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## Mathematics

Assessment Unit M3

*assessing*

Module M3: Mechanics 3

[AMM31]



MONDAY 16 JUNE, MORNING

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### TIME

1 hour 30 minutes.

### INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all seven** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

### INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take  $g = 9.8 \text{ ms}^{-2}$ , unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

- 1 **Fig. 1** below shows two light inextensible strings AB and PC used to keep a small smooth ring hanging in equilibrium. AB passes through the ring and PC is attached to the ring at P. A is attached to a fixed point on a vertical wall and B is attached to a fixed point on a horizontal ceiling. The ring can be modelled as a particle of weight 40 N.

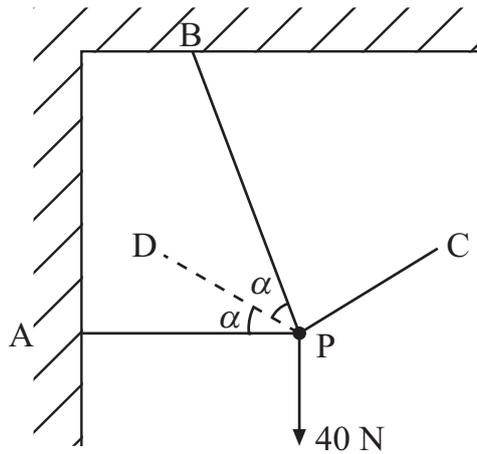


Fig. 1

AP is horizontal and the angle APB is  $2\alpha$ , where  $\cos \alpha = 0.8$   
PD bisects the angle APB.

- (i) Explain briefly why the resultant,  $R$ , of the forces in AP and PB acts along PD. [1]

The tension in AP is  $S$  newtons.

- (ii) Find, in terms of  $S$ , the magnitude of  $R$ . [2]

The tension,  $T$ , in PC is a minimum when PC makes an angle  $\theta$  with the horizontal.

- (iii) Explain briefly why  $\sin \theta = 0.8$  [2]

- (iv) Find the minimum value of  $T$ . [3]

- 2 The forces  $\mathbf{F}_1$ ,  $\mathbf{F}_2$  and  $\mathbf{F}_3$  newtons act on a particle, P, moving it along the line whose vector equation is

$$\mathbf{r} = \lambda \mathbf{b}$$

where  $\lambda$  is a scalar constant and  $\mathbf{b}$  is measured in metres.

Given that

$$\mathbf{F}_1 = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}$$

- (i) show that the work done by  $\mathbf{F}_1$  as P is moved along the line is 0J. [2]
- (ii) Explain what the result in (i) says about the directions of  $\mathbf{F}_1$  and  $\mathbf{b}$ . [1]

A and B are the two points on the line where  $\lambda$  takes the values 0 and 2 respectively.  
The total work done by the three forces as the particle moves from A to B is  $W$  joules.

Given that

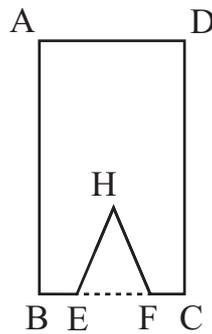
$$\mathbf{F}_2 = \begin{pmatrix} 8 \\ 2a \\ 9 \end{pmatrix} \quad \text{and} \quad \mathbf{F}_3 = \begin{pmatrix} -a \\ a \\ -a \end{pmatrix}$$

where  $a$  is a scalar constant,

- (iii) find  $W$  in terms of  $a$ . [4]
- (iv) If  $W = 200$ , find  $a$ . [1]

- 3 An engineering component is made from a rectangular metal lamina ABCD of width 32 cm and height 60 cm.

**Fig. 2** below shows the first stage of production where an isosceles triangle EHF of base 16 cm and sides 17 cm is cut from the lamina with  $BE = FC = 8$  cm.



**Fig. 2**

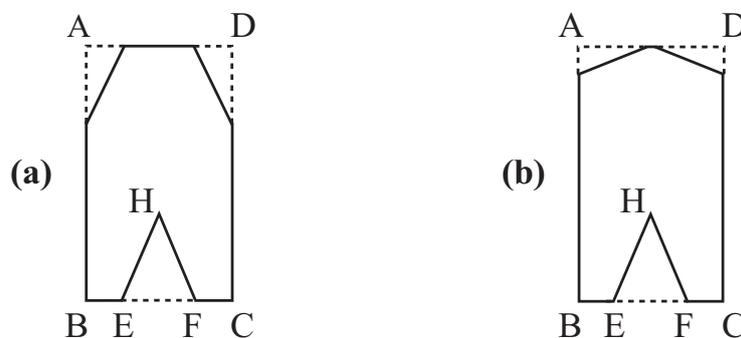
- (i) If the mass of metal that is removed is 0.2 kg show that the mass remaining is 3 kg. [3]
- (ii) Show that the centre of mass at the end of this first stage is  $1\frac{2}{3}$  cm above the centre of the original lamina ABCD. [5]
- (iii) If the lamina is freely suspended from A, find the angle that AB makes with the vertical. [3]

At the second stage of production the corners at A and D are removed by cutting off two identical right angled triangles with perpendicular sides 8 cm and 15 cm.

