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ADVANCED
General Certificate of Education
2014

Mathematics

Assessment Unit C3

assessing

Module C3: Core Mathematics 3

[AMC31]

AMC31

THURSDAY 15 MAY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Farmer Fred's field floods. The area under water adjoins a dyke. Fred takes measurements, in metres, 3 m apart, from the water's edge to the opposite boundary as shown in **Fig. 1** below:

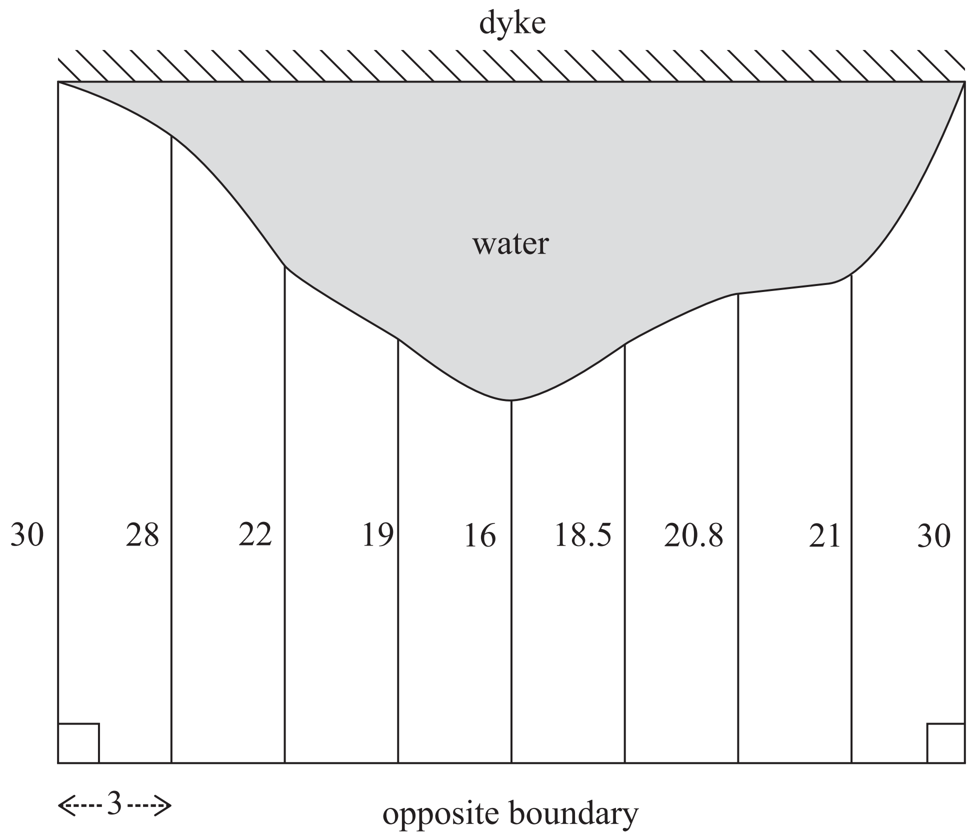

Fig. 1

Use Simpson's Rule to find an approximation to the surface area of the field which is not under water. [4]

2 A curve is defined parametrically by

$$x = at \qquad y = \frac{a}{1 + t^2}$$

Derive a Cartesian equation of this curve. [3]

3 Given the functions:

$$f(x) = \frac{2x^2 - x - 15}{2x - 5}$$

and

$$g(x) = \frac{x - 3}{4x^2 - 25}$$

write $\frac{f(x)}{g(x)}$ in the form $(ax + b)^2$, where a and b are integers. [5]

4 **(a)** **(i)** Express

$$\frac{2}{x^2 - 1}$$

in partial fractions. [5]

(ii) Hence, or otherwise, show that

$$\frac{1}{\sec\theta - 1} - \frac{1}{\sec\theta + 1} \equiv 2\cot^2\theta$$ [4]

(b) **(i)** Expand

$$(1 + 5x)^{\frac{2}{5}}$$

in a binomial series up to and including the term in x^3 [4]

