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ADVANCED
General Certificate of Education
2011

Mathematics

Assessment Unit M3

assessing

Module M3: Mechanics 3

[AMM31]

AMM31

MONDAY 20 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A particle P of mass 1 kg moves from the origin O to a point A through a zero gravity zone. The displacement of A from O is **s** metres where

$$\mathbf{s} = 6\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$$

P is acted on by two forces $\mathbf{F_1}$ newtons and $\mathbf{F_2}$ newtons where

$$\mathbf{F_1} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \qquad \mathbf{F_2} = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$$

W_1 is the work done by $\mathbf{F_1}$ in moving the particle from O to A and W_2 is the work done by $\mathbf{F_2}$ in moving the particle from O to A.

(i) Find **(a)** W_1

(b) W_2 [4]

P passes through O when $t = 0$ s with initial velocity **u** m s^{-1} where

$$\mathbf{u} = \begin{pmatrix} 6 \\ 2 \\ -4 \end{pmatrix}$$

and reaches A when $t = 2$ s.

(ii) Find the **velocity** of P at A. [4]

(iii) Verify that the total work done on the particle is 84 J and that this satisfies the Work–Energy Principle. [4]

2 [Take **i** to be a unit vector direction East and **j** to be a unit vector direction North.]

A burglar is running along a straight path with velocity $(4\mathbf{i} + 3\mathbf{j})\,\mathrm{m\,s^{-1}}$
A police sergeant is 120 m East and 80 m South of the burglar's position when she spots him.
At time $t = 0$ s she starts running with velocity $(\mathbf{i} + 5\mathbf{j})\,\mathrm{m\,s^{-1}}$

(i) Find the velocity of the sergeant relative to the burglar. [3]

(ii) Show that the sergeant will catch the burglar and find the time at which this will occur. [6]

3 An elastic string is attached to a fixed point A and hangs vertically in equilibrium supporting a particle of mass 0.1 kg.
The string is of natural length 0.5 m and modulus of elasticity 1.8 N. The extension in the string is $\frac{49}{180}$ m.
Initially the particle is pulled vertically downwards a further distance of 0.2 m and released from rest.
After t seconds the particle is x metres below the equilibrium position as shown in **Fig. 1** below.

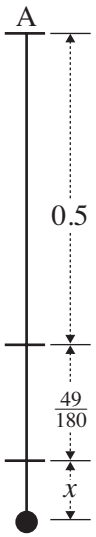

Fig. 1

(i) Show that the equation of motion of the particle is

$$\ddot{x} = -36x$$

and that this represents S.H.M. [6]

(ii) Find the value of t when the particle is first 0.1 m above the equilibrium position. [7]

4 A variable force

$$F = 15 + 12x - 3x^2$$

acts on a particle P of mass $\frac{8}{9}$ kg as it moves along a smooth, straight, horizontal track, where x is P's distance from a fixed point O.
Fig. 2 below shows the graph of F.

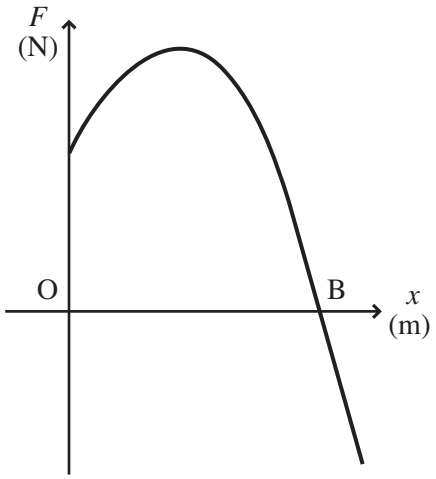

Fig. 2

The graph crosses the x-axis at B.

(i) Find the work done by F as it moves the particle from O to B. [7]

The particle passes through O with a speed of $8\,\text{m}\,\text{s}^{-1}$

(ii) Use the Work–Energy Principle to find P's maximum speed. [4]

5 **Fig. 3** below shows a circular steel plate that forms one end of a storage cylinder.
The plate has a radius of 0.5 m.
Its centre is at C where the x and y-axes meet.
Two discs to make holes for inspection hatches are cut in the plate.
One disc of radius 0.1 m centre A (0, 0.3).
The other disc of radius 0.2 m centre B (0.2, –0.2).

