



Rewarding Learning

ADVANCED

General Certificate of Education

2011

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

WEDNESDAY 1 JUNE, MORNING

**MARK
SCHEME**

1 (i) $\vec{AB} = \vec{AO} + \vec{OB} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{i} + 6\mathbf{j}$
 $= -\mathbf{i} + 7\mathbf{j}$

M1W1

(ii) $|\vec{AB}| = \sqrt{(-1)^2 + (7)^2} = \sqrt{50} = 5\sqrt{2}$

M1W1

(iii) $\vec{OA} \cdot \vec{OB} = (3\mathbf{i} - \mathbf{j}) \cdot (2\mathbf{i} + 6\mathbf{j})$

$= 6 - 6$

M1

$= 0$

W1

(iv) 90°

MW1

7

2 (i) $x + xy - 12$

$1 + y + \frac{dy}{dx} x$

M2W2

(ii) at (2, 5)

$1 + 5 + 2 \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -3$

MW1

Eqn: tangent $y - 5 = -3(x - 2)$

M1

$y = -3x + 11$

W1

$y + 3x = 11$

7

3 (i) $2 \cos x + 4 \sin x = R \cos (x - \alpha)$

$$\cos \alpha = 2/R$$

$$\sin \alpha = 4/R$$

$$\tan \alpha = 2 \Rightarrow \alpha = 63.43^\circ$$

M1W1

$$= 63.4^\circ \text{ (3.s.f.)}$$

$$R = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$$

M1W1

$$2\sqrt{5} \cos (x - 63.4^\circ)$$

(ii) $2 \cos x + 4 \sin x = 3$

$$\sqrt{20} \cos (x - 63.4^\circ) = 3$$

M1

$$\cos (x - 63.4^\circ) = \frac{3}{\sqrt{20}}$$

$$x - 63.4^\circ = 47.8695^\circ \text{ or } 312.13^\circ$$

MW2

$$x = 111^\circ \text{ or } 15.6^\circ$$

W2

9

4 (i) Volume = $\int_{-12}^3 \pi y^2 \, dx$

M2

$$= \int_{-12}^3 \pi (144 - x^2) \, dx$$

MW1

$$= \pi \left[144x - \frac{x^3}{3} \right]_{-12}^3$$

MW1

$$= \pi \left[(432 - 9) - (-1728 + 576) \right]$$

M1

$$= 1575 \pi \text{ cm}^3$$

W2

(ii) The bowl needs a flat bottom

MW1

8

5 (i) $\sin(A+B) = \sin A \cos B + \cos A \sin B$ M1

$$\sin(A+A) = \sin A \cos A + \cos A \sin A$$
 M1

$$= 2 \sin A \cos A$$
 W1

(ii) LHS $\tan A + \cot A$

$$\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$
 M1W1

$$\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$
 MW1

$$\frac{1}{\frac{1}{2} \sin 2A}$$
 M2

$$\frac{2}{\sin 2A} = \text{R.H.S.}$$
 W1

6 $\frac{dx}{dt} = k(3-x)(4-x)$

$$\int_0^2 \frac{dx}{(3-x)(4-x)} = \int_0^{10} k dt$$
 M2W2

$$\frac{1}{(3-x)(4-x)} = \frac{A}{3-x} + \frac{B}{4-x}$$
 MW1

$$1 = A(4-x) + B(3-x)$$
 MW1

Let $x = 4$ $1 = -B \Rightarrow B = -1$ MW2

$x = 3$ $1 = A$

$$\frac{1}{3-x} - \frac{1}{4-x}$$

$$\int_0^2 \left(\frac{1}{3-x} - \frac{1}{4-x} \right) dx = \int_0^{10} k dt$$
 MW1

$$\left[-\ln |3-x| + \ln |4-x| \right]_0^2 = \left[kt \right]_0^{10}$$

W3

$$\left[\ln \left| \frac{4-x}{3-x} \right| \right]_0^2 = 10k$$

$$\ln 2 - \ln \frac{4}{3} = 10k$$

$$k = \frac{1}{10} \ln \frac{3}{2}$$

W2

$$[k = 0.0405]$$

14

7 (i) $x=1$ $y=2$

MW2

(ii) $f : x \rightarrow \frac{2x+3}{x-1}$

Let $y = \frac{2x+3}{x-1}$

M1

$$y(x-1) = 2x+3$$

M1

$$yx - 2x = y + 3$$

$$x = \frac{y+3}{y-2}$$

M1W1

$$f^{-1} : x \rightarrow \frac{x+3}{x-2}$$

W1

domain $x > 2$

MW1

8

$$8 \quad (i) \quad \int \underset{\substack{\uparrow \\ \text{int}}}{x} \ln \underset{\substack{\uparrow \\ \text{diff}}}{x} dx$$

M1

$$= \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \frac{1}{x} dx$$

W3

$$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + c$$

W2

$$(ii) \quad (a) \quad \int x \ln x^2 dx$$

$$= \int 2x \ln x dx$$

M1W1

$$= x^2 \ln x - \frac{x^2}{2} + d$$

MW1

$$(b) \quad \int x \ln 3x dx$$

$$= \int x (\ln 3 + \ln x) dx$$

M1W1

$$= \int x \ln 3 + x \ln x dx$$

$$= \frac{x^2}{2} \ln 3 + \frac{x^2}{2} \ln x - \frac{x^2}{4} + g$$

MW2

13

Total

75