

GCE A2

Mathematics

Summer 2010

Mark Schemes

Issued: October 2010

**NORTHERN IRELAND GENERAL CERTIFICATE OF SECONDARY EDUCATION (GCSE)
AND NORTHERN IRELAND GENERAL CERTIFICATE OF EDUCATION (GCE)**

MARK SCHEMES (2010)

Foreword

Introduction

Mark Schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of 16- and 18-year-old students in schools and colleges. The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes therefore are regarded as a part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

The Council hopes that the mark schemes will be viewed and used in a constructive way as a further support to the teaching and learning processes.

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Rewarding Learning

**ADVANCED
General Certificate of Education
2010**

Mathematics

Assessment Unit C3

Module C3: Core Mathematics 3

[AMC31]

WEDNESDAY 2 JUNE, AFTERNOON

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

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Positive marking:

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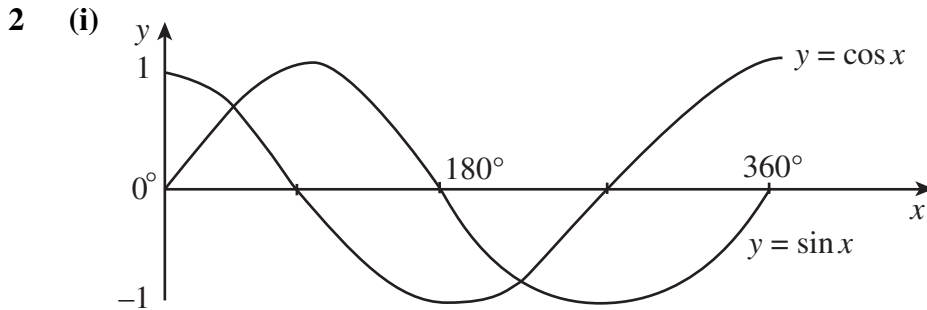
When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

1 $5x + 3 < 2$ $5x + 3 > -2$
 $5x < -1$ $5x > -5$
 $-1 < x < -\frac{1}{5}$

M1

W3

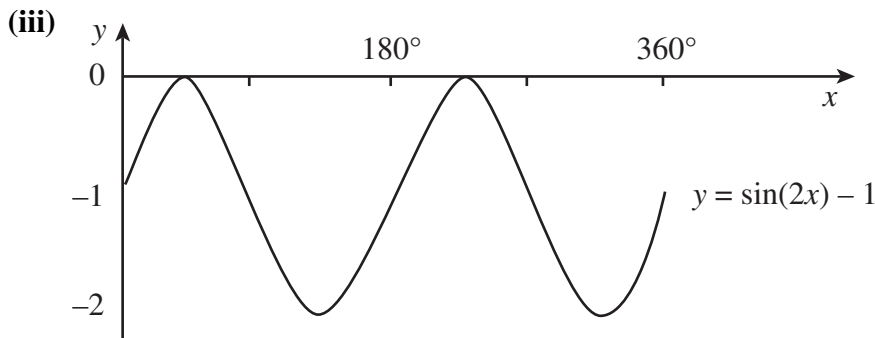
4



MW2

(ii) $\sin x = \cos(x - 90^\circ)$
 $a = 90^\circ$

MW1



MW2

5

3 (a) $y = x^2 \ln x$
 $u = x^2 \quad \frac{du}{dx} = 2x$
 $v = \ln x \quad \frac{dv}{dx} = \frac{1}{x}$
 $\frac{dy}{dx} = x^2 \frac{1}{x} + 2x \ln x$
 $= x + 2x \ln x$

M1

W2

(b) $\int 3x^2 + e^{-x} - \operatorname{cosec} x \cot x + \frac{3}{x} dx$
 $x^3 - e^{-x} + \operatorname{cosec} x + 3 \ln x + c$

MW5

8

<p>4 $x + 1 = \tan t$ $y - 1 = \cot^2 t$ $= \frac{1}{\tan^2 t}$ $y - 1 = \frac{1}{(x + 1)^2}$</p>	<p>MW1 MW1 M1 M1W1</p>	<p>5</p>
<p>5 (a) $\frac{2x - 7}{(x - 3)^2} = \frac{A}{x - 3} + \frac{B}{(x - 3)^2}$ $2x - 7 = A(x - 3) + B$ Put $x = 3$ $-1 = B$ Compare coeff of x $2 = A$ $= \frac{2}{x - 3} - \frac{1}{(x - 3)^2}$</p>	<p>M1W1 M1 M1W1 MW1</p>	
<p>(b) $\frac{1}{(3 - x)^2} = (3 - x)^{-2}$ $= \frac{1}{9} \left[\left(1 - \frac{x}{3} \right)^{-2} \right]$ $= \frac{1}{9} \left[1 + (-2) \left(-\frac{x}{3} \right) + \frac{(-2)(-3)}{2} \left(-\frac{x}{3} \right)^2 + \dots \right]$ $= \frac{1}{9} \left[1 + \frac{2}{3}x + \frac{1}{3}x^2 + \dots \right]$ $= \frac{1}{9} + \frac{2}{27}x + \frac{1}{27}x^2 + \dots$</p>	<p>MW1 M1W1 MW3 W1</p>	<p>13</p>

