Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2014

Mathematics

Assessment Unit C2

assessing

Module C2: AS Core Mathematics 2

[AMC21]

AMC21

FRIDAY 17 JANUARY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Use the Trapezium Rule with 5 strips to find an approximate value for

$$\int_1^2 \frac{1}{x^2 + 4}\,dx$$

[6]

2 **(a)** A sequence is defined by the recurrence relation

$$u_{n+1} = a - \frac{u_n}{3}$$

where $u_1 = 12$, $n \geqslant 1$ and a is a constant.

(i) Given $u_2 = 32$, find a. [1]

(ii) This sequence tends to a limit L, find L. [2]

(b) **(i)** Sketch the graph of

$$y = 4^{-x}$$

[2]

(ii) Solve the equation

$$4^{-x} = 10$$

[2]

3 **(a)** A ship sails 50 miles from port P to port Q on a bearing of 100°
It then sails 40 miles from port Q to port R on a bearing of 210° as shown in **Fig. 1** below.

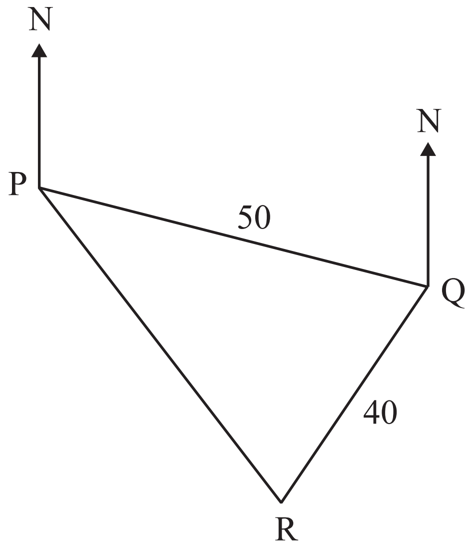

Fig. 1

(i) Find the distance PR. [3]

(ii) Find the bearing of R from P. [3]

(b) The sector AOB of a circle of radius r subtends an angle of $\frac{2\pi}{7}$ radians at its centre, as shown in **Fig. 2** below.

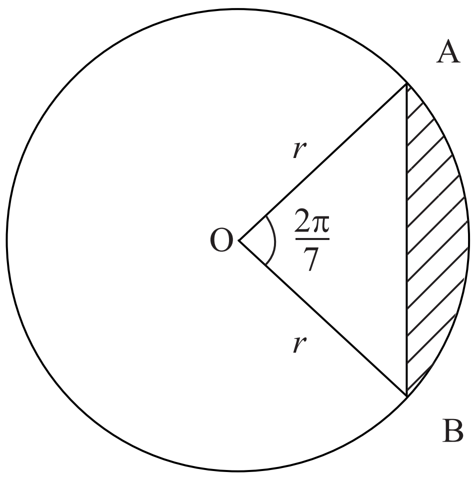

Fig. 2

Find, in terms of r, the area of the shaded segment. [5]

4 **(i)** Sketch the graph of

$$y = \tan 3x$$

for $0 \leqslant x \leqslant \pi$ [2]

(ii) State the period of this graph. [1]

(iii) Solve the equation

$$\sin 3x = \sqrt{3} \cos 3x$$

for $0 \leqslant x \leqslant \pi$ [5]

5 **(a)** Find

$$\int \frac{2x + 3}{\sqrt{x}} \, dx$$ [4]

(b) The rudder for a boat can be modelled by the area enclosed by the graph of

$$y = \sqrt[3]{x - 8}$$

the y-axis and the line $y = 1$, as shown shaded in **Fig. 3** below.

