Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2014

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

FRIDAY 10 JANUARY, MORNING

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for correct working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidates's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

		AVAILABLE MARKS
1 **(i)** $\mathbf{AB} = \begin{pmatrix} 1 & x & 2 \\ 2 & 3 & 0 \end{pmatrix}\begin{pmatrix} 0 & 6 \\ 1 & x \\ -1 & -3 \end{pmatrix}$		
$= \begin{pmatrix} x-2 & x^2 \\ 3 & 12+3x \end{pmatrix}$	MW2	
(ii) **AB** singular $\Rightarrow$ det **AB** = 0	M1	
$\Rightarrow (x-2)(12+3x) - 3x^2 = 0$	MW1	
$\Rightarrow 12x - 24 + 3x^2 - 6x - 3x^2 = 0$	W1	
$\Rightarrow 6x - 24 = 0$		
$\Rightarrow x = 4$	W1	6
2 **(a)** $\tan\theta = \frac{1}{\sqrt{3}}$		
$\Rightarrow \theta = 30°$	MW1	
Therefore the transformation matrix is $\begin{pmatrix} \cos 60° & \sin 60° \\ \sin 60° & -\cos 60° \end{pmatrix}$	M1	
$= \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$	W1	
(b) $\begin{pmatrix} 1 & 2 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} t \\ mt \end{pmatrix}$	M1	
$\Rightarrow x + 2mx = t$ and $4x - mx = mt$	MW1	
Divide to give $\frac{1+2m}{4-m} = \frac{1}{m}$	M1	
$\Rightarrow m + 2m^2 = 4 - m$	W1	
$\Rightarrow 2m^2 + 2m - 4 = 0$		
$\Rightarrow m^2 + m - 2 = 0$		
$\Rightarrow (m+2)(m-1) = 0$		
$\Rightarrow m = 1, -2$	W1	
Hence the lines are $y = x$ and $y = -2x$	W1	9

AVAILABLE MARKS

3 **(a)** $3 \times 3 = 1$; $7 \times 7 = 1$ – these are already in set
$3 \times 7 = 5 \Rightarrow a = 5$ MW1

(b) **(i)**

$\times_{18}$	1	5	7	11	13	17
1	1	5	7	11	13	17
5	5	7	17	1	11	13
7	7	17	13	5	1	11
11	11	1	5	13	17	7
13	13	11	1	17	7	5
17	17	13	11	7	5	1

MW3

(ii)

$5^1 = 5$ or $11^1 = 11$
$5^2 = 7$ $11^2 = 13$
$5^3 = 17$ $11^3 = 17$
$5^4 = 13$ $11^4 = 7$
$5^5 = 11$ $11^5 = 5$
$5^6 = 1$ $11^6 = 1$ MW1
Hence 5 has period 6 Hence 11 has period 6 MW1

(iii) $\{1, 17\}$ MW2

(iv) $\{1, 7, 13\}$ MW2 10

4 **(i)** $|\mathbf{P} - \lambda\mathbf{I}| = 0$ M1

$$\Rightarrow \begin{vmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{vmatrix} = 0$$ M1

$\Rightarrow (3-\lambda)[(5-\lambda)(3-\lambda)-1] + 1[-1(3-\lambda)+1] + [1-(5-\lambda)] = 0$ M1
$\Rightarrow (3-\lambda)[15-8\lambda+\lambda^2-1] - 3 + \lambda + 1 + 1 - 5 + \lambda = 0$ W1
$\Rightarrow (3-\lambda)[\lambda^2-8\lambda+14] + 2\lambda - 6 = 0$
$\Rightarrow (3-\lambda)[\lambda^2-8\lambda+14] - 2(3-\lambda) = 0$
$\Rightarrow (3-\lambda)[\lambda^2-8\lambda+12] = 0$ W1
$\Rightarrow (3-\lambda)(\lambda-2)(\lambda-6) = 0$ W1
$\Rightarrow \lambda = 6, 3, 2$ W1

AVAILABLE MARKS

(ii) $\begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 6\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ M1

$\Rightarrow 3x - y + z = 6x$ M1

$-x + 5y - z = 6y$

$x - y + 3z = 6z$

$\Rightarrow -3x - y + z = 0$ ①

$-x - y - z = 0$ ②

$x - y - 3z = 0$ ③

① – ② $\Rightarrow -2x + 2z = 0 \quad \Rightarrow x = z$

② – ③ $\Rightarrow -2x + 2z = 0 \quad \Rightarrow x = z$

Use ② $\Rightarrow -z - y - z = 0 \quad \Rightarrow y = -2z$ W1

Hence a possible eigenvector is $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ W1

