



Rewarding Learning

ADVANCED
General Certificate of Education
January 2012

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]



WEDNESDAY 1 FEBRUARY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all seven** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$



Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 (i) Show that the sum of the series

$$\frac{5}{1 \times 2 \times 3} + \frac{8}{2 \times 3 \times 4} + \frac{11}{3 \times 4 \times 5} + \dots + \frac{3n+2}{n(n+1)(n+2)}$$

is given by

$$2 - \frac{1}{n+1} - \frac{2}{n+2} \quad [8]$$

(ii) Hence find the sum of the series

$$\frac{5}{1 \times 2 \times 3} + \frac{8}{2 \times 3 \times 4} + \frac{11}{3 \times 4 \times 5} + \dots \quad [1]$$

2 Find the first 2 non-zero terms of the Maclaurin series for $\tan^{-1}(x)$. [6]

3 (a) Find the equation of an ellipse with focus F(4, 0) and associated directrix $x = 6.25$ [6]

(b) Find the equation of the parabola with focus F(4, 1) and directrix $x = -1$ [4]

4 Find the general solution of the equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = 3 \cos 2x \quad [12]$$

5 (i) Using partial fractions show that

$$\frac{3}{(1+2x^2)(1-2x)} \equiv \frac{2x+1}{1+2x^2} + \frac{2}{1-2x} \quad [6]$$

(ii) Hence find a series expansion for

$$\frac{3}{(1+2x^2)(1-2x)}$$

up to and including the term in x^4 [5]

(iii) State the range of values of x for which the expansion is valid. [3]

6 Use the principle of mathematical induction to show that for all non-negative integers n

$$\frac{d^n}{dx^n} \left[e^x \cos \sqrt{3}x \right] = 2^n e^x \cos \left(\sqrt{3}x + \frac{n\pi}{3} \right) \quad [8]$$

$\left[\text{Note that } \frac{d^0}{dx^0} f(x) = f(x) \right]$

7 (i) If $Z = e^{i\theta}$ show that

$$\frac{1}{2} \left(Z^n + \frac{1}{Z^n} \right) \equiv \cos n\theta \quad [3]$$

(ii) Given that

$$\cos^6 \theta \equiv a \cos 6\theta + b \cos 4\theta + c \cos 2\theta + d$$

find a , b , c and d . [6]

(iii) Hence, find, in radians, the general solution to the equation

$$\frac{1}{8} \cos 6\theta + \frac{3}{4} \cos 4\theta + \frac{15}{8} \cos 2\theta + \frac{5}{4} = \frac{1}{2} \quad [7]$$

THIS IS THE END OF THE QUESTION PAPER