(ii) Hence, **write down** a similar expansion for

$$(1 - 5x)^{\frac{2}{5}}$$ [2]

(iii) Hence, assuming x^4 and higher powers of x can be ignored, solve the equation

$$\sqrt[5]{(1+5x)^2} + \sqrt[5]{(1-5x)^2} = 1\tfrac{7}{8}$$ [3]

5 **(a)** Find

$$\int\left(\operatorname{cosec}^2 2x + \sqrt{x} - \frac{3}{x} + e^{-3x}\right)dx$$ [5]

(b) The blade of a knife can be modelled as the area bounded by the curves $y = 3\cos x$ and $y = \frac{\pi^2}{4} - x^2$ and the y-axis, as shown shaded in **Fig. 2** below.

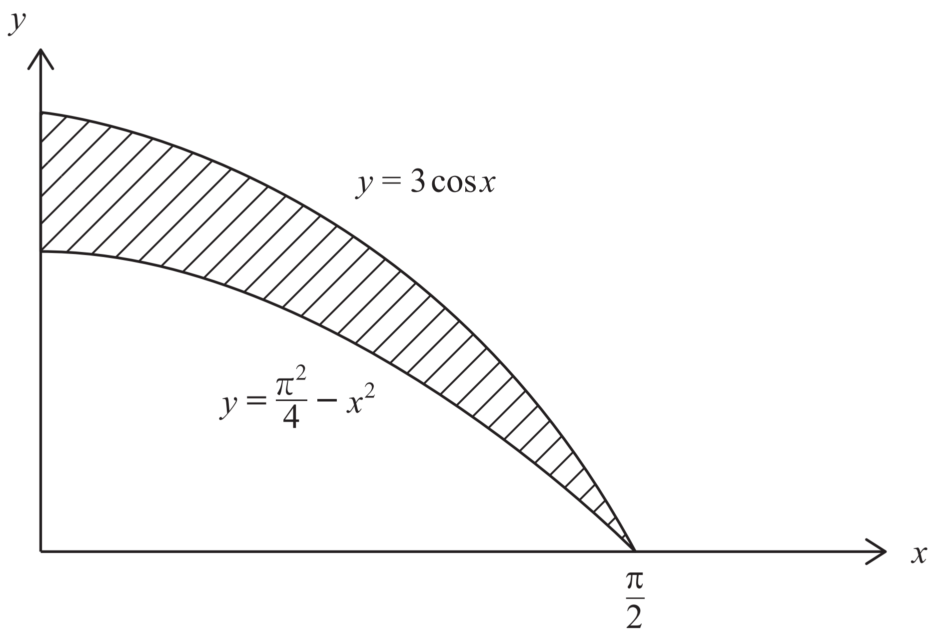

Fig. 2

Calculate the exact area of the blade. [6]

6 **(a)** **(i)** Sketch the graph of

$$y = |\sin x|$$

for $-360° \leqslant x \leqslant 360°$ [2]

(ii) Write down a function whose graph is shown in **Fig. 3** below. [2]

