Rewarding Learning

ADVANCED
General Certificate of Education
2014

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

AMF21

THURSDAY 12 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

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Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Given that

$$f(x) = \ln(1 + \sin x)$$

find the Maclaurin expansion for $f(x)$ up to and including the term in x^2 [6]

2 Find, in radians, the general solution of the equation

$$\sin 3\theta = \sin\theta$$ [6]

3 A geometric series is given by

$$e^{3x} + 3e^{x} + 9e^{-x} + \ldots \qquad x > 0$$

(i) Find an expression for the nth term of the series. [3]

(ii) Find the range of values of x for which the series has a sum to infinity. [4]

(iii) Find the sum to infinity when $x = \ln 2$ [4]

4 **(i)** Given that

$$xy + 3y - x = 0$$

show that

$$(x + 3)\frac{dy}{dx} + y = 1$$ [2]

(ii) Hence, using mathematical induction, prove that for $n \geqslant 2$

$$(x + 3)\frac{d^n y}{dx^n} + n\frac{d^{n-1}y}{dx^{n-1}} = 0$$ [5]

5 **(i)** Express in partial fractions

$$f(x) = \frac{2}{(x-1)^2(x^2+1)}$$ [6]

(ii) Hence find the exact value of

$$\int_2^3 \frac{2}{(x-1)^2(x^2+1)}\,dx$$ [5]

6 **(i)** Show that

$$(Z^n - e^{i\theta})(Z^n - e^{-i\theta}) \equiv Z^{2n} - 2Z^n\cos\theta + 1$$

where Z is a complex number. [2]

(ii) Hence or otherwise find in $(\cos\theta + i\sin\theta)$ form the roots of the equation

$$Z^4 - Z^2\sqrt{2} + 1 = 0$$ [5]

7 **(i)** Show that an equation of the normal to the parabola $y^2 = 4ax$ at the point $P(at^2, 2at)$ is

$$y + tx = 2at + at^3$$ [4]

The normal at P meets the curve again at the point $Q(as^2, 2as)$.

(ii) Show that if $t \neq 0$

$$s = -\left(t + \frac{2}{t}\right)$$ [3]

(iii) Using **(ii)** show that

$$PQ^2 = \frac{16a^2(t^2+1)^3}{t^4}$$ [4]

8 A particle P moves in a straight line so that its displacement x centimetres from a fixed point at time t seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = \lambda \sin 2t$$

where λ is a constant.

(i) Given that $x = 0$ and $\frac{dx}{dt} = \frac{\lambda}{4}$ when $t = 0$, find x in terms of t and λ. [14]

(ii) As t becomes large describe what happens to the solution found in part **(i)**. [2]

THIS IS THE END OF THE QUESTION PAPER