Rewarding Learning

ADVANCED
General Certificate of Education
January 2011

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

AMF21

WEDNESDAY 2 FEBRUARY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Show that the sum of the series

$$1 \times 2 \times 5 + 2 \times 3 \times 6 + \ldots + n\,(n+1)\,(n+4)$$

is given by

$$\frac{1}{12}n\,(n+1)\,(n+2)\,(3n+17)$$ [6]

2 Write

$$\frac{2x^2 - x + 1}{(x^2+1)\,(x^2+2)}$$

in partial fractions. [6]

3 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = \sin x$$ [12]

4 **(i)** Using Maclaurin's theorem, derive a series expansion for $\cos\theta$ up to and including the term in θ^4 [5]

(ii) Hence, and using a binomial expansion, find a series expansion for

$$\frac{\cos 3x}{\sqrt{1-x^2}}$$

up to and including the terms in x^4 [8]

5 Prove by mathematical induction that

$$a_n = 5^n + 3$$

is divisible by 4 for each non-negative integer n. [7]

6

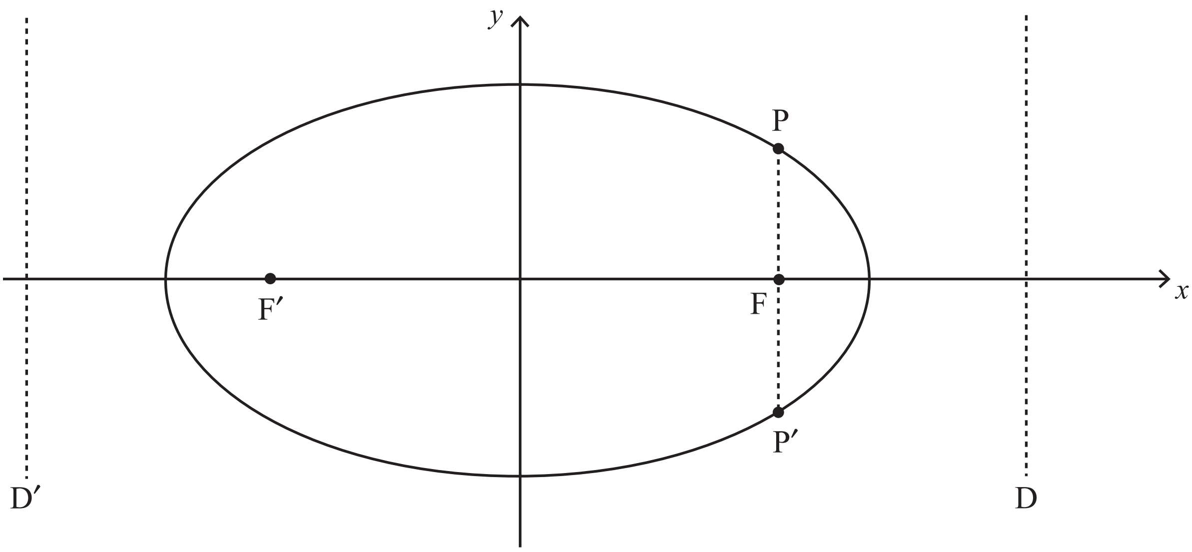

Fig. 1

Fig. 1 above shows an ellipse with equation

$$\frac{x^2}{17^2} + \frac{y^2}{8^2} = 1$$

The foci of the ellipse are F′, F and its directrices are D′ and D.

(i) Show that the equation of the directrix D is $x = \frac{289}{15}$ [3]

(ii) Find the coordinates of the focus F. [2]

(iii) Derive the equation of the tangent to the ellipse at a general point $(17 \cos \theta, 8 \sin \theta)$. [5]

PP′ is a latus rectum of the ellipse.

(iv) Show that the tangent at P meets the x-axis on the directrix D. [6]

7 **(i)** If $z = \cos\theta + i\sin\theta$ is a complex number, show that

$$\cos\theta = \frac{1}{2}(z + z^{-1})$$ [2]

(ii) Hence find numbers a, b and c such that

$$\cos^4\theta = a\cos 4\theta + b\cos 2\theta + c$$ [7]

(iii) Hence, or otherwise, find the general solution of

$$2\cos 4\theta + 8\cos 2\theta + 5 = 0$$ [6]