<p>6 (a) $f(x) = 4e^{-x} - x$ $f'(x) = -4e^{-x} - 1$ $x_0 = 1.3$ $x_1 = 1.3 - \frac{-0.20987}{-2.09013} = 1.1996$ $x_2 = 1.1996 - \frac{-0.00478}{-2.20478} = 1.20$</p>	<p>M1W2</p> <p>M1W1</p> <p>W2</p>	
<p>(b) (i) $\frac{1}{2} N_0 = N_0 e^{-k5730}$ $\ln \frac{1}{2} = \ln e^{-5730k}$ $k = \frac{\ln 2}{5730} = 0.000121$</p> <p>(ii) $N_0 e^{-1000 \left(\frac{\ln 2}{5730} \right)}$ $= 88.6$</p>	<p>M1</p> <p>M1</p> <p>MW1</p> <p>MW1</p> <p>W1</p>	<p>12</p>
<p>7 (a) $4 \sin x + 1 = 3 \operatorname{cosec} x$ $4 \sin^2 x + \sin x - 3 = 0$ $(4 \sin x - 3)(\sin x + 1) = 0$ $\sin x = \frac{3}{4}$ or $\sin x = -1$ $x = 48.6^\circ, 131^\circ$ or 270°</p>	<p>M2W1</p> <p>MW1</p> <p>MW3</p>	
<p>(b) LHS $= \frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta}$ $= \frac{1 - (1 - 2\sin^2\theta)}{2\sin\theta \cos\theta}$ $= \frac{2\sin^2\theta}{2\sin\theta \cos\theta}$ $= \tan\theta = \text{RHS}$</p>	<p>M1W2</p> <p>MW2</p> <p>W1</p>	<p>13</p>

8 (i) To find area under curve

$$A = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sec^2 x \, dx$$

M2W1

$$= [\tan x]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$$

MW1

$$A = \sqrt{3} - (-\sqrt{3})$$

$$= 2\sqrt{3}$$

MW1

$$\text{Shaded area} = 2 \times \frac{\pi}{3} \times 4 - 2\sqrt{3}$$

M1W1

$$= \frac{8}{3}\pi - 2\sqrt{3}$$

MW1

(ii) To find gradient of tangent

$$y = \sec^2 x$$

$$\frac{dy}{dx} = 2 \sec x \sec x \tan x$$

M1W2

$$= 2 \sec^2 x \tan x$$

MW1

$$x = -\frac{\pi}{6}, m = 2 \times \frac{2}{\sqrt{3}} \times \frac{2}{\sqrt{3}} \times \left(-\frac{1}{\sqrt{3}}\right) = \frac{-8}{3\sqrt{3}}$$

MW1

$$x = -\frac{\pi}{6}, y = \sec^2 x = \frac{4}{3}$$

M1

$$y - \frac{4}{3} = -\frac{8}{3\sqrt{3}} \left(x - \left(-\frac{\pi}{6}\right)\right)$$

$$y + \frac{8}{3\sqrt{3}}x = \frac{4}{3} - \frac{4\pi}{9\sqrt{3}}$$

$$(y + 1.54x = 0.527)$$

W1

15

Total

75



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Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

MONDAY 24 MAY, AFTERNOON

MARK SCHEME

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		AVAILABLE MARKS
5 (a)	$u = x - 2 \quad \frac{du}{dx} = 1$	MW1
	$\int \frac{3x}{\sqrt{x-2}} dx = \int \frac{3u+6}{\sqrt{u}} du$	M1W1
	$= \int 3u^{\frac{1}{2}} + 6u^{-\frac{1}{2}} du$	MW1
	$= 2u^{\frac{3}{2}} + 12u^{\frac{1}{2}} + c$	MW2
	$= 2(x-2)^{\frac{3}{2}} + 12(x-2)^{\frac{1}{2}} + c$	MW1
	$= (x-2)^{\frac{1}{2}}(2x+8) + c$	
(b)	$u = 4x \quad \frac{dv}{dx} = \cos 2x$	M1
	$\frac{du}{dx} = 4 \quad v = \frac{1}{2} \sin 2x$	W2
	$I = \left[2x \sin 2x \right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} 2 \sin 2x dx$	M1W1
	$I = \left[2x \sin 2x + \cos 2x \right]_0^{\frac{\pi}{4}}$	MW1
	$I = \frac{\pi}{2} - 1$	MW1
		14

6 (i) $\frac{dx}{dt} = 4t^3 \quad \frac{dy}{dt} = 4t - 8$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{4t - 8}{4t^3} = \frac{t - 2}{t^3}$$

MW2

M1

W1

(ii) $\frac{dy}{dx} = \frac{t - 2}{t^3} = 0$

$$t = 2$$

$$x = 10 \text{ and } y = -2$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \times \frac{dt}{dx}$$

$$\frac{d}{dt} \left(\frac{dy}{dx} \right) = -2t^{-3} + 6t^{-4}$$

$$\frac{d^2y}{dx^2} = \left(\frac{-2}{t^3} + \frac{6}{t^4} \right) \times \frac{1}{4t^3} = \frac{6 - 2t}{4t^7}$$

$$t = 2 \quad \frac{d^2y}{dx^2} = \frac{2}{512} \Rightarrow +ve \therefore \text{min}$$

(10, -2) is a min

M1

W1

MW2

M2

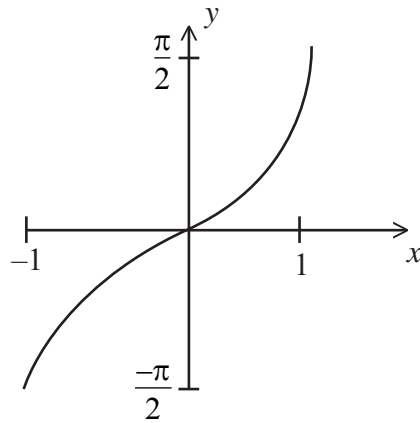
W2

MW1

AVAILABLE
MARKS

13

7 (a)



$$-1 \leq x \leq 1$$

(b) $2 \sin \theta \cos \theta = \cos \theta$
 $\cos \theta (2 \sin \theta - 1) = 0$
 $\cos \theta = 0 \quad \sin \theta = \frac{1}{2}$
 $\theta = \pm \frac{\pi}{2} \quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

(c)
$$\text{LHS} = \frac{1 + \frac{\sin^2 x}{\cos^2 x}}{1 - \frac{\sin^2 x}{\cos^2 x}}$$

$$= \frac{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}$$

$$= \frac{1}{\cos^2 x - \sin^2 x}$$

$$= \frac{1}{\cos 2x}$$

$$= \sec 2x = \text{RHS}$$

M1MW1
MW1

MW1
MW2

MW2

M1W1

M1W1

M1W1

MW1

Total

AVAILABLE
MARKS

15

75



Rewarding Learning

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Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

