface of a glass bowl can be modelled by rotating the curve

$$y = \mathrm{e}^x + 1$$

between the lines $x = 0$ and $x = 1$ through 2π radians about the x-axis.

(i) Find the maximum volume that the bowl can hold. [7]

(ii) State one assumption made in the modelling. [1]

5 At time $t = 0$ hours a small block of ice starts to melt.
The volume, $V\,\mathrm{cm}^3$, of the solid ice decreases with time, at a rate which is proportional to the square root of the volume of ice remaining at that time.
This can be modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = k\sqrt{V}$$

Initially the volume of ice is $64\,\mathrm{cm}^3$ and one hour later the volume of ice is $48\,\mathrm{cm}^3$
If the ice started to melt at 12.00 (noon), find the time at which the ice has completely melted. [10]

6 The expression $(7\sin x - 24\cos x)$ can be written in the form

$$R\sin(x - \alpha)$$

where R is an integer and $0 \leqslant \alpha \leqslant \frac{\pi}{2}$

(i) Find R and α. [4]

(ii) Hence find

$$\int \frac{1}{(7\sin x - 24\cos x)^2}\,dx$$ [3]

7 The function f is defined as

$$f: x \to \tan x \qquad -\frac{\pi}{4} \leqslant x \leqslant \frac{\pi}{4}$$

(i) Write down the inverse function f^{-1} and state its domain and range. [4]

The function g is defined as

$$g: x \to |x| \qquad x \in \mathbb{R}$$

(ii) Find the composite function gf, stating its range. [4]

(iii) Hence sketch the graph of $y = gf(x)$ [3]

8 The parametric equations of a curve are

$$x = 2t - \sin 2t \qquad y = 4\cos t$$

where $\frac{\pi}{2} \leqslant t \leqslant \frac{3\pi}{2}$

Find the **exact** coordinates of the point on the curve at which the gradient is $\sqrt{2}$ [11]

THIS IS THE END OF THE QUESTION PAPER