Rewarding Learning

ADVANCED
General Certificate of Education
2011

Mathematics

Assessment Unit S4

assessing

Module S2: Statistics 2

[AMS41]

WEDNESDAY 22 JUNE, MORNING

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for correct working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidates's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

			AVAILABLE MARKS
1	– Assign 2-digit numbers to each of the 62 pupils – say 00 to 61.	M1	
	– Use random No. table to read off numbers 2 digits at a time – start anywhere and move in any direction.	M1	
	– Nos. outside the range of assigned numbers are ignored.	M1	
	– Nos. that are duplicated are ignored.	M1	
	– When six valid two digit numbers have been selected then stop. Those are the lucky few!	M1	5

2 **(i)**

$$r = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)\left(\sum y^2 - \frac{(\sum y)^2}{n}\right)}}$$ M1

$$= \frac{47775 - \frac{674 \times 696}{10}}{\sqrt{\left(46542 - \frac{674^2}{10}\right)\left(49280 - \frac{696^2}{10}\right)}}$$ W1 W2

$$= 0.89447\ldots = 0.894 \text{ (3 s.f.)}$$ W1

(ii) There appears to be a strong positive correlation between performances in History and in English. M2 — 7

3 **(i)** $y = a + bx$

$$b = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}$$ M1

$$= \frac{14500 - \frac{150 \times 502}{5}}{5500 - \frac{150^2}{5}}$$ W1

$$= -0.56$$ W1

$$a = \bar{y} - b\bar{x}$$ M1

$$= \frac{502}{5} - (-0.56) \times \frac{150}{5}$$ W1

$$= -117.2$$ W1

$$y = 117.2 - 0.56x$$

(ii) $x = 25$ $\quad \hat{y} = 117.2 - 0.56 \times 25$ M1

$$= 103.2 \text{(N)}$$ W1

$$= 103 \text{ (3 s.f.)}$$

(iii) The value 65 lies outside the range of temperatures in the test. M1 — 9

			AVAILABLE MARKS
4	$\text{CI} = \bar{x} \pm 1.96\sqrt{\dfrac{\hat{\sigma}^2}{n}}$		
	$\bar{x} = \dfrac{1996}{80} = 24.95$	M1 1.96 MW1 MW1	
	$\hat{\sigma}^2 = \dfrac{1}{79}\left(49975 - \dfrac{1996^2}{80}\right) = 2.21...$	M1 W1	
	$\text{CI} = 24.95 \pm 1.96\sqrt{\dfrac{2.21...}{80}}$ $= (24.62404..., 25.27596...)$ $= (24.6, 25.3)$ (3 s.f.)	W2	7
5 **(i)**	$\bar{x} = \dfrac{399.2}{8} = 49.9$	MW1	
	$\hat{\sigma}^2 = \dfrac{1}{7}\left(19925.12 - \dfrac{399.2^2}{8}\right)$	M1	
	$= 0.72$	W1	
(ii)	$H_0: \mu = 50$	M1	
	$H_1: \mu < 50$	M1	
	One-tailed test		
	t-test with 7 degrees of freedom	M2	
	$t_{crit} = -1.895$	MW1	
	$t_{test} = \dfrac{49.9 - 50}{\sqrt{\dfrac{0.72}{8}}}$	MW1 MW1	
	$= -0.\dot{3}$	W1	
	As $t_{test} > t_{crit}$ we do not reject H_0.	M1	
	There is insufficient evidence at 5% to suggest that the amount of aftershave in the bottles is less than 50 mls on average.	M1	13

AVAILABLE MARKS

6 **(i)** An alternative hypothesis is a statement declaring the situation believed to be true should the null hypothesis not be true. M2

(ii) $\bar{x} = \dfrac{1285}{50} = 25.7$ MW1

$\hat{\sigma} = \sqrt{\dfrac{1}{49}\left(33183 - \dfrac{1285^2}{50}\right)} = 1.798\ldots$ MW1

$= 1.80\ (3\text{ s.f.})$

H_0: $\mu = 25$ M1

H_1: $\mu \neq 25$ M1

2 tailed test

$|z_{\text{crit}}| = 1.96$ MW1

$z_{\text{test}} = \dfrac{25.7 - 25}{\dfrac{1.80}{\sqrt{50}}}$ W1 W1

$= 2.75$ W1

Since $|z_{\text{test}}| > |z_{\text{crit}}|$ we reject H_0 in favour of H_1. M1

There is sufficient evidence at 5% level to suggest that the average television viewing time per week is not 25 hours. M1 — 12

7 Let X be mass of a sweet

$X \sim N(6, 0.9^2)$

Let Y be mass of a wrapper

$Y \sim N(0.2, 0.02^2)$

Let T be total mass in a tube of sweets

$T = X_1 + X_2 + \ldots + X_{10} + Y_1 + Y_2 + \ldots + Y_{10}$ M1

$E(T) = 10 \times 6 + 10 \times 0.2 = 62$ MW1

$\text{Var}(T) = 10 \times 0.9^2 + 10 \times 0.02^2 = 8.104$ M1 W1

$T \sim N(62, 8.104)$

$P(60 < T < 65) = P\left(\dfrac{60-62}{\sqrt{8.104}} < z < \dfrac{65-62}{\sqrt{8.104}}\right)$

$= P(-0.703 < z < 1.054)$ M1 W1

$= \Phi(1.054) - \Phi(-0.703)$

$= \Phi(1.054) + \Phi(0.703) - 1$ M1

$= 0.8540 + 0.7589 - 1$ W2

$= 0.6129 = 0.613\ (3\text{ s.f.})$ W1 — 10

		AVAILABLE MARKS
8 **(i)** Type – Normal Mean – 502 ml Variance – $\frac{18}{6} = 3\,\text{ml}^2$ $\bar{X}_6 \sim \text{N}(502, 3)$	MW3	
(ii) $\text{P}(\bar{X}_6 < 500) = \text{P}\left(z < \frac{500-502}{\sqrt{3}}\right)$		
$= \text{P}(Z < -1.155)$	MW1	
$= 1 - \text{P}(Z < 1.155)$	M1	
$= 1 - \Phi(1.155)$		
$= 1 - 0.8759$	W1	
$= 0.1241$	W1	
$= 0.124$ (3 s.f.)		
(iii) Let T be r.v. "No. of packs with average contents less than 500 ml"		
$T \sim \text{B}(8, 0.1241)$	MW1	
$\text{P}(T \geqslant 2) = 1 - [\text{P}(T = 0) + \text{P}(T = 1)]$	M1	
$\text{P}(T = 0) = 0.8759^8 = 0.346447$		
$\text{P}(T = 1) = 8 \times 0.8759^7 \times 0.1241 = 0.392684$	M1 W1	
$1 - [0.346447 + 0.392684]$		
$= 0.260869 = 0.261$ (3 s.f.)	W1	12
	Total	**75**