Rewarding Learning

ADVANCED
General Certificate of Education
January 2011

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

FRIDAY 28 JANUARY, MORNING

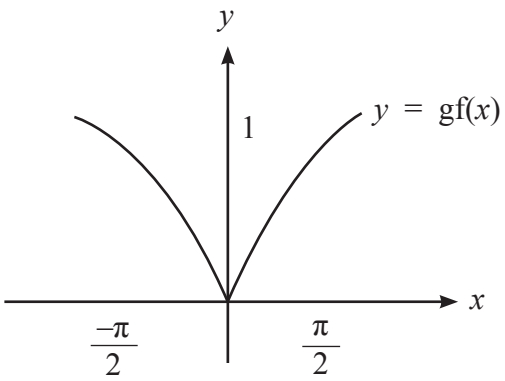
MARK SCHEME

		AVAILABLE MARKS
1 **(a)** **(i)** $\frac{1}{(x+1)(x-1)} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$	MW1	
$1 = A\ (x-1) + B\ (x+1)$	MW1	
Let $x = 1$ $\quad 1 = 2B \Rightarrow B = \frac{1}{2}$	MW1	
$x = -1 \quad 1 = 2A \Rightarrow A = -\frac{1}{2}$	W1	
(ii) $\frac{1}{2}\int \frac{1}{(x-1)} - \frac{1}{(x+1)}\ dx$	MW1	
$= \frac{1}{2}\ln \mid x-1 \mid - \frac{1}{2}\ln \mid x+1 \mid + c$	MW3	
(b) $\int x \cos x \ dx$		
$= x \sin x - \int \sin x\ dx$	M1W2	
$= x \sin x + \cos x + c$	W2	13

			AVAILABLE MARKS
2	Using dot product		
	$\mathbf{p} \bullet \mathbf{q} = \lvert \mathbf{p} \rvert \lvert \mathbf{q} \rvert \cos\theta$	M1	
	$(\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \bullet (4\mathbf{i} + 5\mathbf{j} + \mathbf{k})$	M1	
	$= 4 - 5 - 2 = -3$	W1	
	$\lvert \mathbf{i} - \mathbf{j} - 2\mathbf{k} \rvert = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$	M1W1	
	$\lvert 4\mathbf{i} + 5\mathbf{j} + \mathbf{k} \rvert = \sqrt{4^2 + 5^2 + 1^2} = \sqrt{42}$	MW1	
	$-3 = \sqrt{6}\ \sqrt{42}\ \cos\theta$ $\dfrac{-3}{\sqrt{252}} = \cos\theta$		
	$\theta = 100.89°$ $(101°)$	W1	7
3	$\cos x = 2[\sin x \cos 60° + \cos x \sin 60°]$	M1W1	
	$\cos x = 2\left[\sin x \dfrac{1}{2} + \cos x \sqrt{\dfrac{3}{2}}\right]$	MW1	
	$\cos x = \sin x + \sqrt{3}\cos x$ $\left(1 - \sqrt{3}\right)\cos x = \sin x$		
	$1 - \sqrt{3} = \tan x$	M1W1	
	$x = -36.2°$ or $144°$	MW2	7

		AVAILABLE MARKS
4 (a) (i) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$	M1	
$1 + x^2 = y^2$	W1	
(ii) $y^2 = 1 + x^2$		
$2y\ \frac{dy}{dx} = 2x$	M2W2	
$\frac{dy}{dx} = \frac{x}{y}$		
$y^2 = 1 + 1$		
$= 2$		
$y = \pm\sqrt{2}$	MW2	
$\frac{dy}{dx} = \frac{1}{\pm\sqrt{2}}$	MW1	9
5 (i) LHS $\tan x \sec^4 x = \tan x \sec^2 x \sec^2 x$	MW1	
$= \tan x\,(1 + \tan^2 x)\sec^2 x$	M1	
$= \tan x \sec^2 x + \tan^3 x \sec^2 x$	W1	
$=$ RHS		
(ii) $\int_0^{\frac{\pi}{4}} \tan x \sec^2 x + \tan^3 x \sec^2 x\ dx$	MW1	
$u = \tan x\ \frac{du}{dx} = \sec^2 x$	MW1	
$\int_0^{\frac{\pi}{4}} (\tan x + \tan^3 x)\sec^2 x\ dx$		
$\int (u + u^3)\sec^2 x\ \frac{du}{\sec^2 x}$	M1W1	
$= \left[\frac{u^2}{2} + \frac{u^4}{4}\right]$	MW1	
$= \left[\frac{\tan^2 x}{2} + \frac{\tan^4 x}{4}\right]_0^{\frac{\pi}{4}}$	M1	
$= \frac{3}{4}$	MW1	10

			AVAILABLE MARKS
6	$(1+x^2)\,\frac{dy}{dx} = x\,(1+y)$		
	$\int \frac{dy}{1+y} = \int \frac{x\,dx}{(1+x^2)}$	M2W1	
	$\ln \mid 1+y \mid = \frac{1}{2} \ln \mid 1+x^2 \mid + c$	W2	
	$x = 0$ when $y = 0 \;\therefore\; c = 0$	M1W1	
	$\ln \mid 1+y \mid = \frac{1}{2} \ln \mid 1+x^2 \mid$		
	$1+y = \sqrt{1+x^2}$	M2W1	
	$y = \sqrt{1+x^2} - 1$	W1	11
7	Volume $= \int_0^9 \pi y^2 \, dx$	M2W1	
	$= \pi \int_0^9 e^{\frac{x}{3}} \, dx$	W1	
	$= \pi \left[3e^{\frac{x}{3}} \, dx \right]_0^9$	MW1	
	$= 3\pi (e^3 - 1)$	W2	7

		AVAILABLE MARKS
8 **(i)** $y = \sin x$		
$x = \sin^{-1} y$	MW1	
$f^{-1} : x \rightarrow \sin^{-1} x$	MW1	
domain $-1 \leq x \leq 1$	W1	
range $-\frac{\pi}{2} \leq f^{-1}(x) \leq \frac{\pi}{2}$	W1	
(ii) $x \rightarrow \sin x \rightarrow \lvert \sin x \rvert$	M1W1	
$gf : x \rightarrow \lvert \sin x \rvert$		
range $0 \leq gf(x) \leq 1$	W2	
(iii) 	MW3	11
Total		**75**