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ADVANCED  
General Certificate of Education  
2012

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## Mathematics

Assessment Unit C4  
*assessing*  
Module C4: Core Mathematics 4  
[AMC41]



FRIDAY 1 JUNE, MORNING

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### TIME

1 hour 30 minutes.

### INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.  
Answer **all eight** questions.  
Show clearly the full development of your answers.  
Answers should be given to three significant figures unless otherwise stated.  
You are permitted to use a graphic or scientific calculator in this paper.

### INFORMATION FOR CANDIDATES

The total mark for this paper is 75  
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.  
A copy of the **Mathematical Formulae and Tables booklet** is provided.  
Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$ .



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**Answer all eight questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

- 1 (a)** For each relation below state if it is a one–one function, a many–one function or a mapping.

**(i)** a:  $x \rightarrow \pm\sqrt{x}$   $x \in \mathbb{R}$   $x \geq 0$  [1]

**(ii)** b:  $x \rightarrow \frac{1}{x}$   $x \in \mathbb{R}$   $x \neq 0$  [1]

**(iii)** c:  $x \rightarrow x^2$   $x \in \mathbb{R}$  [1]

- (b)** The function  $f$  is defined by

$$f(x) = x^2 - 3 \quad x \in \mathbb{R}$$

and the function  $g$  is defined by

$$g(x) = 2x + 1 \quad x \in \mathbb{R}$$

- (i)** Find the composite function  $gf(x)$ , stating its domain. [3]

- (ii)** Find the values of  $x$  such that  $gf(x) = 3x$  [3]

- 2 (i)** Sketch the graph of

$$y = 4 - x^2$$

clearly showing where it crosses the  $x$ -axis. [2]

A paperweight can be modelled as the solid formed when the area bounded by the curve

$$y = 4 - x^2$$

and the  $x$ -axis and the  $y$ -axis is rotated through  $360^\circ$  about the  $x$ -axis.

- (ii)** Find the volume of the paperweight. [7]

- 3 During a science experiment, students create two waves in a ripple tank. The first wave can be modelled by the equation

$$h_1 = 6 \cos t \quad 0 \leq t \leq 2\pi$$

The second wave can be modelled by the equation

$$h_2 = 5 \sin t \quad 0 \leq t \leq 2\pi$$

where  $h_1$  and  $h_2$  are the heights of the waves, measured in millimetres, at any time  $t$  seconds from the start of the experiment.

The two waves join together.

- (i) Express the height  $h_1 + h_2$  of the resultant wave in the form

$$r \cos(t - \alpha) \quad 0 \leq \alpha \leq \frac{\pi}{2} \quad r \in \mathbb{R} \quad [4]$$

- (ii) Hence find the times when the resultant wave has a height of 3 mm. [4]

- 4 (i) Given that

$$x^2 + 6xy + y^2 + 32 = 0$$

use implicit differentiation to show that

$$\frac{dy}{dx} = \frac{-(x+3y)}{3x+y} \quad [5]$$

- (ii) Hence find the coordinates of the stationary points on the curve

$$x^2 + 6xy + y^2 + 32 = 0 \quad [6]$$

- 5 Solve the differential equation

$$\cos^2 4x \frac{dy}{dx} = y$$

given that  $y = e^3$  when  $x = \frac{\pi}{16}$  [8]

- 6 (i) Find the vector equation of the line  $l$  through the points  $(2, 4, 3)$  and  $(1, 2, 3)$ . [4]

The point  $P$  lies on the line  $l$ .  
 $O$  is the point  $(0, 0, 0)$ .

- (ii) Hence, using (i), write down the position vector  $\overrightarrow{OP}$  [1]

$OP$  is perpendicular to the line  $l$ .

- (iii) Find the coordinates of the point  $P$ . [6]

- 7 Find the **exact** value of

$$\int_1^3 \ln x \, dx \quad [7]$$

- 8 (i) Prove the identity

$$\tan 3\theta \equiv \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \quad [7]$$

- (ii) Hence solve the equation

$$\tan 3\theta = \tan \theta \quad 0^\circ \leq \theta \leq 360^\circ \quad [5]$$

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**THIS IS THE END OF THE QUESTION PAPER**

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