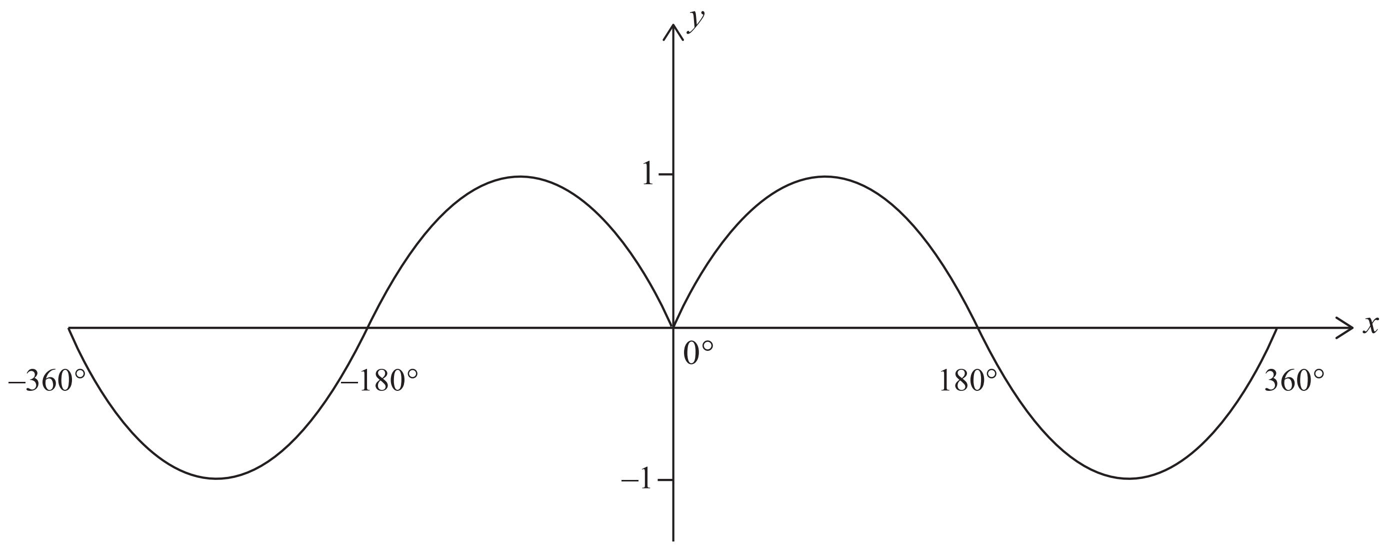

Fig. 3

(b) Solve the equation

$$\operatorname{cosec}\left(2\theta + \frac{\pi}{3}\right) = 2$$

where $-\pi \leqslant \theta \leqslant \pi$ [7]

7 Find the exact equation of the tangent to the curve

$$y = \tan\left(2x - \frac{\pi}{6}\right)\cos\left(x + \frac{\pi}{12}\right)$$

at the point where $x = \frac{\pi}{4}$ [10]

8 A potential well in nuclear physics may be modelled by the function

$$y = -\frac{\ln x}{e^x}$$

shown sketched in **Fig. 4** below:

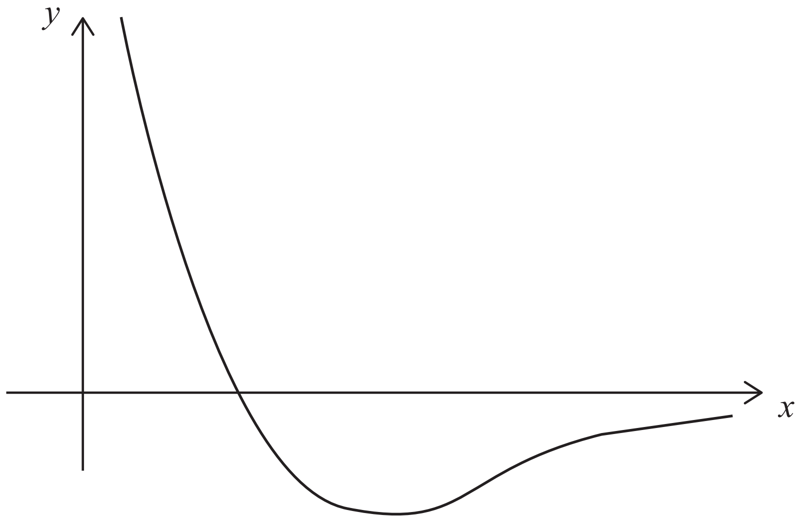

Fig. 4

(i) Find $\frac{dy}{dx}$ [3]

(ii) Show that the x coordinate of the stationary point on the curve $y = -\frac{\ln x}{e^x}$ satisfies the equation

$$\ln x = \frac{1}{x}$$ [3]

(iii) Hence, using the Newton–Raphson method twice with starting value 2, find an approximation to the coordinates of the stationary point. [7]

THIS IS THE END OF THE QUESTION PAPER