



Rewarding Learning

ADVANCED
General Certificate of Education
January 2013

Mathematics

Assessment Unit C3

assessing

Module C3: Core Mathematics 3

[AMC31]

WEDNESDAY 23 JANUARY, MORNING



TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all eight** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that

$$\ln z \equiv \log_e z$$



Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

- 1** A radioactive substance decays at a rate which can be modelled by the equation

$$B = 5000e^{-0.007t}$$

where B is the number of particles of the substance remaining at any time t seconds.

Find the value of t when 3000 particles remain.

[4]

- 2** Solve

$$\operatorname{cosec}(\theta + 40^\circ) = 5$$

where $-180^\circ \leq \theta \leq 180^\circ$

[4]

- 3 (i)** Show that the expression

$$\frac{x-1}{3x+5} \div \frac{x^2+3x-4}{9x^2-25}$$

simplifies to

$$\frac{3x-5}{x+4}$$

[4]

- (ii)** Using the result of **(i)**, find

$$\frac{d}{dx} \left(\frac{x-1}{3x+5} \div \frac{x^2+3x-4}{9x^2-25} \right)$$

giving your answer in its simplest form.

[5]

- 4 (a) The reed of a musical instrument can be modelled by the area under the curve

$$y = \frac{1}{x^4 - 1}$$

between the lines $x = 2$ and $x = 4$ and the x -axis, as shown in **Fig. 1** below.

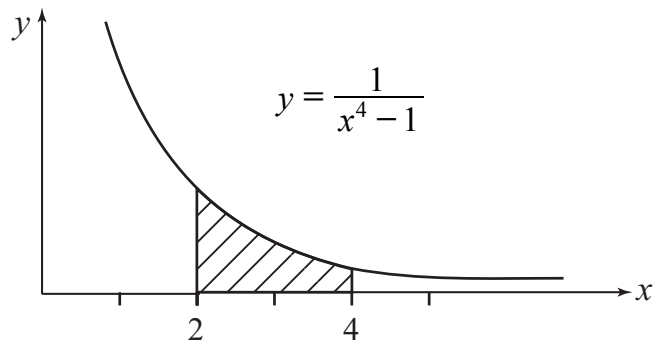


Fig. 1

Use Simpson's rule with 4 strips to find an approximation for the area of the reed,

$$\int_2^4 \frac{1}{x^4 - 1} dx \quad [6]$$

- (b) Find

$$\int \left(\frac{7}{x} - \operatorname{cosec}^2 3x - 6x \right) dx \quad [4]$$

- 5 (i) Solve

$$\ln x - 1 = 0 \quad [2]$$

- (ii) Sketch the graph of

$$y = |\ln x - 1| \quad [4]$$

- (iii) Find the **exact** solutions of

$$|\ln x - 1| = 2 \quad [5]$$

6 (i) Prove the identity

$$\sin \theta + \cos \theta \cot \theta \equiv \operatorname{cosec} \theta \quad [4]$$

(ii) Hence, or otherwise, find a Cartesian equation of the parametric curve

$$x = 2 + \cos t \quad y = \sin t + \cos t \cot t \quad [5]$$

(iii) Write down the equation of the horizontal axis of symmetry of this curve. [1]

7 Find the equation of the tangent to the curve

$$y = e^{3x} \cot\left(\frac{1}{2}x + \frac{\pi}{4}\right)$$

at the point where $x = 0$ [8]

8 (i) Express

$$\frac{2 + 7x}{(2 - x)(2 + 3x)}$$

in partial fractions. [5]

(ii) Find the first three terms in the binomial expansion of

$$(2 + kx)^{-1} \quad [7]$$

(iii) Hence, using the results from parts (i) and (ii), find the first three terms in the binomial expansion of

$$\frac{2 + 7x}{(2 - x)(2 + 3x)} \quad [7]$$

THIS IS THE END OF THE QUESTION PAPER