The right angles in the triangles coincide with the right angles at A and D.

The centre of mass is now at the centre of the original lamina ABCD.

**Fig. 3** below shows the correct shape of the component after the second stage of production together with an incorrect shape.



**Fig. 3**

- (iv) State which shape is the correct one, giving a brief explanation for your choice. [2]

- 4 The X-potential motor has been developed by aliens for a new spacecraft. The thrust  $T(x)$  Meganewtons (i.e.  $10^6$  newtons) produced by the motor is given by

$$T(x) = e^{x-1} + e^{1-x}$$

where  $x$  metres is the distance travelled from rest.

The motor is used to propel a two tonne spacecraft from a position of rest in gravity free space but needs to act only for the first two metres of travel.

- (i) Show that the work done by the motor over the first 2 m travelled by the spacecraft is approximately  $4.70 \times 10^6$  J. [5]
- (ii) Find the speed of the spacecraft when it has travelled the first two metres. [2]

- 5 A particle P of mass 2 kg is attached to one end of a light elastic string of natural length 5 m and modulus of elasticity 16 g N. The other end of the string is attached to a fixed point A on a rough horizontal surface. P is held at a point B on the surface with the string just taut. It is then given a speed of  $21 \text{ m s}^{-1}$  in the direction AB.

- (i) Evaluate, in terms of  $d$ , the work done on the particle by the tension in the string as P moves a distance  $d$  metres beyond B. [2]

The coefficient of friction between the particle and the surface is 0.5

- (ii) Use the Work–Energy Principle to find how far P moves beyond B, before it comes to rest. [6]
- (iii) Determine whether this position of rest is instantaneous or permanent. [2]

- 6 Take Glasgow airport to be 200 km NE of Belfast airport. Planes on the Belfast–Glasgow route operate at  $400 \text{ km h}^{-1}$  in still air. One day the wind was blowing from W  $15^\circ$  N at  $50 \text{ km h}^{-1}$ . The pilot set the course on the Belfast–Glasgow route at  $\theta^\circ$  anti-clockwise from its intended flight path as shown in Fig. 4 below.



Fig. 4

- (i) Explain briefly why the plane's course was set in this direction. [1]

The plane covered the flight path at  $v \text{ km h}^{-1}$

- (ii) By considering the velocity diagram for the flight show that

$$v^2 - 50v - 157500 = 0 \quad \text{equation (a)} \quad [5]$$

- (iii) Find the time taken to fly from Belfast to Glasgow. [2]

The plane was unable to land at Glasgow and immediately had to return to Belfast. The return flight path was covered at  $w \text{ km h}^{-1}$

- (iv) By considering the velocity diagram for this flight show that

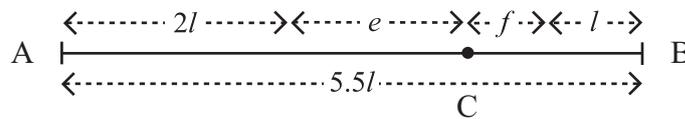
$$w^2 + 50w - 157500 = 0 \quad \text{equation (b)} \quad [3]$$

- (v) Show that if  $w = -v$  then equation (b) transforms to equation (a). [1]

The plane left Belfast at noon.

- (vi) Find the time at which it landed back in Belfast. [2]

- 7 **Fig. 5** below shows a particle of mass  $m$  attached to one end of each of two elastic strings,  $S_1$  and  $S_2$ . The other ends of the strings are attached to fixed points on a smooth horizontal surface –  $S_1$  to A and  $S_2$  to B.



**Fig. 5**

$S_1$  has natural length  $2l$  and modulus of elasticity  $\lambda$ .

$S_2$  has natural length  $l$  and modulus of elasticity  $2\lambda$ .

The distance AB is  $5.5l$

When the particle is in equilibrium at C the extension in  $S_1$  is  $e$  and the extension in  $S_2$  is  $f$ .

- (i) Show that  $e = 2l$  [5]

The particle is displaced a small distance  $d$  to the right of C so that both strings remain taut. It is released from rest where  $d < 0.25l$

At time  $t$  its displacement from C is  $x$ .

- (ii) Show that the equation of motion, in terms of  $x$ , is of the standard form for S.H.M. [5]

- (iii) Given that  $\lambda = 6.4ml\pi^2$ , show that the period of the motion is 0.5 [2]

The particle is released from rest at  $t = 0$  and  $x = 0.2l$

- (iv) Write down in its simplest form the expression for  $x$  as a function of time. [1]

- (v) Briefly explaining your answer, state the maximum distance the particle could be displaced to the right of C and still move in S.H.M. [2]

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**THIS IS THE END OF THE QUESTION PAPER**

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