TUESDAY 22 JUNE, AFTERNOON

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1	$z = \cos\theta + i \sin\theta$ $z^3 = \cos 3\theta + i \sin 3\theta$ $z^3 = \cos^3 \theta + 3 \cos^2 \theta i \sin \theta$ $+ 3 \cos \theta i^2 \sin^2 \theta + i^3 \sin^3 \theta$ <p>Compare imaginary parts</p> $\sin 3\theta = 3 \cos^2 \theta \sin \theta + (-1) \sin^3 \theta$ $= 3(1 - \sin^2 \theta) \sin \theta - \sin^3 \theta$ $= 3 \sin \theta - 4 \sin^3 \theta$	<p>MW1</p> <p>MW1</p> <p>M1</p> <p>W1</p> <p>W1</p>
2	$\sum_{n+1}^{2n} = \sum_{n+1}^{2n} (4k^3 - k)$ $= 4 \sum_{n+1}^{2n} k^3 - \sum_{n+1}^{2n} k$ $= \left(4 \sum_1^{2n} k^3 - 4 \sum_1^n k^3 \right)$ $- \left(\sum_1^{2n} k - \sum_1^n k \right)$ $= 4 \cdot \frac{1}{4} (2n)^2 (2n+1)^2 - 4 \frac{1}{4} n^2 (n+1)^2$ $- \frac{1}{2} 2n(2n+1) + \frac{1}{2} n(n+1)$ $= n^2 \left(4 [4n^2 + 4n + 1] - [n^2 + 2n + 1] \right)$ $- (2n^2 + n) + \frac{1}{2} n^2 + \frac{1}{2} n$ $= 15n^4 + 14n^3 + \frac{3}{2} n^2 - \frac{1}{2} n$	<p>MW1</p> <p>W1</p> <p>M1</p> <p>MW1</p> <p>MW1</p> <p>W1</p>
		5
		6

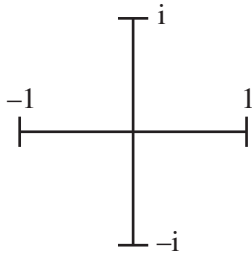
- 3 (i) $f(x) = (1+x)^n$ $f'(x) = n(1+x)^{n-1}$ M1W1
 $f''(x) = n(n-1)(1+x)^{n-2}$ $f'''(x) = n(n-1)(n-2)(1+x)^{n-3}$
 $f(0) = 1$ $f'(0) = n$ $f''(0) = n(n-1)$ $f'''(0) = n(n-1)(n-2)$ M1
 $f(x) \doteq f(0) + xf'(0) = \frac{x^2}{2}f''(0) + \frac{x^3}{6}f'''(0) + \dots$
 $(1+x)^n \doteq 1 + nx + \frac{n(n-1)}{2}x^2 +$
 $+ n\frac{(n-1)(n-2)}{6}x^3 + \dots$ M1W1
- (ii) $\frac{1+x}{(1+2x^2)(1-2x)} = \frac{Ax+B}{1+2x^2} + \frac{C}{1-2x}$ M1W1
 $1+x = (Ax+B)(1-2x) + C(1+2x^2)$ MW1
 Put $x = \frac{1}{2}$ $\frac{3}{2} = \frac{3}{2}C \Rightarrow C = 1$
 Compare coeff x^2 , $0 = -2A + 2C \Rightarrow A = 1$
 Compare coeff x , $1 = A - 2B \Rightarrow B = 0$ M1W2
 $\frac{x}{1+2x^2} + \frac{1}{1-2x}$
- (iii) $\frac{1+x}{(1+2x^2)(1-2x)} = \frac{x}{1+2x^2} + \frac{1}{1-2x}$
 $(1+2x^2)^{-1} = 1 + (-1)(2x^2) + \frac{(-1)(-2)}{2}(2x^2)^2 + \dots$ M1W1
 $x(1+2x^2)^{-1} = x - 2x^3 + \dots$
 $(1-2x)^{-1} = 1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(2x)^2$
 $+ (-1)\frac{(-2)(-3)}{6}(-2x)^3 + \dots$ MW1
 $= 1 + 2x + 4x^2 + 8x^3 + \dots$ W1
 $\therefore \text{Result} = 1 + 3x + 4x^2 + 6x^3 + \dots$ W1

		Marks	Available Marks	
4	(i) Aux.Eq ⁿ $m^2 - 10m + 16 = 0$ $(m-2)(m-8) = 0$ $y = Ae^{2x} + Be^{8x}$ Try $y = ke^{3x}$ for particular integral $9ke^{3x} - 30ke^{3x} + 16ke^{3x} = e^{3x}$ $\Rightarrow k = -\frac{1}{5}$ $y^{gs} = Ae^{2x} + Be^{8x} - \frac{1}{5}e^{3x}$	MW1	11	
		M1W1		
		M1		
		MW1		
		MW1		
		MW1		
		(ii) Aux.Eq ⁿ $m^2 + pm + 16$ is a perfect square $\Rightarrow p = 8$ and $k = -4$ } or $p = -8$ and $k = 4$ }		MW1
				W3
		5		(i) Let $P(k) \equiv 2^{k+1} \sin x \cos x \dots \cos 2^k x \equiv \sin(2^{k+1} x)$ When $n = 0$ LHS = $2 \sin x \cos x$ RHS = $\sin 2x$ $\therefore P(0)$ true ① Assume $P(k)$ true Consider $2^{k+2} \sin x \cos x \dots \cos(2^{k+1} x)$ $= 2(2^{k+1} \sin x \cos x \dots \cos 2^k x) \cos 2^{k+1} x$ $= 2 \sin(2^{k+1} x) \cos(2^{k+1} x)$ by $P(k)$ $= \sin 2^{k+2} x \therefore P(k+1)$ true ② ① and ② $\Rightarrow P(n)$ true $\forall n \in \mathbb{Z} \quad n \geq 0$
M1				
M1				
MW1				
MW1				
M1				
(ii) $2^3 \sin x \cos x \cos 2x \cos 4x = \frac{\sqrt{2}}{2}$	MW1			
	M1W1			
	M2W1			
	by (i) $\sin 8x = \frac{1}{\sqrt{2}}$		W1	
	$8x = n\pi + (-1)^n \frac{\pi}{4}$			
	$x = \frac{n\pi}{8} + (-1)^n \frac{\pi}{32}$			

