



Rewarding Learning

ADVANCED
General Certificate of Education
January 2013

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

MONDAY 28 JANUARY, MORNING



TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all eight** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$.



Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find in terms of π the general solution of the equation

$$1 + \frac{\cos 2\theta}{\cos \theta} = 0 \quad [6]$$

2 (i) Show that

$$\sum_{r=1}^n r(r+2) = \frac{n}{6}(n+1)(2n+7) \quad [4]$$

(ii) Using this result, find the sum

$$3 \ln 5 + 4 \ln 5^2 + 5 \ln 5^3 + \dots + 12 \ln 5^{10}$$

leaving your answer in terms of $\ln 5$ [3]

3 (i) Given that

$$f(x) = \frac{5x^2 + 12x + 14}{(x+2)^2(x^2+1)} \quad x \neq -2$$

express $f(x)$ in partial fractions. [8]

(ii) Hence or otherwise show that $f(x) > 0$ [3]

4 (a) Using small angle approximations, show that as $x \rightarrow 0$

$$\frac{1 - \cos 4x + x \sin 2x}{x^2} \rightarrow 10 \quad [3]$$

(b) Using Maclaurin's theorem, find a series expansion for $\cos 4x$ up to and including the term in x^4 [5]

- 5 Using the principle of mathematical induction, prove that if n is a positive integer then

$$3^{2n} + 11$$

is divisible by 4

[7]

- 6 A particle moves so that its displacement x metres from a fixed point O at time t seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 6x = 20 \cos t$$

Find the general solution of this equation.

[10]

- 7 (i) Show that the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point P ($a \cos \theta$, $b \sin \theta$) is

$$\frac{x \cos \theta}{a} + \frac{y \sin \theta}{b} = 1$$

[5]

The equation of the tangent to this ellipse at Q ($-a \sin \theta$, $b \cos \theta$) is

$$\frac{y \cos \theta}{b} - \frac{x \sin \theta}{a} = 1$$

The tangents at P and Q intersect at the point R.

- (ii) Find in terms of θ the coordinates of R.

[6]

- (iii) Show that as θ varies the locus of R is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$$

[2]

8 (i) Using De Moivre's theorem, show that

$$z^n - \frac{1}{z^n} = 2i \sin n\theta \quad [3]$$

where $z = \cos \theta + i \sin \theta$

(ii) Using (i) with $n = 1$ or otherwise, show that

$$16 \sin^5 \theta = \sin 5\theta - 5 \sin 3\theta + 10 \sin \theta \quad [5]$$

(iii) Hence evaluate

$$\int_0^{\frac{\pi}{3}} \sin^5 \theta \, d\theta \quad [5]$$

THIS IS THE END OF THE QUESTION PAPER
