Rewarding Learning

ADVANCED
General Certificate of Education
January 2011

Mathematics

Assessment Unit M2

assessing

Module M2: Mechanics 2

[AMM21]

MONDAY 31 JANUARY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 At time $t = 0$ seconds a particle P is passing through a fixed point O with a velocity of $(8\mathbf{i} - 2\mathbf{j})\ \mathrm{m\,s^{-1}}$
P has a **constant** acceleration of

$$(2\mathbf{i} - 4\mathbf{k})\ \mathrm{m\,s^{-2}}$$

for $0 \le t \le 4$

(i) Find the velocity of P when $t = 4$ [3]

When $t > 4$ seconds the acceleration of P is given by

$$(t\mathbf{i} + 8t^{-2}\,\mathbf{j} - 4\mathbf{k})\ \mathrm{m\,s^{-2}}$$

(ii) Find the velocity of P when $t = 8$ [6]

2 One end of a light inextensible string of length L metres is attached to a fixed point C.
A small brass ball, B, of mass 1.5 kg is attached to the other end of the string.
B moves in a horizontal circle with constant angular velocity 5 rad $\mathrm{s^{-1}}$ as shown in **Fig. 1** below.

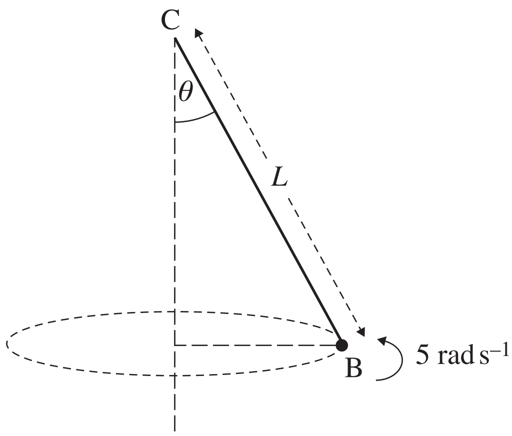

Fig. 1

The tension in the string is 20 N.
The string makes an angle θ with the downward vertical.

(i) Find θ. [3]

(ii) Find L. [6]

3 Fred, mass 80 kg, uses a smooth zip line to cross a river as shown in **Fig. 2** below. In doing so he drops through a vertical distance of h metres. He lands on the other side of the river with a speed of $16\,\text{m}\,\text{s}^{-1}$

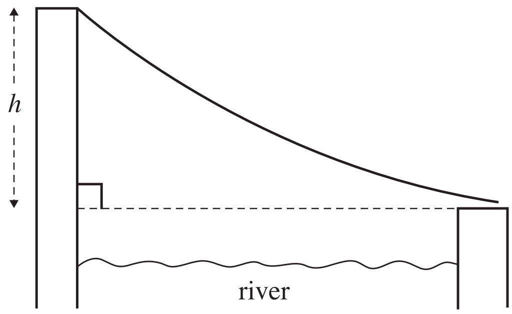

Fig. 2

(i) Find Fred's kinetic energy on landing. [2]

(ii) **Hence** find h. [6]

(iii) State one modelling assumption you have made when answering this question. [1]

4 A lorry of mass 15 tonnes is travelling along a straight horizontal road.
The lorry has a constant speed of $16\,\text{m}\,\text{s}^{-1}$ and the driving force being developed by its engine is 15 625 N.
There is a constant resistance to motion of R newtons.
Model the lorry as a particle.

(i) Find R. [3]

The lorry now **ascends** a hill which is inclined at 3° to the horizontal as shown in **Fig. 3** below. The resistance to motion remains unchanged.

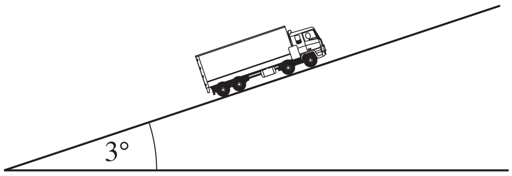

Fig. 3

(ii) Draw a diagram showing the external forces acting on the lorry. [2]

When the lorry is accelerating at $0.1\,\text{m}\,\text{s}^{-2}$ it has speed $10\,\text{m}\,\text{s}^{-1}$

(iii) Find the power now being developed by the lorry's engine. [7]

5 A lobster pot, mass 20 kg, is placed on the surface of the sea.
When the lobster pot has dropped x metres vertically through the water its speed is $v\,\text{m}\,\text{s}^{-1}$
The lobster pot experiences an upward resistance of $2v^2$ newtons throughout its motion.

(i) Show that the equation of motion of the lobster pot may be described by the differential equation

$$v\frac{dv}{dx} = \frac{98 - v^2}{10}$$ [4]

When the lobster pot has dropped a distance S metres its speed is 6 m s^{-1}

(ii) Find S. [8]

6 **[Take g = 10 m s^{-2} in this question]**
A ball is kicked, with speed 15 m s^{-1}, from a point O on horizontal ground.
The angle of projection is θ, where $\sin\theta = 0.6$, above the horizontal.
A vertical wall is set at right angles to the plane of the trajectory of the ball and is 15 m from O as shown in **Fig. 4** below.

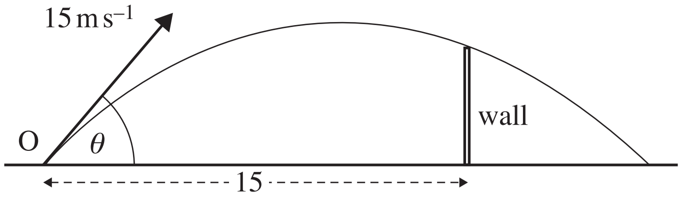

Fig. 4

The ball just clears the wall.

(i) Find the time taken for the ball to reach the wall. [3]

(ii) Find the height of the wall. [3]

(iii) Find the speed of the ball as it clears the wall. [5]

7 **[Take g = 10 m s^{-2} in this question]**
A car, mass *m* kilograms, climbs a hill 500 m long.
The top of the hill is 25 m vertically above the horizontal level at the bottom of the hill as shown in **Fig. 5** below.

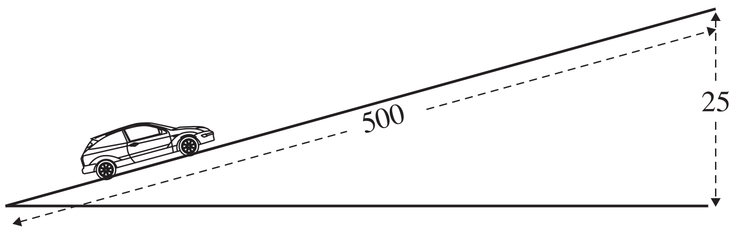

Fig. 5

The car's engine exerts a constant force of 8 kN.
The coefficient of friction between the car and the road surface is 0.2
Model the car as a particle.

(i) Draw a diagram showing all the external forces acting on the car. [2]

At the bottom of the hill the car has a speed of 4 m s^{-1}
At the top of the hill the car has a speed of 6 m s^{-1}

(ii) Using the work-energy principle, find *m*. [11]

THIS IS THE END OF THE QUESTION PAPER