Rewarding Learning

ADVANCED
General Certificate of Education
2014

Mathematics

Assessment Unit M3
assessing
Module M3: Mechanics 3

[AMM31]

MONDAY 16 JUNE, MORNING

MARK
SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

AVAILABLE MARKS

1 **(i)** The forces in AP and PB are equal hence their resultant acts along PD — M1

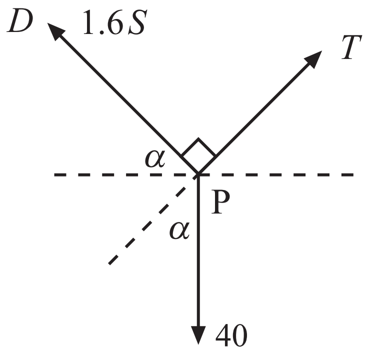
(ii) $R = 2\,S\cos\alpha = 1.6\,S$ — M1 W1

(iii) $\mathrm{D\hat{P}C} = 90°$ — M1

$\therefore$ PC makes $(90 - \alpha)°$ with the horizontal

$\therefore \sin\theta = \cos\alpha = 0.8$ — W1

(iv)



Resolving $\perp_R$ PD — M1

$T = 40\cos\alpha$ — MW1

$= 32\,\mathrm{N}$ — W1

8

2 **(i)** $W = \mathbf{F}_1 \,.\, \lambda\mathbf{b}$ — M1

$$= \lambda\begin{pmatrix}3\\5\\4\end{pmatrix}\cdot\begin{pmatrix}3\\-5\\4\end{pmatrix}$$

$$= \lambda\,(9 - 25 + 16) = 0$$ — W1

(ii) $\mathbf{F}_1$ and $\mathbf{b}$ are perpendicular — M1

(iii) Work done by $\mathbf{F}_1 = 0$

$$\mathbf{F}_2 + \mathbf{F}_3 = \begin{pmatrix}8-a\\3a\\9-a\end{pmatrix}$$ — MW1

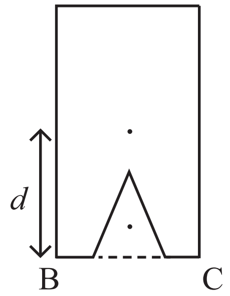
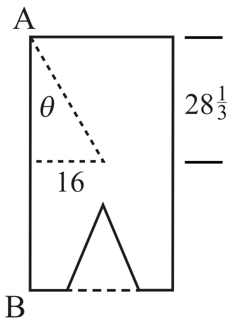
$$W = (\mathbf{F}_2 + \mathbf{F}_3)\,.\,\mathbf{r} = 2(24 - 3a + 15a + 36 - 4a)$$ — M1 MW1

$$= 2(60 + 8a)$$ — W1

(iv) $60 + 8a = 100$

$a = 5$ — MW1

8

				AVAILABLE MARKS
3	**(i)** 	Pythagoras cut out $8 \times 15 \rightarrow 0.2$ full sheet $32 \times 60 \rightarrow 3.2$ component $3.2 - 0.2 = 3$	MW1 MW1 MW1	
	(ii) Moments about BC 	$3.2 \times 30 = 0.2 \times 5 + 3 \times d$ $95 = 3\,d$ $d = 31\frac{2}{3}$ $d = 30 + 1\frac{2}{3}$	M1 M1 MW1 W1 W1	
	(iii) 	$\tan\theta = \dfrac{16}{28\frac{1}{3}}$ $\theta = 29.45 \rightarrow 29.5°$	M1 W1 W1	
	(iv) Bottom has effectively $2 \times (8, 15, 17)$ triangles removed $\therefore$ top 2 need to be removed in a complementary way i.e. 8 cm sides along AD so **(a)**		 MW1 MW1	13
4	**(i)** $W = \int_0^2 T\,dx$ $= \int_0^2 (e^{x-1} + e^{1-x})\,dx$ $= \left[e^{x-1} - e^{1-x}\right]_0^2$ $= (e^1 - e^{-1}) - (e^{-1} - e^1)$ $= 2(e^1 - e^{-1})$ $= 4.70 \times 10^6$ J		M1 W1 MW1 MW1 W1	
	(ii) $\frac{1}{2} \times 2 \times 10^3\, v^2 = 4.7 \times 10^6$ $v^2 = 4.7 \times 10^3$ $v = 68.55 \rightarrow 68.6$ m s^{-1}		MW1 W1	7