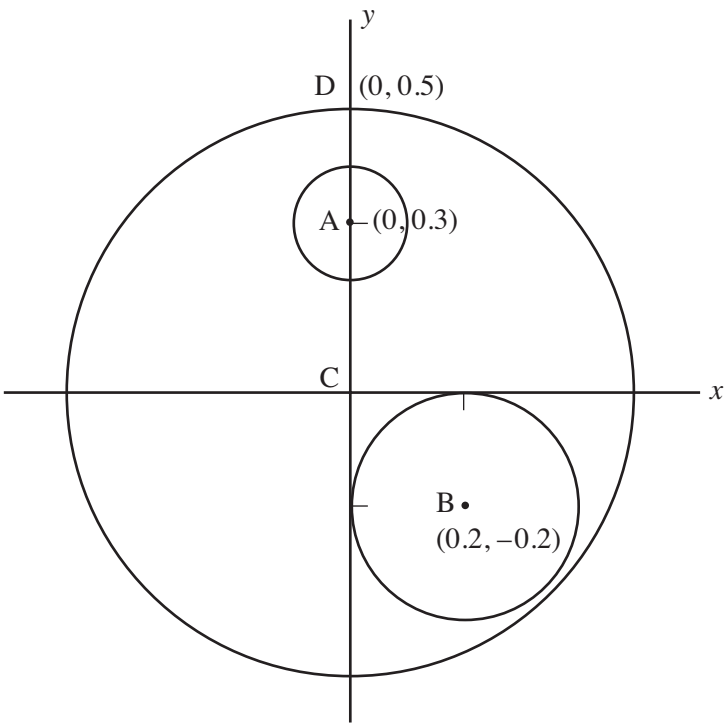

Fig. 3

Model this plate as a lamina of uniform density.
The mass of steel removed from the smaller hole is m kg.

(i) Show that when both discs have been removed from the plate its mass is then $20m$ kg. [3]

The centre of mass of the plate is now at G $(\bar{x}, \bar{y})$.

(ii) Show that $\bar{x} = -0.04$ m and find $\bar{y}$. [8]

The plate has an actual mass of 20 kg and rests on rough ground with A vertically above C.
It is kept in equilibrium by a force P acting at D (0, 0.5) and parallel to the x-axis.

(iii) Find P. [4]

6 A particle B of mass m kg is attached to two springs S_1 and S_2 whose other ends are attached to two fixed points A and C respectively.
A is at the bottom and C at the top of a line of greatest slope of a smooth plane inclined at α to the horizontal as shown in **Fig. 4** below.

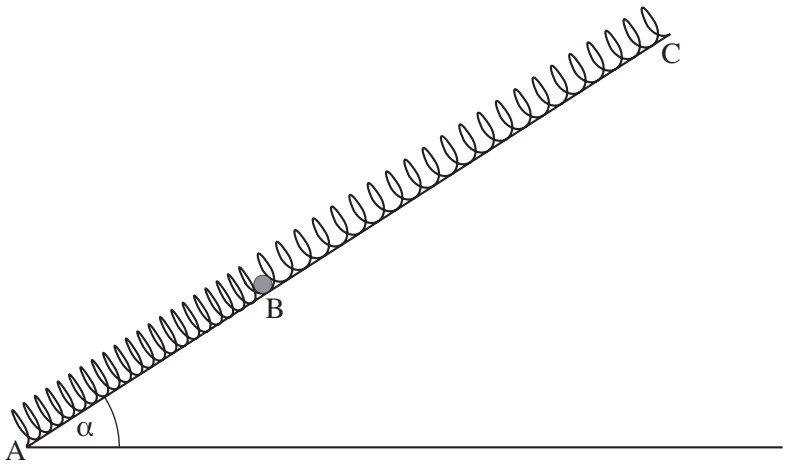

Fig. 4

Both springs are of natural length l metres and modulus of elasticity $0.5mg$ newtons.
$AC = 2.5l$ metres
The system rests in equilibrium with S_1 extended x metres and S_2 extended y metres.

(i) Show that $x + y = 0.5l$ [2]

(ii) By considering the forces acting on B, find x in terms of l and α. [7]

(iii) If S_1 is compressed show that

$$\sin\alpha > 0.25$$ [2]

(iv) If $\sin\alpha = 0.75$, find the elastic energy stored in S_1 in terms of l, m and g. [4]

THIS IS THE END OF THE QUESTION PAPER

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