



Rewarding Learning

ADVANCED
General Certificate of Education
2013

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

THURSDAY 30 MAY, MORNING



TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all seven** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 (i) Show that

$$\sum_{r=1}^n (2r-1)^2 = \frac{n}{3}(4n^2-1) \quad [4]$$

(ii) Using this result, evaluate

$$\sum_{r=5}^{40} (2r-1)^2 \quad [3]$$

2 Find, in terms of π , the general solution of the equation

$$\tan^4 x - 4 \tan^2 x + 3 = 0 \quad [6]$$

3 (i) Using Maclaurin's theorem, show that a series expansion for $\ln(1+ax)$, where a is a constant, up to and including the term in x^3 is

$$\ln(1+ax) = ax - \frac{a^2x^2}{2} + \frac{a^3x^3}{3} \quad [6]$$

(ii) Hence or otherwise find a series expansion for

$$\ln \left\{ \frac{(1-3x)^2}{1+2x} \right\}$$

up to and including the term in x^3 [5]

(iii) For the series expansion in **(ii)** find the coefficient of x^n [2]

- 4 A man adds 10 grams of sugar to a cup of hot tea. The mass of sugar, M grams, undissolved in the tea t seconds after being added is modelled by the differential equation

$$\frac{dM}{dt} + \frac{2M}{10-t} = \frac{1}{5} \quad 0 \leq t < 10$$

(i) Find M in terms of t . [9]

(ii) Find the mass of sugar undissolved after 5 seconds. [2]

- 5 Use the principle of mathematical induction to show that for $n \geq 1$

$$\sum_{r=1}^n r^2 \geq \frac{n(n+1)^2}{4} \quad [8]$$

- 6 (i) Show that an equation of the tangent to the parabola $y^2 = 4ax$ at the point $P(at^2, 2at)$ is

$$x - ty + at^2 = 0 \quad [4]$$

(ii) Find the coordinates of the point Q where this tangent cuts the y -axis. [2]

(iii) If O is the origin find the point of intersection of the perpendicular bisectors of the lines PQ and OQ . [5]

(iv) As t varies find the equation of the locus of this point of intersection. [3]

7 (i) Using De Moivre's theorem, show that if $z = \cos \theta + i \sin \theta$ then

$$2 \cos n\theta = z^n + \frac{1}{z^n} \quad [3]$$

(ii) Hence or otherwise show that if $z = \cos \theta + i \sin \theta$ the equation

$$3z^4 - 5z^3 + 8z^2 - 5z + 3 = 0$$

can be transformed into the equation

$$6 \cos^2 \theta - 5 \cos \theta + 1 = 0 \quad [4]$$

(iii) Using (ii) or otherwise solve the equation

$$3z^4 - 5z^3 + 8z^2 - 5z + 3 = 0 \quad [9]$$

THIS IS THE END OF THE QUESTION PAPER