AVAILABLE MARKS

5 **(i)** $W_T = -\frac{1}{2}\frac{\lambda}{l}d^2$ M1

$= -\frac{1}{2}\cdot\frac{16g}{5}d^2 = -1.6g\,d^2$ W1

(ii) $W_F = -\mu Mgd = -\frac{1}{2}\times 2\times gd = -gd$ M1 W1

By W.E.P. M1

$0 - \frac{1}{2}\times 2\times 21^2 = -1.6\,gd^2 - gd$ MW1 W1

$1.6\,gd^2 + gd - 441 = 0$

$1.6\,d^2 + d - 45 = 0$

$(1.6\,d + 9)(d - 5) = 0$

$\therefore d = 5$ as $d > 0$ W1

(iii) $T = \frac{16g}{5}\times 5 = 16\,g$

$\mu R = 0.5\times 2\,g = g$ MW1

$T > \mu R \therefore$ P moves MW1 10

6 **(i)** The plane needs a vel component $\perp_R$ the route to counter the wind component $\perp_R$ the route. M1

(ii)

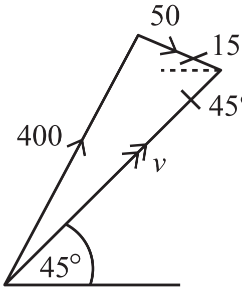


Diagram MW1

60° angle MW1

Apply cos rule about 60° angle M1

$400^2 = v^2 + 50^2 - 2\times 50\,v\times\frac{1}{2}$ MW1

$v^2 - 50\,v - 157500 = 0$ W1

(iii) $v^2 - 50v - 157\,500 = 0$

$v = \frac{50 \pm \sqrt{2500 - 4\times 1\times(-15\,700)}}{2}$ MW1

$= 422.65$ or -372.65

flight time = 0.473 hours MW1

(iv)

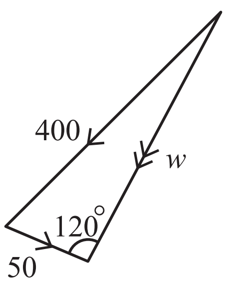


Diagram and angle 120° MW1

$400^2 = w^2 + 50^2 - 2\times 50\times w\times\left(-\frac{1}{2}\right)$ MW1

$w^2 + 50\,w - 157500 = 0$ W1

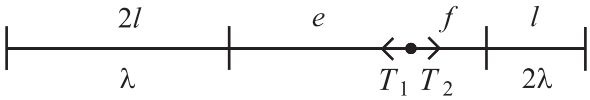
(v) $w \to -v \therefore (-v)^2 + 50(-v) - 157\,500 = 0$

$\therefore v^2 - 50\,v - 157\,500 = 0$ MW1

(vi) flight time $= \frac{200}{372.65} + 0.473$ M1

arrives at 1.01 pm W1 14

7 (i)



at equilibrium, $T_1 = T_2$ — M1

$$\frac{\lambda e}{2l} = \frac{2\lambda f}{l}$$ MW2

$$e = 4f$$

$$e + f = 2.5l$$ MW1

$$5f = 2.5\,l$$

$$f = 0.5\,l$$

$$e = 2l$$ W1

(ii) $m\,\ddot{x} = T_2 - T_1$ — M1

$$= \frac{2\lambda(0.5l - x)}{l} - \frac{\lambda(2l + x)}{2l}$$ MW2

$$= \frac{\lambda}{2l}(2l - 4x - 2l - x)$$

$$\ddot{x} = -\frac{5\lambda}{2lm}\,x = -\omega^2 x \text{ as } \frac{5\lambda}{2lm} > 0$$ W1 W1

(iii) $\ddot{x} = \frac{-5}{2lm}\,\frac{32\,ml\,\pi^2}{5}\,x = -16\pi^2 x$

$$= -(4\pi)^2\,x \qquad \therefore \omega = 4\pi$$ MW1

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = \frac{1}{2} = 0.5$$ MW1

(iv) $x = 0.2\,l\cos 4\pi t$ — M1

(v) $0.5\,l$ — MW1

because then S_2 would become slack and the oscillatory motion would not be simple harmonic. — M1 — 15

AVAILABLE MARKS

Total **75**