



Rewarding Learning

ADVANCED
General Certificate of Education
2011

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

[AMM41]

WEDNESDAY 22 JUNE, MORNING



TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all six** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take $g = 9.8 \text{ m s}^{-2}$, unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$



6205.02R

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

- 1** A uniform triangular lamina OAB is bounded by the x -axis, the line $x = a$ and the line $y = kx$. O is the origin and A is the point $(a, 0)$. The centre of mass of the lamina is G as shown in **Fig. 1** below.

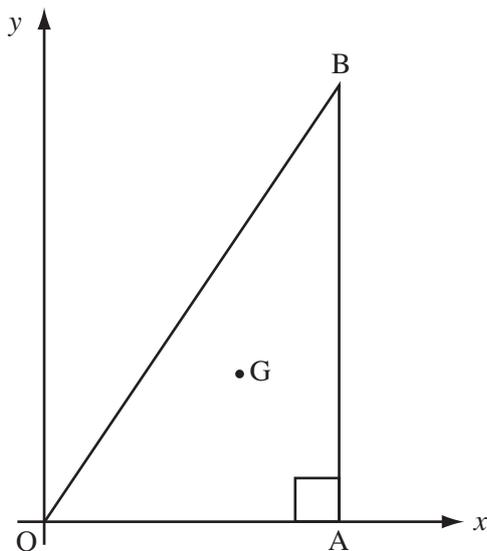


Fig. 1

The mass per unit area of the lamina is σ .

- (i) Find in terms of a , k , and σ , the mass of the lamina and by using integration, its moments about the x - and y -axes. [8]
- (ii) Find the coordinates of G. [2]
- (iii) Verify that for this triangular lamina the coordinates of G can be found by summing the coordinates of the three vertices and dividing by three. [2]

- 2 The star-ship FANTASIA has a proton beam engine. The zoomeration, Z , of the star-ship is believed to depend on the power, P , speed, S and density, D , of the beam. The dimensions of these quantities are shown in the table below.

Quantity	Dimension
Z	$[L^3][T^{-4}]$
P	$[M][L^{-2}][T]$
S	$[M^2][T^{-2}]$
D	$[M^{-1}][L^3][T^{-1}]$

Boris Blastov conjectures that

$$Z = kP^a S^b D^c$$

where k is a dimensionless constant.

- (i) Use the method of dimensions to find a , b and c . [8]

During a trial of the engine the following measurements were made in S.I. units:

$$Z = 1.2 \times 10^{11}, P = 2 \times 10^2, S = 3 \times 10^9, D = 5 \times 10^3$$

- (ii) Show that $k = 1.6 \times 10^{12}$ [3]

During further trials, when P was doubled Z increased by a factor of 5

- (iii) State whether or not Boris's conjecture could be correct. Briefly explain your answer. [2]

- 3 Three spheres A, B, and C of equal radius and masses $4m$, $2m$ and m respectively are at rest in a straight line on a smooth horizontal surface. The coefficient of restitution between each pair of spheres is 0.5

A is projected towards B with speed u and collides directly with it.

After the collision A and B move off in the same direction, B with speed w and A with speed v .

- (i) Show that $w = u$ and find v . [6]

A collision now occurs between B and C.

- (ii) Find the speeds of B and C after this collision. [4]

- (iii) State if any further collisions will occur, briefly explaining your answer. [1]

- 4 In a distant galaxy, the planet Fido has two moons, Ria and Via.

The masses of the planet and its moons are M , m_1 and m_2 respectively.

The orbit of Ria has a radius r and the coplanar orbit of Via has a radius $2.08r$

The universal gravitational constant is G .

Ria moves around its orbit at speed v_1 and Via moves at v_2

- (i) Show that $v_1 = 1.44v_2$ approximately. [5]

The angular velocities of Ria and Via are ω_1 and ω_2 respectively.

- (ii) By using the result in (i), or otherwise, show that $\omega_1 = 3\omega_2$ approximately. [3]

The period of Via is 63 days.

- (iii) Find the period of Ria. [3]

- 5 The “Dip” is part of a competitive cycle trail where competitors must keep their bikes in contact with the track at all times.

Fig. 2 below shows the vertical section along the “Dip”.

