



Rewarding Learning

ADVANCED  
General Certificate of Education  
2013

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## Mathematics

Assessment Unit M2

*assessing*

Module M2: Mechanics 2

[AMM21]



THURSDAY 13 JUNE, MORNING

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### TIME

1 hour 30 minutes.

### INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all seven** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or a scientific calculator in this paper.

### INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take  $g = 9.8 \text{ m s}^{-2}$ , unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$

**Answer all seven questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

- 1** A particle of mass 4 kg is acted upon by a force,  $\mathbf{P}$ , so that its position vector  $\mathbf{r}$  relative to a fixed point O, at time  $t$  seconds is given by

$$\mathbf{r} = (4t^2 - 5t)\mathbf{i} + (16 - t^2)\mathbf{j} \text{ metres}$$

- (i)** Find an expression for the velocity of the particle at any time  $t$ . [2]

- (ii)** Show that the acceleration of the particle is constant. [3]

- (iii)** Find  $\mathbf{P}$ . [2]

- 2** A ball of mass 0.5 kg is travelling across a smooth horizontal surface. The ball is then struck by a bat.

Immediately before being struck the ball had a velocity of  $(-2\mathbf{i} + 4\mathbf{j}) \text{ ms}^{-1}$  and immediately after being struck the velocity of the ball is  $(5\mathbf{i} - 10\mathbf{j}) \text{ ms}^{-1}$

- (i)** Find the magnitude of the impulse exerted on the ball by the bat. [4]

At the moment of being struck the position vector of the ball was  $(-2\mathbf{i} - 2\mathbf{j}) \text{ m}$ .

- (ii)** Find the position vector of the ball after 3 seconds. [4]

- 3** The mass of a car is 800 kg.

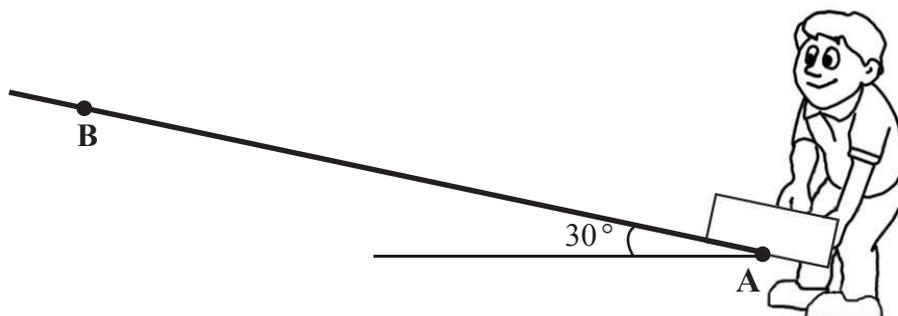
The engine of the car works at a constant rate of 20 kW, and the motion of the car is subject to a constant resistance of magnitude 600 N.

- (i)** Find the acceleration of the car when it is travelling at  $25 \text{ ms}^{-1}$  on a level road. [4]

The car now ascends a straight road, inclined at  $5^\circ$  to the horizontal, with the same power output and against the same constant resistance.

- (ii)** Find the maximum speed at which the car can ascend the road. [4]

- 4 A postman places a parcel of mass 4 kg at the bottom of a rough ramp which is inclined at  $30^\circ$  to the horizontal as shown in **Fig. 1** below.



**Fig. 1**

The coefficient of friction between the ramp and the parcel is 0.7

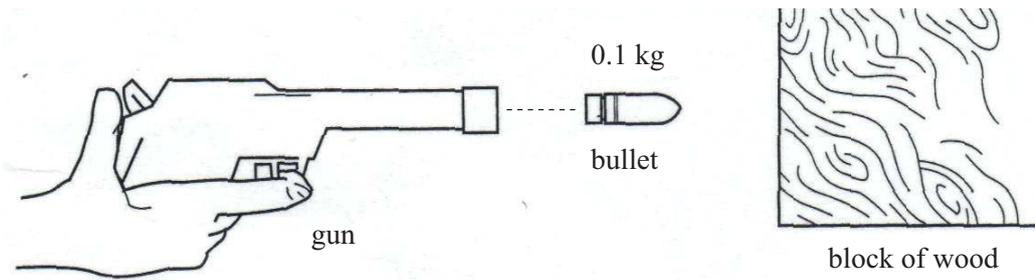
The postman pushes the parcel up the ramp giving it an initial speed of  $5 \text{ ms}^{-1}$  at point A.  
The parcel stops at point B.  
The distance  $AB = d$  metres.

Model the parcel as a particle.

Take the potential energy at A to be zero.

