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ADVANCED
General Certificate of Education
2014

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

THURSDAY 22 MAY, MORNING

MV18

TIME

1 hour 30 minutes, plus your additional time allowance.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(i)** Write

$$\frac{2x - 7}{(5 - x)(1 + x)}$$

in partial fractions. [6 marks]

(ii) Hence find

$$\int \frac{2x - 7}{(5 - x)(1 + x)}\,\mathrm{d}x$$ [4 marks]

2 The points A and B have position vectors:

$$\overrightarrow{OA} = 3\mathbf{i} - 4\mathbf{j}$$

and $$\overrightarrow{OB} = 7\mathbf{i} + 5\mathbf{j}$$

relative to a fixed origin O.

(i) Find $\overrightarrow{AB}$. [2 marks]

The point C has position vector

$$\overrightarrow{OC} = 3\mathbf{i} - 2\mathbf{j}$$

(ii) Find a vector equation of the line through C, parallel to AB. [3 marks]

(iii) Show that the point with position vector $(11\mathbf{i} + 16\mathbf{j})$ lies on this line. [4 marks]

3 Use the substitution $u = 3x - 5$ to evaluate

$$\int_2^3 6x\sqrt{3x-5}\,\mathrm{d}x$$ [9 marks]

4 The curved surface of a glass bowl can be modelled by rotating the curve

$y = e^x + 1$

between the lines $x = 0$ and $x = 1$ through 2π radians about the x-axis.

(i) Find the maximum volume that the bowl can hold. [7 marks]

(ii) State one assumption made in the modelling. [1 mark]

5 At time $t = 0$ hours a small block of ice starts to melt.
The volume, V cm^3, of the solid ice decreases with time, at a rate which is proportional to the square root of the volume of ice remaining at that time.
This can be modelled by the differential equation

$$\frac{dV}{dt} = k\sqrt{V}$$

Initially the volume of ice is 64 cm^3 and one hour later the volume of ice is 48 cm^3
If the ice started to melt at 12.00 (noon), find the time at which the ice has completely melted. [10 marks]

6 The expression ($7 \sin x - 24 \cos x$) can be written in the form

$R \sin (x - \alpha)$

where R is an integer and $0 \leqslant \alpha \leqslant \frac{\pi}{2}$

(i) Find R and α. [4 marks]

(ii) Hence find

$$\int \frac{1}{(7 \sin x - 24 \cos x)^2} \, dx$$ [3 marks]

7 The function f is defined as

f: $x \rightarrow \tan x \qquad -\frac{\pi}{4} \leqslant x \leqslant \frac{\pi}{4}$

(i) Write down the inverse function f^{-1} and state its domain and range. [4 marks]

The function g is defined as

g: $x \rightarrow |x| \qquad x \in \mathbb{R}$

(ii) Find the composite function gf, stating its range. [4 marks]

(iii) Hence sketch the graph of $y = gf(x)$ [3 marks]

8 The parametric equations of a curve are

$$x = 2t - \sin 2t \qquad y = 4\cos t$$

where $\frac{\pi}{2} \leqslant t \leqslant \frac{3\pi}{2}$

Find the **exact** coordinates of the point on the curve at which the gradient is $\sqrt{2}$ [11 marks]

THIS IS THE END OF THE QUESTION PAPER