6 (i)	$a = \frac{6}{2} = 3$	MW1	AVAILABLE MARKS
	With vertex(0, 0) eq ⁿ $y^2 = 12x$	MW1	
	vertex (3,0) $y^2 = 12(x - 3)$	MW1	
(ii)	$y^2 = 36t^2$	M1	
	$12(x - 3) = 12(3t^2 + 3 - 3) = 36t^2$ equal	W1	
(iii)	$\frac{dy}{dt} = 6$ $\frac{dx}{dt} = 6t$ $\frac{dy}{dx} = \frac{6}{6t}$ $\frac{dy}{dx} = \frac{1}{t}$	M1W2	
	$y - 6t = \frac{1}{t}(x - 3t^2 - 3)$	M1W1	
	$ty - 6t^2 = x - 3t^2 - 3$		
	$ty - x = 3t^2 - 3$	W1	
(iv)	Tangent at R $-\frac{y}{t} - x = \frac{3}{t^2} - 3$	M1W1	
	$\Rightarrow -ty - t^2x = 3 - 3t^2$		
	Tangent at T $ty - x = 3t^2 - 3$	M1	
	Add $-(1 + t^2)x = 0$		
	$\therefore x = 0$	W1	

15

7 (i)



(ii) $z = 1e^{\frac{i\pi}{4}}$ $r = 1$

$k = \frac{\pi}{4}$

(iii) $(z^8 - 1) \div (z^4 - 1)$ for $z^4 - 1$

$z^8 - 1 = (z^4 - 1)(z^4 + 1)$

Equation $z^4 + 1 = 0$

Alternative solution

$\left[z - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \right) \right] \left[z - \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right) \right]$

$= z^2 - \sqrt{2}z + 1$

$\left[z - \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \right) \right] \left[z - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right) \right]$

$= z^2 + \sqrt{2}z + 1$

$(z^2 - \sqrt{2}z + 1)(z^2 + \sqrt{2}z + 1)$

$= z^4 + \sqrt{2}z^3 + z^2$

$\quad - \sqrt{2}z^3 - 2z^2 - \sqrt{2}z$

$\quad \quad + z^2 + \sqrt{2}z + 1$

$= z^4 + 1$

So eqⁿ is $z^4 + 1 = 0$

AVAILABLE MARKS

MW1

MW1

MW1

MW1

MW1

M1W1

MW1

M1

W1

W1

MW1

Total

8

75



Rewarding Learning

**ADVANCED
General Certificate of Education
2010**

Mathematics

Assessment Unit F3

assessing

Module FP3: Further Pure Mathematics 3

[AMF31]

THURSDAY 27 MAY, MORNING

**MARK
SCHEME**

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

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			AVAILABLE MARKS
1	$u = e^x$	$\frac{du}{dx} = e^x$	MW1
	$\frac{dv}{dx} = \cos x$	$v = \sin x$	MW1
	$\int e^x \cos x dx = e^x \sin x - \int e^x \sin x dx$		MW1
	$u = e^x$	$\frac{du}{dx} = e^x$	W1
	$\frac{dv}{dx} = \sin x$	$v = -\cos x$	
	$\int e^x \cos x dx = e^x \sin x - \left[-e^x \cos x + \int e^x \cos x dx \right]$		
	$\int e^x \cos x dx = e^x \sin x + e^x \cos x - \int e^x \cos x dx + c$		MW1
	$2 \int e^x \cos x dx = e^x \sin x + e^x \cos x + c$		
	$\int e^x \cos x dx = \frac{e^x}{2} (\sin x + \cos x) + c'$		MW1
			6

- 2 (i) Origin in plane – necessary. }
 $2 - 0 - 2 = 0$ A in plane
 $4 - 6 + 2 = 0$ B in plane }

M1W1

AVAILABLE
MARKS

(ii) plane Π_2 $\mathbf{r} \cdot \mathbf{n} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} = d$

M1

$$d = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} = 12 - 6 - 1 = 5$$

W1

(iii) Π_2 Cartesian equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} = 3x - 2y - z = 5$

M1W1

$$3x - 2y - z = 5 \quad (\text{a})$$

M1

$$\frac{x - 2y + 2z = 0}{2x - 3z = 5} \quad (\text{b})$$

$$x = \frac{3z + 5}{2}$$

W1

Substitute in (b)