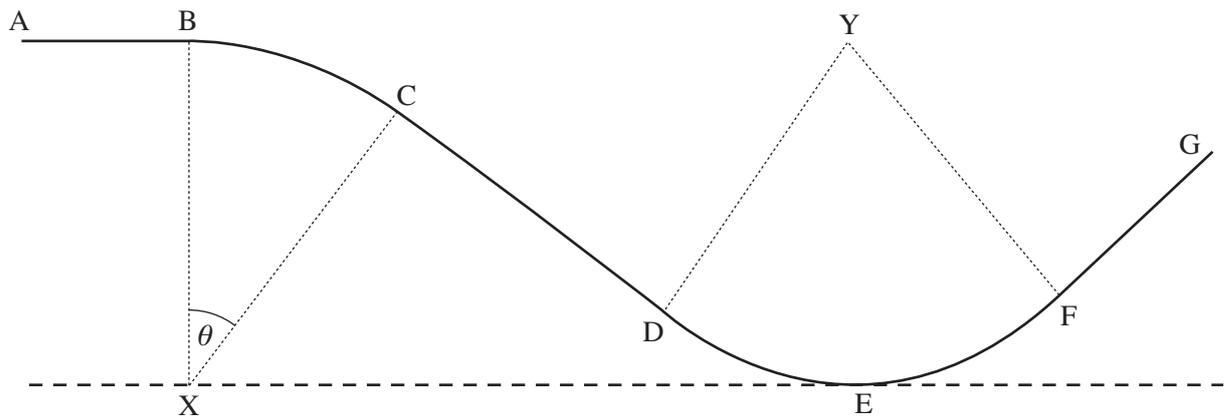


Fig. 2

There is a horizontal approach AB, a circular arc BC, centre X, a straight incline CD and another circular arc DEF, centre Y, followed by an upward incline FG.

Both arcs are of radius 5 m. BC subtends θ° at its centre X where $\cos \theta = 0.8$

A, B and Y are at the same horizontal level.

E is 5 m vertically below the level of A and B.

Decla approaches the “Dip” along AB reaching B with a speed of 4 m s^{-1}

Model Decla and her bike as a particle of mass 70 kg.

- (i) Find the force exerted by the track on the bike and rider at E. [7]

At C, Decla and her bike experience a normal reaction, N newtons, from the track.

- (ii) Find N . [6]

- (iii) Explain briefly why the bike remains in contact with the track over the arc BC. [2]

- (iv) State what further modelling assumption you have made in answering this question. [1]

6 The triangle ABC has sides:

$$\begin{aligned} AB &= 0.6 \text{ m} \\ BC &= 0.8 \text{ m} \\ AC &= 1.0 \text{ m} \end{aligned}$$

D is the point on AC such that BD is perpendicular to AC.

Forces of 12 N, 9 N and 8 N act along the sides BA, BC and AC respectively as shown in **Fig. 3** below.

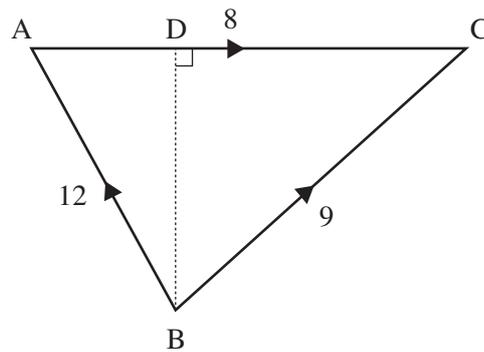


Fig. 3

The resultant of these forces is R .

(i) Show that R passes through D and find its magnitude.

[7]

The forces are now replaced and 3 N acts along BA, 5 N along AC and 4 N along CB as shown in **Fig. 4** below.

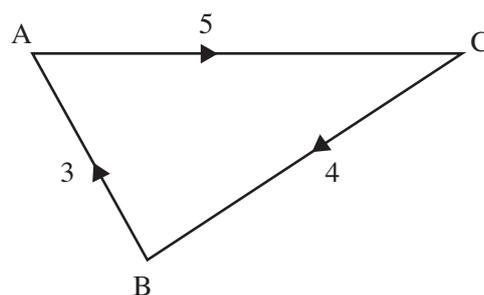


Fig. 4

(ii) Show that this system reduces to a couple and find its moment.

[5]

THIS IS THE END OF THE QUESTION PAPER

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