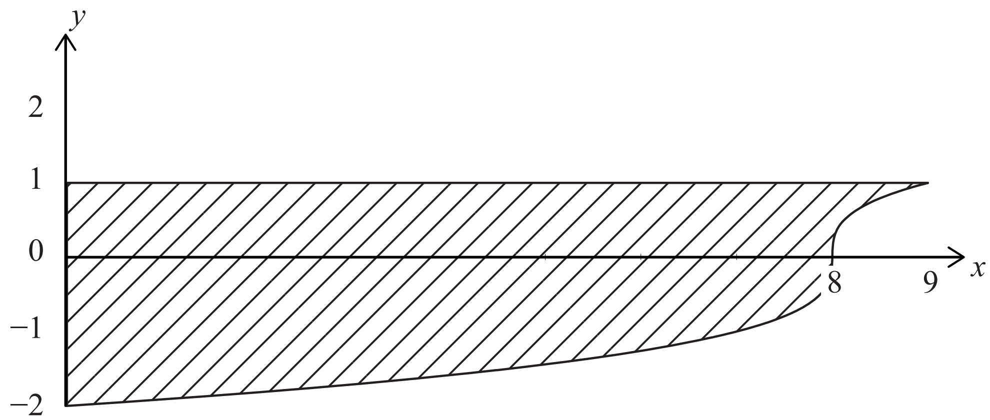

Fig. 3

Find the area of the rudder. [6]

6 **(a)** In the Binomial expansion of

$$\left(1 + \frac{x}{3}\right)^n$$

the coefficient of x^2 is 4

Find the value of n, where n is a positive integer. [6]

(b) An arithmetic progression has a common difference of $\frac{1}{2}$
The 5th, 13th and 19th terms of this arithmetic progression form the first three terms of a geometric progression.

(i) Find the first term of the AP. [7]

(ii) State the common ratio of the GP. [1]

7 The chain on a bicycle passes over two sprockets. The sprockets can be modelled as two circles A and B as shown in **Fig. 4** below.

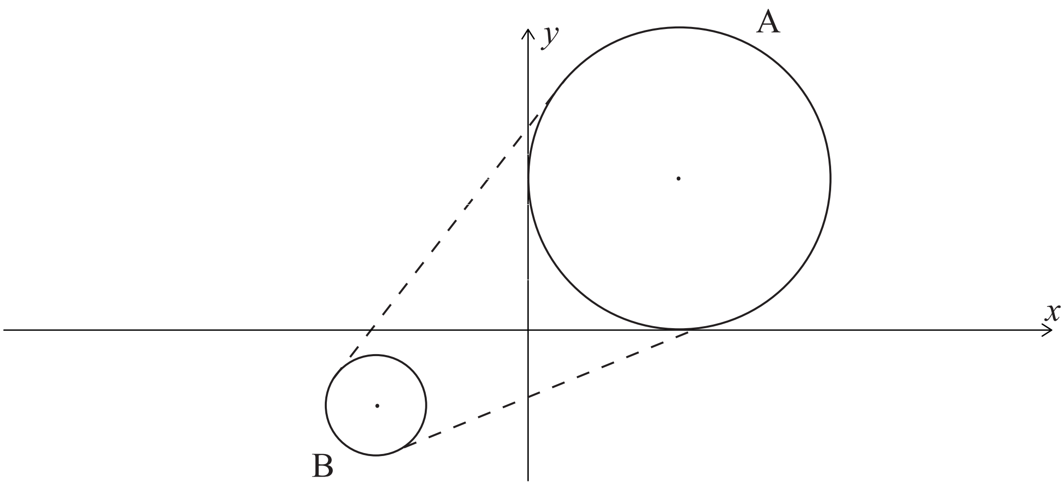

Fig. 4

Circular sprocket A has a radius of 6 cm and touches both the x- and y-axes.

(i) Find the equation of circle A. [3]

The equation of circle B is

$$x^2 + 12x + y^2 + 6y + 41 = 0$$

(ii) Find the length of the radius and the coordinates of the centre of circle B. [4]

(iii) Find the shortest distance between the two sprockets. [3]

8 Solve the equation

$$3\log_8 x = 5 + 2\log_x 8$$ [9]

THIS IS THE END OF THE QUESTION PAPER