$$\frac{3z + 5}{2} - 2y + 2z = 0$$

$$4y = 7z + 5$$

$$z = \frac{4y - 5}{7}$$

W1

$$\therefore \frac{2x - 5}{3} = \frac{4y - 5}{7} = z$$

W1

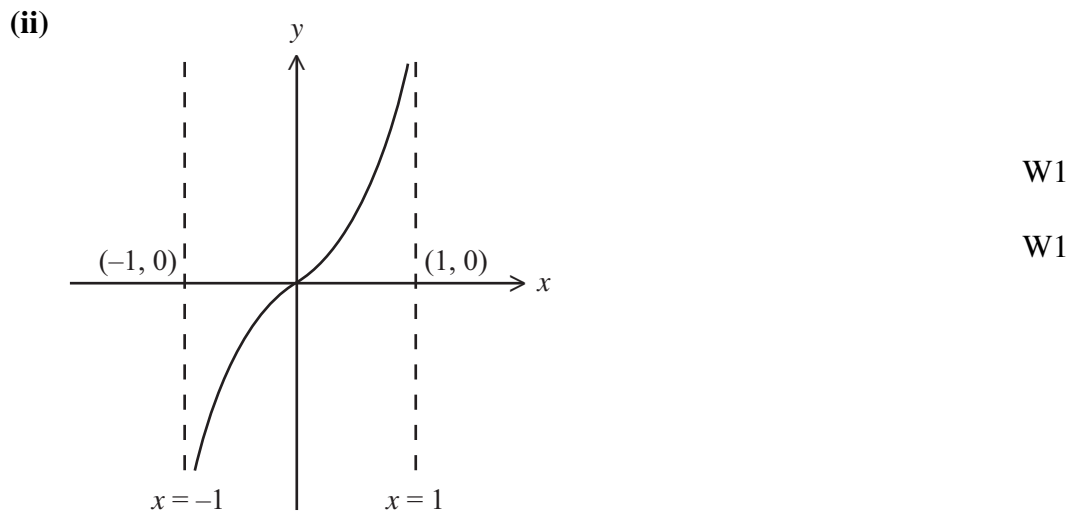
10

		AVAILABLE MARKS	
3	(i) $u = x^n \quad \frac{du}{dx} = nx^{n-1}$	MW1	
	$\frac{dv}{dx} = e^{-x} \quad v = -e^{-x}$	MW1	
	$I_n = \int_0^1 x^n e^{-x} dx = \left[-x^n e^{-x} \right]_0^1 + \int_0^1 nx^{n-1} e^{-x} dx$	M1	
	$= -e^{-1} + n \int_0^1 x^{n-1} e^{-x} dx$	W1	
	$\therefore I_n = -e^{-1} + nI_{n-1}$	W1	
	(ii) $I_4 = \int_0^1 x^4 e^{-x} dx = \left[4I_3 - e^{-1} \right]$	M1	
	$= 4 \left[3I_2 - e^{-1} \right] - e^{-1} = 12 \left(2I_1 - e^{-1} \right) - 5e^{-1}$	W1	
	$= 24 \left(I_0 - e^{-1} \right) - 17e^{-1} = 24I_0 - 41e^{-1}$	W1	
	$I_0 = \int_0^1 e^{-x} dx = \left[-e^{-x} \right]_0^1 = \left[-e^{-1} + 1 \right]$	MW1	
	$\therefore I_4 = 24 - 65e^{-1}$	W1	10
4	$u = e^x \quad \frac{du}{dx} = e^x = u \quad \frac{du}{u} = dx$	MW1	
	$5 \cosh x + 4 \sinh x = \frac{5(e^x + e^{-x})}{2} + \frac{4(e^x - e^{-x})}{2}$	M1	
	$= \frac{9e^x + e^{-x}}{2} = \frac{9u + \frac{1}{u}}{2}$		
	$= \frac{9u^2 + 1}{2u}$	W2	
	$\int \frac{dx}{5 \cosh x + 4 \sinh x} = \int \frac{2u}{9u^2 + 1} \cdot \frac{du}{u}$	M1	
	$= \int \frac{2du}{9u^2 + 1} = \int \frac{2}{(3u)^2 + 1} du$	W1	
	$= \frac{2}{3} \tan^{-1}(3u) + c$	M1	
	$= \frac{2}{3} \tan^{-1}(3e^x) + c$	W1	8

5 (i) $y = \cos^{-1} x$ $\cos y = x$
 $-\sin y \frac{dy}{dx} = 1$ M1W1
 $\sin^2 y = 1 - \cos^2 y = 1 - x^2$ MW1
 $\therefore \frac{dy}{dx} = \frac{-1}{\sin y} = \frac{-1}{\sqrt{1-x^2}}$ W1

(ii) $y = \cos^{-1} 4x$ $\frac{dy}{dx} = \frac{-4}{\sqrt{1-16x^2}}$ M1W1
 $x = \frac{1}{8}$ $y = \cos^{-1} \frac{1}{2} = \frac{\pi}{3}$ MW1
 $x = \frac{1}{8}$ $\frac{dy}{dx} = \frac{-4}{\sqrt{1-16 \cdot \frac{1}{64}}} = \frac{-8}{\sqrt{3}} = \frac{-8\sqrt{3}}{3}$ MW1
 $y - \frac{\pi}{3} = -\frac{8\sqrt{3}}{3} \left(x - \frac{1}{8} \right)$ MW1
 $x = 0$ $y = \text{OP} = \frac{\pi}{3} + \frac{\sqrt{3}}{3} = \frac{\pi + \sqrt{3}}{3}$ MW1

6 (i) $y = \tanh^{-1} x$ $\tanh y = x = \frac{\sinh y}{\cosh y}$ MW1
 $\therefore x = \frac{e^y - e^{-y}}{e^y + e^{-y}} = \frac{e^{2y} - 1}{e^{2y} + 1}$ M1W1
 $x(e^{2y} + 1) = e^{2y} - 1$
 $1 + x = e^{2y}(1 - x)$ W1
 $e^{2y} = \frac{1+x}{1-x}; 2y = \ln \left(\frac{1+x}{1-x} \right)$
 $y = \tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$ W1



10

$$(iii) y = \tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) = \frac{1}{2} \left[\ln(1+x) - \ln(1-x) \right]$$

M1W1

AVAILABLE
MARKS

$$\frac{dy}{dx} = \frac{1}{2} \left[\frac{1}{1+x} - \frac{-1}{1-x} \right]$$

MW1

$$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{2}{1-x^2} = \frac{1}{1-x^2}$$

W1

or

$$\tanh y = x$$

M1

$$\operatorname{sech}^2 y \frac{dy}{dx} = 1$$

W1

$$\operatorname{sech}^2 y \equiv 1 - \tanh^2 y = 1 - x^2$$

MW1

$$(1-x^2) \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{1-x^2}$$

W1

$$(iv) x = \tanh \left[\ln(\sqrt{6x}) \right]$$

$$\tanh^{-1} x = \ln(\sqrt{6x})$$

M1

$$\frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) = \frac{1}{2} \ln(6x)$$