- (i) Find the kinetic energy of the parcel at A. [2]
- (ii) Find, in terms of  $d$ , the work done against gravity in moving the parcel from A to B. [3]
- (iii) Find, in terms of  $d$ , the work done against friction in moving the parcel from A to B. [4]
- (iv) Hence, find  $d$ . [2]

- 5 **Fig. 2** below illustrates a bullet of mass 0.1 kg being fired from a gun directly at a block of wood.



**Fig. 2**

The bullet leaves the gun and hits the block of wood with an initial speed of  $100 \text{ ms}^{-1}$ . When travelling through the block of wood, the bullet is subject to a resistive force of  $20v^2$  newtons, where  $v \text{ ms}^{-1}$  is the velocity of the bullet when it has penetrated a distance  $x$  metres into the wood.

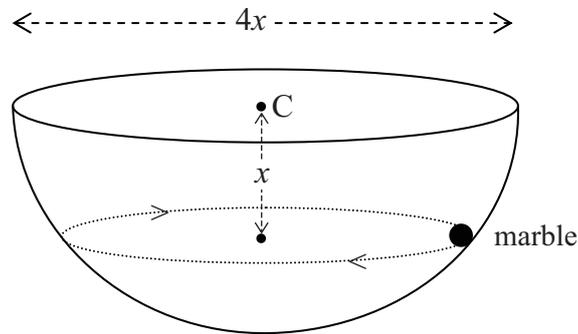
- (i) Show that the equation of the motion of the bullet through the wood can be modelled by

$$-200v = \frac{dv}{dx} \quad [3]$$

- (ii) Find  $x$  when the speed of the bullet has been reduced to  $25 \text{ ms}^{-1}$  [8]

- (iii) Explain briefly why this model could **not** be used to find the distance travelled by the bullet through the block of wood before the bullet comes to rest. [Assume the bullet does not exit the block.] [2]

- 6 A glass marble of mass  $m$  kg is moving in a horizontal circle around the inside surface of a smooth hemispherical bowl of diameter  $4x$  metres.  
The centre of the circular path of the marble is at a distance of  $x$  metres below the centre,  $C$ , of the bowl, as shown in **Fig. 3** below.



**Fig. 3**

- (i) Draw a diagram showing the external forces acting on the marble. [2]
- (ii) Show that the marble travels in a circle of radius  $\sqrt{3}x$  metres. [1]
- (iii) Find, in terms of  $m$  and  $g$ , the magnitude of the reaction between the marble and the bowl. [4]
- (iv) Find, in terms of  $x$  and  $g$ , an expression for the speed of the marble. [5]

7 [In this question take  $g = 10 \text{ m s}^{-2}$ ]

A particle is projected from a point O at an angle of  $\theta$  **below** the horizontal with a speed of  $u \text{ m s}^{-1}$

The position P of the particle when it has travelled  $x$  metres horizontally and dropped a distance of  $y$  metres vertically below the level of O is shown in **Fig. 4** below.

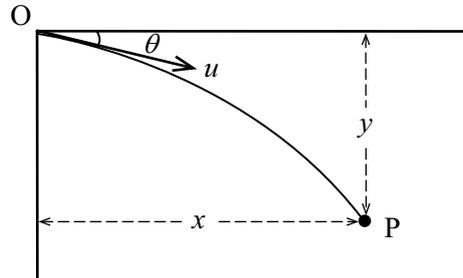


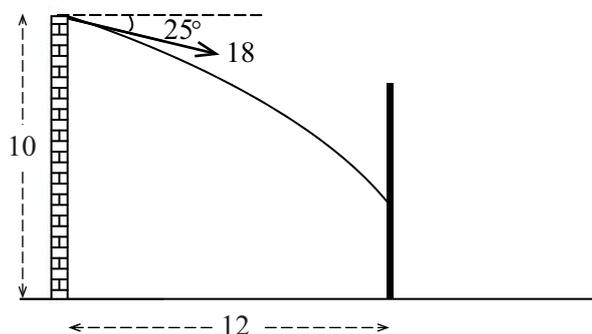
Fig. 4

(i) Prove that the equation of the path of **this** projectile is given by

$$y = x \tan \theta + \frac{gx^2}{2u^2} (1 + \tan^2 \theta)$$

[8]

A ball is projected with an initial speed of  $18 \text{ ms}^{-1}$  at an angle of  $25^\circ$  below the horizontal from a point on the top of a vertical wall. The point of projection is 10 m vertically above horizontal ground. The ball hits a vertical fence which is at a horizontal distance of 12 m from the wall as shown in **Fig. 5** below.



**Fig. 5**

**(ii)** Calculate the height, above the ground, of the point at which the ball hits the fence. [3]

The fence is 6 m high. A ball is now projected from the same point with the same initial speed of  $18 \text{ ms}^{-1}$  at an angle of  $\theta$  below the horizontal, so that it just clears the fence.

**(iii)** Find the minimum value of  $\theta$ . [5]

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**THIS IS THE END OF THE QUESTION PAPER**

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