



Rewarding Learning

ADVANCED  
General Certificate of Education  
2011

---

## Mathematics

Assessment Unit F2

*assessing*

Module FP2: Further Pure Mathematics 2

[AMF21]



TUESDAY 31 MAY, MORNING

---

### TIME

1 hour 30 minutes.

### INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all seven** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

### INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is  $\ell n z$  where it is noted that  $\ell n z \equiv \log_e z$



6231.03R

**Answer all seven questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

- 1** Find, in radians, the general solution of the equation

$$2 \sec^2 \theta - 3 \tan \theta - 1 = 0 \quad [6]$$

- 2** If  $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$ , prove by mathematical induction that

$$\mathbf{A}^n = \begin{pmatrix} 1 & 0 \\ 5n & 1 \end{pmatrix}$$

where  $n$  is a positive integer. [5]

- 3 (i)** Find the sum of the series

$$\frac{1}{1 \times 4} + \frac{1}{2 \times 5} + \frac{1}{3 \times 6} + \dots + \frac{1}{n(n+3)} \quad [8]$$

- (ii)** Hence find

$$\sum_{r=1}^{\infty} \frac{1}{r(r+3)} \quad [1]$$

- 4 (i) Find the equation of the parabola with focus (2, 2) and directrix  $x = 8$  [7]

The latus rectum of a parabola is the chord parallel to the directrix through the focus.

- (ii) Find the length of the latus rectum of the parabola in part (i). [3]

- 5 Solve the differential equation

$$\frac{dy}{dx} + y \cot x = \cos^3 x$$

given that  $y = \frac{3}{8\sqrt{2}}$  when  $x = \frac{\pi}{4}$  [10]

- 6 (i) Use Maclaurin's theorem to find the first four terms in the expansion of

$$\frac{1}{1+x}$$

where  $|x| < 1$  [4]

- (ii) Write

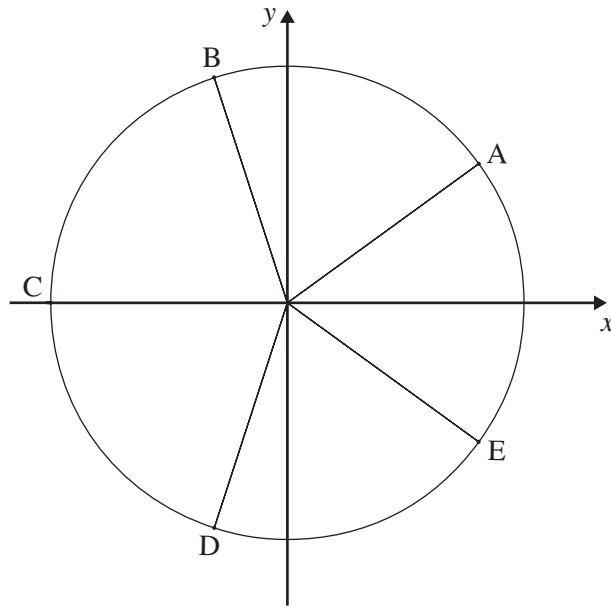
$$\frac{2x^2 + x + 27}{(9 + x^2)(1 - x)}$$

in partial fractions. [6]

- (iii) Hence, or otherwise, derive the first four terms in the expansion of

$$\frac{2x^2 + x + 27}{(9 + x^2)(1 - x)} [6]$$

- 7 (i) Find, in the form  $re^{i\theta}$ , the values of the 5 roots of the equation  $z^5 + 32 = 0$ , which are shown in **Fig. 1** below.



**Fig. 1**

[6]

- (ii) Show that a quadratic equation whose roots are A and E is given by

$$z^2 - 4z \cos \frac{\pi}{5} + 4 = 0$$

[4]

A quadratic equation whose roots are B and D is given by

$$z^2 + 4z \cos \frac{2\pi}{5} + 4 = 0$$

- (iii) Explain why

$$(z + 2) \left( z^2 + 4z \cos \frac{2\pi}{5} + 4 \right) \left( z^2 - 4z \cos \frac{\pi}{5} + 4 \right) = 0$$

is an equation with roots A, B, C, D, E.

[1]

- (iv) By comparing the coefficients of  $z^4$  in the equations in parts (iii) and (i) show that

$$\cos \frac{\pi}{5} = \frac{1 + \sqrt{5}}{4}$$

[8]