W1

$$\frac{1+x}{1-x} = 6x$$

MW1

$$6x^2 - 5x + 1 = 0$$

W1

$$(3x-1)(2x-1) = 0$$

$$x = \frac{1}{3} \text{ or } x = \frac{1}{2}$$

W1

16

$$7 (i) \text{ Area of triangle} = \frac{1}{2} |\mathbf{a} \times \mathbf{b}|$$

M1

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1 & 2 \\ 1 & 2 & -1 \end{vmatrix} = -5\mathbf{i} + \mathbf{j} - 3\mathbf{k}$$

M1W2

$$|\mathbf{a} \times \mathbf{b}| = \sqrt{35} \quad \text{Area of triangle} = \frac{\sqrt{35}}{2}$$

MW1

(ii) line $(\mathbf{r} - \mathbf{a}) \times \mathbf{m} = \mathbf{0}$ has equation $\mathbf{r} = -\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} + \mathbf{k})$	MW1	<table border="1" style="width: 100%; height: 100%; border-collapse: collapse;"> <thead> <tr style="background-color: #333; color: white;"> <th style="padding: 5px;">AVAILABLE MARKS</th> </tr> </thead> <tbody> <tr> <td style="height: 200px;"></td> </tr> <tr> <td style="text-align: center; border-top: 1px solid black; border-bottom: 1px solid black;">15</td> </tr> <tr> <td style="text-align: center;">75</td> </tr> </tbody> </table>	AVAILABLE MARKS		15	75
AVAILABLE MARKS						
15						
75						
line $(\mathbf{r} - \mathbf{b}) \times \mathbf{n} = \mathbf{0}$ has equation $\mathbf{r} = -\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \mu(-\mathbf{i} - \mathbf{j} + 4\mathbf{k})$	MW1					
Intersect $\mathbf{r} = -\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} + \mathbf{k}) = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \mu(-\mathbf{i} - \mathbf{j} + 4\mathbf{k})$	M1					
Equate $\left. \begin{matrix} -1 + \lambda = 1 - \mu \\ 1 = 2 - \mu \end{matrix} \right\} -\mu = 1 \quad \lambda = 1$	MW2					
$2 + \lambda = 1 + 4\mu$						
LHS = $2 + \lambda = 3$ RHS = $-1 + 4\mu = 3$	MW1					
Position vector of C is $\mathbf{c} = \mathbf{j} + 3\mathbf{k}$						
Accept C is the point (0, 1, 3)	W1					

(iii) Volume = $\frac{1}{6} \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$		
$= \frac{1}{6} (\mathbf{j} + 3\mathbf{k}) \cdot (-5\mathbf{i} + \mathbf{j} - 3\mathbf{k}) $	M1W1	
$= \frac{1}{6} 1 - 9 = \frac{8}{6} = \frac{4}{3}$	W1	15
Total		75



Rewarding Learning

**ADVANCED
General Certificate of Education
2010**

Mathematics

Assessment Unit M2
assessing
Module M2: Mechanics 2

[AMM21]

FRIDAY 11 JUNE, MORNING

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

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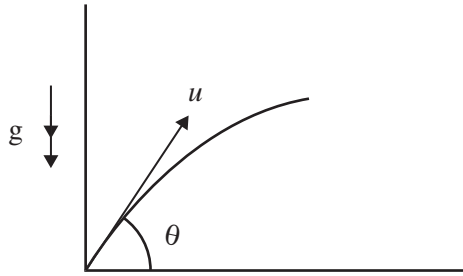
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		AVAILABLE MARKS
1	(i) $\mathbf{F}_1 + \mathbf{F}_2 = (3\mathbf{i} + \mathbf{j} - \mathbf{k})\text{N}$	MW1
	$\mathbf{F} = m\mathbf{a}$	M1
	$3\mathbf{i} + \mathbf{j} - \mathbf{k} = 2\mathbf{a}$	
	$\mathbf{a} = \frac{3}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} - \frac{1}{2}\mathbf{k}$	W1
	(ii) Resultant Force = $3\mathbf{i} + \mathbf{j} - \mathbf{k}$	
	$\mathbf{F} \cdot \mathbf{F}_1 = \mathbf{F} \mathbf{F}_1 \cos \theta$	M1
	$(3\mathbf{i} + \mathbf{j} - \mathbf{k}) \cdot (2\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = 6 - 2 - 1 = 3$	MW1
	$ \mathbf{F} = \sqrt{9+1+1} = \sqrt{11}$	M1W1
	$ \mathbf{F}_1 = \sqrt{4+4+1} = 3$	MW1
	$\cos \theta = \frac{3}{3\sqrt{11}} = \frac{1}{\sqrt{11}}$	
	$\theta = 72.5^\circ$	W1
		9
2	(i) $\text{KE} = \frac{1}{2}mv^2$	M1
	At start KE $\frac{1}{2} \times 0.05 \times 100 = 2.5\text{J}$	W1
	At end KE $\frac{1}{2} \times 0.05 \times 16 = 0.4\text{J}$	W1
	Change in KE = $2.5\text{J} - 0.4\text{J}$ Decrease of 2.1 J	W1
	(ii) Work done by gravity = mgd	M1
	$= 0.05 \times 9.8 \times 2$	
	$= 0.98\text{J}$	W1
	(iii) By work – energy principle	
	Work done by resultant force = Change in KE	M1
	Resultant force = $mg - R$	MW1
	$(mg - R)d = -2.1$	
	$[(0.05 \times 9.8) - R](2) = -2.1$	W2
	$R = 1.54\text{N}$	W1
		11

3 (i)