(iii) $\begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}$ M1

$= 2\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ W1

$\begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$ M1

$= 3\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ W1

(iv) $\mathbf{U} = \begin{pmatrix} 1 & 1 & 1 \\ -2 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix}$ MW1 16

AVAILABLE MARKS

5 **(i)** $x^2 + y^2 + 6x - 10y + 9 = 0$

$\Rightarrow (x+3)^2 + (y-5)^2 = 25$

Hence $C_1(-3, 5)$ and $r_1 = 5$ MW2

$x^2 + y^2 - 2x - 6y + 1 = 0$

$\Rightarrow (x-1)^2 + (y-3)^2 = 9$

Hence $C_2(1, 3)$ and $r_2 = 3$ MW2

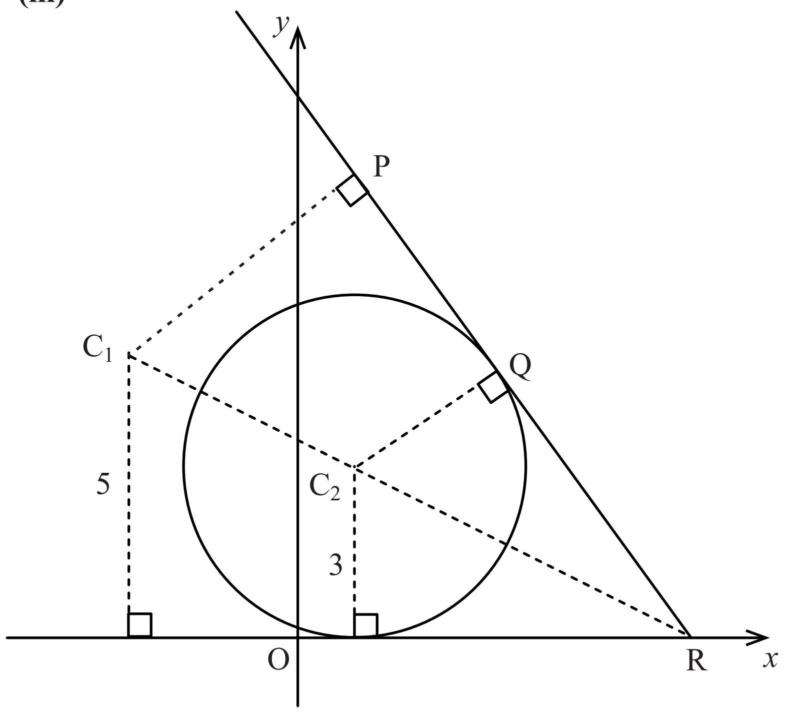
(ii) Distance between centres $C_1 C_2 = \sqrt{4^2 + 2^2}$ M1

$= \sqrt{20}$ W1

Also, $r_1 + r_2 = 5 + 3 = 8$ $\quad r_1 - r_2 = 5 - 3 = 2$ MW1

Since $C_1 C_2 < r_1 + r_2$ and $C_1 C_2 > r_1 - r_2$, then the circles intersect MW1

(iii)



