



Rewarding Learning

ADVANCED  
General Certificate of Education  
2011

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## Mathematics

Assessment Unit C4

*assessing*

Module C4: Core Mathematics 4

[AMC41]



WEDNESDAY 1 JUNE, MORNING

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### TIME

1 hour 30 minutes.

### INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all eight** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

### INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$ .



6137

**Answer all eight questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

**1** Given that the points A and B have position vectors:

$$\vec{OA} = 3\mathbf{i} - \mathbf{j}$$

$$\text{and } \vec{OB} = 2\mathbf{i} + 6\mathbf{j}$$

find:

**(i)** the vector  $\vec{AB}$ ; [2]

**(ii)** the magnitude of  $\vec{AB}$ ; [2]

**(iii)**  $\vec{OA} \cdot \vec{OB}$  [2]

**(iv)** Hence write down the angle AOB. [1]

**2 (i)** Differentiate implicitly with respect to  $x$

$$x + xy - 12 \quad [4]$$

**(ii)** Hence find the equation of the tangent to the curve

$$x + xy - 12 = 0$$

at the point (2, 5). [3]

3  $2 \cos x + 4 \sin x$  can be written in the form

$$R \cos (x - \alpha)$$

where  $\alpha$  is acute and  $R$  is real.

(i) Find  $R$  and  $\alpha$ .

[4]

(ii) Hence solve the equation

$$2 \cos x + 4 \sin x = 3$$

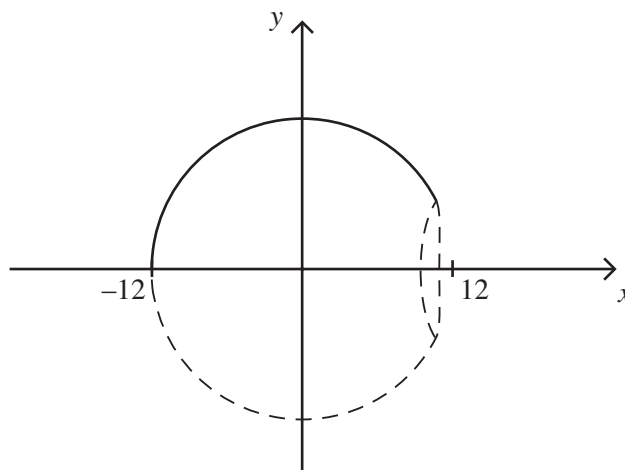
where  $0^\circ \leq x \leq 360^\circ$

[5]

4 The surface of a goldfish bowl can be modelled by part of the curve

$$y = \sqrt{144 - x^2}$$

being rotated through  $2\pi$  radians about the  $x$ -axis as shown in **Fig. 1** below.



**Fig. 1**

The radius of the bowl is 12 cm and it is to be filled to a depth of 15 cm.

(i) Find the volume of water in the bowl.

[7]

(ii) State one criticism of the model.

[1]

5 (i) Starting with the appropriate compound angle formula prove that

$$\sin 2A \equiv 2 \sin A \cos A \quad [3]$$

(ii) Show that

$$\tan A + \cot A \equiv \frac{2}{\sin 2A} \quad [6]$$

6 The amount  $x$  of a substance present in a certain chemical reaction after time  $t$  can be modelled by the differential equation

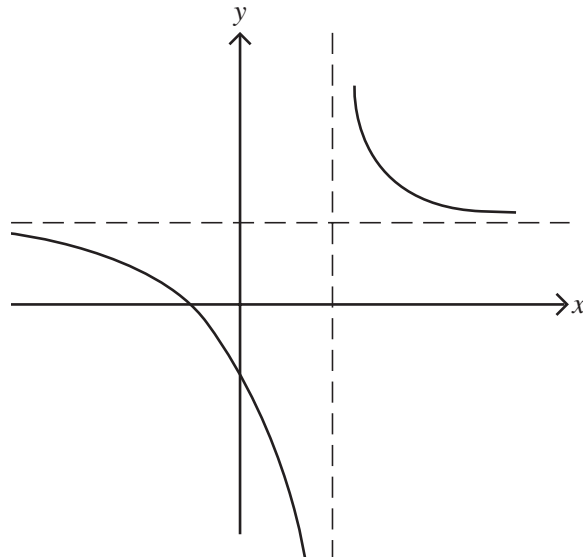
$$\frac{dx}{dt} = k(3 - x)(4 - x)$$

where  $k$  is a constant and  $x = 0$  when  $t = 0$

Given that  $x = 2$  when  $t = 10$ , find the value of  $k$ . [14]

7 **Fig. 2** below shows a sketch of the graph whose equation is

$$y = \frac{2x + 3}{x - 1}$$



**Fig. 2**

(i) Write down the equations of the asymptotes to this graph. [2]

The function  $f$ , with domain  $x > 1$ , is defined by

$$f: x \rightarrow \frac{2x + 3}{x - 1}$$

(ii) Find the inverse function  $f^{-1}$ , stating its domain. [6]

8 (i) Using integration by parts, show that

$$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + c \quad [6]$$

(ii) Using (i), find

(a)  $\int x \ln x^2 \, dx$  [3]

(b)  $\int x \ln 3x \, dx$  [4]

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**THIS IS THE END OF THE QUESTION PAPER**

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