At greatest height $v = 0$

M1

$$u = u \sin \theta$$

$$v = 0$$

$$v^2 = u^2 + 2as$$

M1

$$a = -g$$

$$0 = u^2 \sin^2 \theta - 2gs$$

$$s = ?$$

$$s = \frac{u^2 \sin^2 \theta}{2g}$$

W1

(ii) When travelling horizontally $v = 0$ vertically

M1

$$u = u \sin \theta$$

$$v = 0$$

$$v = u + at$$

M1

$$a = -g$$

$$0 = u \sin \theta - gt$$

$$t = ?$$

$$t = \frac{u \sin \theta}{g}$$

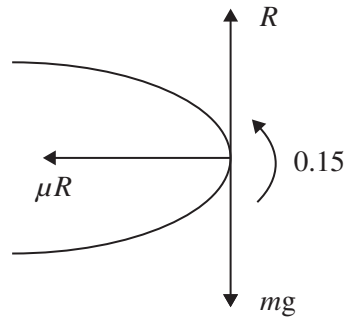
W1

6

AVAILABLE
MARKS

4 (i)

$$r = 100 \text{ m}$$



$$\mu R = mr\omega^2$$

M3

$$\mu mg = mr\omega^2$$

MW1

$$\mu = \frac{100(0.15)^2}{10} = 0.225$$

W1

(ii) Kinetic Energy = $\frac{1}{2}mv^2$

M1

$$v = r\omega$$

M1

$$\text{KE} = \frac{1}{2}m(100)^2(0.15)^2$$

W1

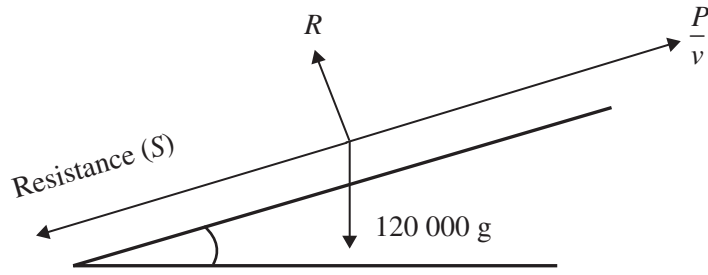
$$= 113 \text{ m J}$$

W1

AVAILABLE
MARKS

9

5 (i)



MW2

(ii) At steady speed $a = 0$

M1

$$P = Fv$$

$$F = \frac{P}{v} = \frac{240\,000}{12} = 20\,000\text{ N}$$

M1W1

Using $F = ma$

M1

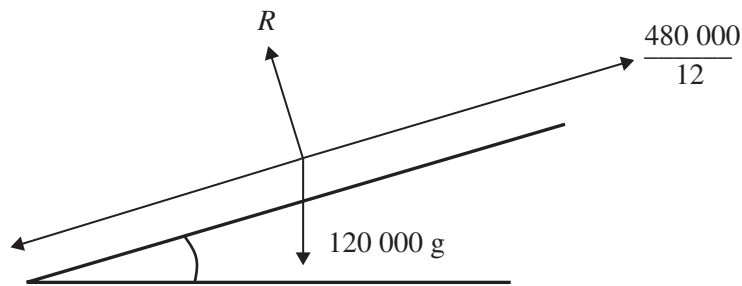
$$20\,000 - S = 120\,000 g \sin \theta$$

W1

$$S = 10\,200\text{ N}$$

W1

(iii)



Using $F = ma$

$$\frac{480\,000}{12} - 10\,200 - \frac{120\,000(9.8)}{120} = 120\,000 a$$

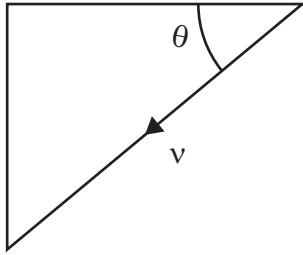
MW3

$$a = 0.167\text{ m s}^{-2}$$

W1

AVAILABLE
MARKS

12

		AVAILABLE MARKS
6 (i)	$\mathbf{a} = \frac{d\mathbf{v}}{dt} = (3t^2 - 6t)\mathbf{i} + (2t - 4)\mathbf{j}$	M1W2
(ii)	<p>For $\mathbf{a} = 0$ \mathbf{i} and \mathbf{j} components equal zero</p> $(3t^2 - 6t) = 0 \quad t = 0 \text{ or } t = 2$ $(2t - 4) = 0 \quad t = 2$ <p>$t = 2$ seconds</p>	M1 MW1 W1
(iii)	$\mathbf{s} = \int \mathbf{v} dt$ $\mathbf{s} = \left(\frac{t^4}{4} - t^3 \right) \mathbf{i} + \left(\frac{t^3}{3} - 2t^2 \right) \mathbf{j} + \mathbf{c}$ <p>At $t = 0$, $\mathbf{s} = 3\mathbf{j} \quad \therefore \mathbf{c} = 3\mathbf{j}$</p> $\mathbf{s} = \left(\frac{t^4}{4} - t^3 \right) \mathbf{i} + \left(\frac{t^3}{3} - 2t^2 + 3 \right) \mathbf{j}$	M1 W2 MW1
(iv)	<p>When $t = 2$</p> $\mathbf{s} = -4\mathbf{i} - \frac{7}{3}\mathbf{j}$ $ \mathbf{s} = \sqrt{16 + \frac{49}{9}}$ $= 4.63$	MW1 M1 W1
(v)	<p>When $t = 2$</p> $\mathbf{v} = -4\mathbf{i} - 4\mathbf{j}$ <div style="display: flex; align-items: center; margin-top: 10px;">  <div style="margin-left: 20px;"> $\theta = \tan^{-1} 1$ </div> </div>	MW1 M1W1
At	<p>low positive \mathbf{i} direction</p>	MW1
		17

7 (i) $a = \frac{1}{(s - 600)^2}$

$$v \frac{dv}{ds} = \frac{1}{(s - 600)^2}$$

$$\int v \, dv = \int \frac{1}{(s - 600)^2} \, ds$$

$$\frac{v^2}{2} = \frac{-1}{s - 600} + c$$

At $v = 0, s = 0, c = -\frac{1}{600}$

$$\frac{v^2}{2} = \frac{1}{600 - s} - \frac{1}{600}$$

$$v = \sqrt{\frac{2}{600 - s} - \frac{1}{300}}$$

$$v = \sqrt{\frac{s}{300(600 - s)}}$$

(ii) By considering the motion at B, velocity and acceleration at B would be infinite using this model.