Tangents intersect on x-axis MW1

Gradient of line $C_1C_2 = \dfrac{2}{-4} = -\dfrac{1}{2}$ MW1

$\Rightarrow$ Equation of line $C_1 C_2$ is $y - 3 = -\dfrac{1}{2}(x - 1)$

$\Rightarrow y = -\tfrac{1}{2}x + 3\tfrac{1}{2}$ MW1

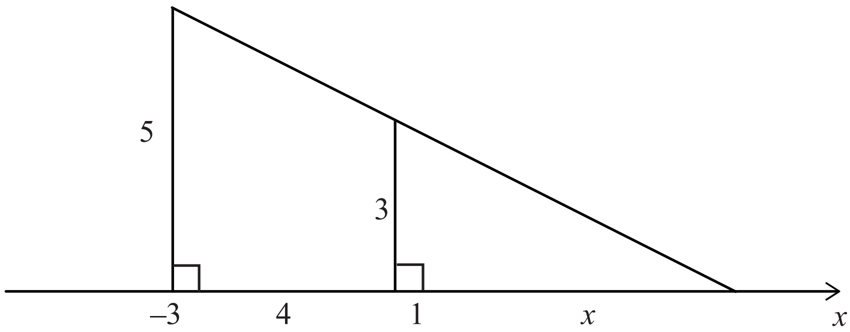
Line cuts x-axis when $y = 0$ M1

$\Rightarrow \tfrac{1}{2}x = 3\tfrac{1}{2}$

$\Rightarrow x = 7$

Hence point of intersection R is (7, 0) W1 13

Alternative solution



Tangents intersect on x-axis MW1

Using similar triangles

$$\frac{3}{5} = \frac{x}{4 + x}$$ MW1

$\Rightarrow 3(4 + x) = 5x$ MW1

$\Rightarrow 12 + 3x = 5x$

$\Rightarrow x = 6$ MW1

$\Rightarrow$ point of intersection is (7, 0) W1

AVAILABLE MARKS

AVAILABLE MARKS

6 **(a)** $z + \frac{1}{z} = \frac{3 + \mathrm{i}}{1 - \mathrm{i}} + \frac{1 - \mathrm{i}}{3 + \mathrm{i}}$ MW1

$= \frac{3 + \mathrm{i}}{1 - \mathrm{i}} \times \frac{1 + \mathrm{i}}{1 + \mathrm{i}} + \frac{1 - \mathrm{i}}{3 + \mathrm{i}} \times \frac{3 - \mathrm{i}}{3 - \mathrm{i}}$ M1

$= \frac{3 + 4\mathrm{i} - 1}{1 + 1} + \frac{3 - 4\mathrm{i} - 1}{9 + 1}$ MW2

$= \frac{2 + 4\mathrm{i}}{2} + \frac{2 - 4\mathrm{i}}{10}$ MW1

$= 1\frac{1}{5} + 1\frac{3}{5}\mathrm{i}$ W1

(b) $(p + q\mathrm{i})^2 = 17 - 6\sqrt{2}\,\mathrm{i}$

$\Rightarrow p^2 - q^2 + 2pq\mathrm{i} = 17 - 6\sqrt{2}\,\mathrm{i}$ M1

$\Rightarrow p \;- q^2 = 17$ ① MW1

and $2pq = -6\sqrt{2}$ ② MW1

② $\Rightarrow q = \frac{-3\sqrt{2}}{p}$ W1

Hence ① $\Rightarrow p^2 - \left(\frac{-3\sqrt{2}}{p}\right)^2 = 17$ M1

$\Rightarrow p^4 - 17p^2 - 18 = 0$ W1

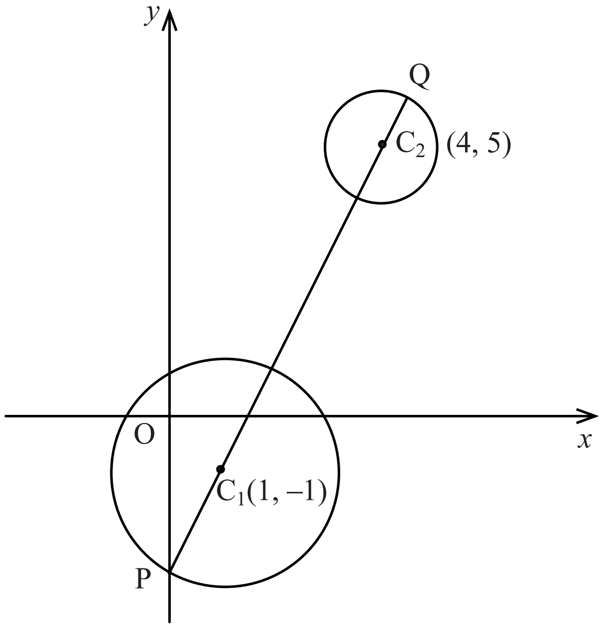
$\Rightarrow (p^2 + 1)(p^2 - 18) = 0$

$\Rightarrow p^2 = -1, 18$

$\Rightarrow p = \pm\sqrt{18} = \pm 3\sqrt{2}$ W1

$\Rightarrow q = \pm 1$ W1

			AVAILABLE MARKS
(c) **(i)**	Circle with centre (1, –1) and radius 2	MW3	
(ii)	Circle with centre (4, 5) and radius 1	MW1	



(iii)	Distance between centres $C_1C_2 = \sqrt{3^2 + 6^2} = \sqrt{45}$	MW1	
	$PQ = 2 + \sqrt{45} + 1$	M1	
	Hence maximum value of $\lvert u - v \rvert = 3 + 3\sqrt{5}$	W1	21
		Total	**75**