



Rewarding Learning

ADVANCED
General Certificate of Education
January 2014

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

MONDAY 27 JANUARY, MORNING



TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all seven** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 (i) Simplify the expression

$$(\sqrt{r} + \sqrt{r-1})(\sqrt{r} - \sqrt{r-1}) \quad [1]$$

(ii) Hence show that
$$\sum_{r=1}^n \frac{1}{\sqrt{r} + \sqrt{r-1}} = \sqrt{n} \quad [3]$$

2 Find, in radians, the general solution of the equation

$$3 \tan^2 \theta = 2 \sin \theta \quad [9]$$

3 (a) A root of the equation

$$z^4 - 6z^3 + 23z^2 - 34z + 26 = 0$$

is $1 + i$

(i) State with a reason one other complex root. [2]

(ii) Find the remaining roots. [5]

(b) Given that n is a positive integer, show that

$$(1 + i)^{4n} - (1 - i)^{4n} = 0 \quad [5]$$

4 Let $y = \tan x$

(i) Show that $\frac{dy}{dx} = 1 + y^2$ and $\frac{d^2y}{dx^2} = 2y(1 + y^2)$ [3]

(ii) Find, in terms of y , an expression for $\frac{d^3y}{dx^3}$ [2]

(iii) Hence or otherwise find the Maclaurin series for $\tan x$ up to and including the term in x^3 [4]

(iv) Using this approximation for $\tan x$ and other small angle approximations, find

$$\lim_{x \rightarrow 0} \left(\frac{x \sin^2 x}{\tan x - x \cos x} \right) \quad [3]$$

5 Use mathematical induction to prove that if $n \geq 1$ and $\sin x \neq 0$

$$\cos x + \cos 3x + \cos 5x + \dots + \cos(2n - 1)x \equiv \frac{\sin 2nx}{2 \sin x} \quad [9]$$

6 The extension x centimetres of a spiral spring at time t seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + n^2x = \cos \omega t$$

where n and ω are constants and $\omega \neq n$

Given that when $t = 0$, $x = 0$ and $\frac{dx}{dt} = V$, express x as a function of t . [12]

7 (i) Find the eccentricity of the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \quad [2]$$

(ii) State the coordinates of the foci and the equations of the directrices. [2]

(iii) Show that an equation for the tangent to this ellipse at the point $P(3 \cos \theta, 2 \sin \theta)$ is

$$\frac{x \cos \theta}{3} + \frac{y \sin \theta}{2} = 1 \quad [4]$$

A line perpendicular to the tangent at P is drawn from the origin and intersects the tangent at Q.

(iv) Find, in terms of θ , the coordinates of Q. [5]

(v) Verify that as θ varies Q lies on the curve

$$(x^2 + y^2)^2 = 9x^2 + 4y^2 \quad [4]$$

THIS IS THE END OF THE QUESTION PAPER