AVAILABLE
MARKS

MW1

M2W1

MW2

M1W1

MW1

MW2

11

Total

75



Rewarding Learning

**ADVANCED
General Certificate of Education
2010**

Mathematics

Assessment Unit M3

assessing

Module M3: Mechanics 3

[AMM31]

TUESDAY 15 JUNE, MORNING

MARK SCHEME

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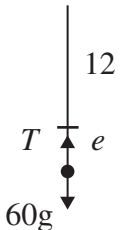
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		AVAILABLE MARKS
1	(i) $AB = 30, AD = 24$ By Pythagoras $BD = 18$	M1W1
	$\triangle s$ similar, scale $\frac{1}{2} \therefore BH = 9$	MW1
(ii)	Let m be the mass of $\triangle ADE$	M1
	$4m \times 3 - m \times -3 = 3md$	M1W2
	$d = 5$	W1
		8
2	(i) $a = 1, a\omega^2 = 9$ and $\omega > 0 \therefore \omega = 3$	MW2 W1
	$T = \frac{2\pi}{\omega} = \frac{2\pi}{3}$	M1W1
(ii)	$v^2 = \omega^2(a^2 - x^2)$	M1
	$= 9(1 - 0.64) = 9 \times 0.36$	W1
	$v > 0 \therefore v = 3 \times 0.6 = 1.8 \text{ ms}^{-1}$	W1
(iii)	Starting at A, $t = 0$	M1
	$x = a \cos \omega t$	MW1
	$-0.8 = 1 \cos 3t$	MW1
	$3t = 2.498$	M1
	$t = 0.833 \text{ s}$	W1
		13

3 (i)  Res \updownarrow $T = mg = 60g$ MW1
 By Hooke's Law $T = \frac{240g}{12}$ M1W1
 $20g e = 60g$
 $e = 3$ W1

(ii) equating energies at 2 pts M1

at clifftop GPE = $60g \times 15$ EPE = 0 KE = 0 W1

at P GPE = $60g d$, EPE = $\frac{1}{2} \frac{240g}{12} (3 + d)^2$ KE = 0 M1W2

$$10g(9 + 6d + d^2) - 60gd = 900g$$

$$90 + 10d^2 = 900$$

$$10d^2 = 810$$

$$d^2 = 81 \text{ and } d > 0 \therefore d = 9$$

W1

(iii) Res \updownarrow $R = T - 60g$ M1

at P, extn is 12 m $\therefore T = \frac{240g}{12} \times 12 = 240g$ MW1

$$R = 240g - 60g$$

$$= 180g \text{ N}$$

W1

AVAILABLE
MARKS

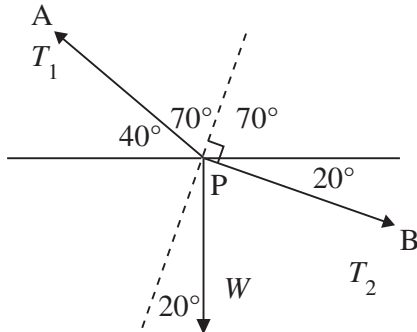
13

		AVAILABLE MARKS
4 (a) (i)	$W = \int_0^d 3(x-2)^2 dx$	M1W1
	$= [(x-2)^3]_0^d$	MW1
	$= (d-2)^3 - (-8)$ $= (d-2)^3 + 8$	W1
(ii)	$(d-2)^3 + 8 = 35$	MW1
	$(d-2)^3 = 27$	
	$d-2 = 3$ $d = 5$	W1
(b) (i)	$D = \mathbf{F} \cdot \mathbf{s}$	M1
	$= 3 \begin{pmatrix} 2 \\ -5 \\ a \end{pmatrix} \cdot \begin{pmatrix} a-1 \\ 1-2a \\ a \end{pmatrix}$	M1
	$= 3(2a-2-5+10a+a^2)$ $= 3(a^2+12a-7)$	W1
(ii)	$3(a^2+12a-7) = 114$	MW1
	$a^2+12a-7 = 38$	
	$a^2+12a-45 = 0$ $(a-3)(a+15) = 0$ $a = 3 \text{ or } -15$	W2
(iii)	$a = 3$	
	$\mathbf{F} = 3 \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}, \mathbf{s} = \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$	MW2
(iv)	For $a = 3$, \mathbf{F} and \mathbf{s} are in the same direction so the resultant \mathbf{R} of the system of forces is in the direction of \mathbf{F} .	MW2

16

5 (i) $T = \frac{\lambda x}{l} \therefore x = \frac{Tl}{\lambda}$ M1MW1
 $= \frac{0.8 \times 92}{184}$
 $= 0.4\text{m}$ W1

(ii)



$$T_1 \cos 70^\circ = W \cos 20^\circ$$
 M1W2

$$W = \frac{92 \cos 70^\circ}{\cos 20^\circ}$$

$$= 33.485$$

$$\rightarrow 33.5\text{N}$$
 W1

(iii)

Res \leftrightarrow

$$T_1 \cos 40^\circ = T_2 \cos 20^\circ$$

$$T_2 = \frac{92 \cos 40^\circ}{\cos 20^\circ}$$

$$= 74.999$$

$$\rightarrow 75.0\text{N}$$
 W1

(iv)

$$x = \frac{1}{4} (0.9 + x)$$
 MW1

$$\frac{3}{4}x = \frac{1}{4} \times 0.9$$

$$x = 0.3$$
 MW1

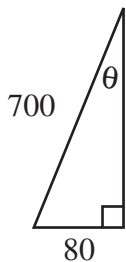
$$75 = \frac{\lambda \times 0.3}{0.9}$$

$$\lambda = 225\text{N}$$
 MW1

AVAILABLE
MARKS

13

6 (i)

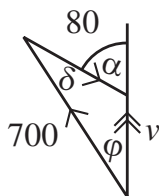


$$\sin \theta^\circ = \frac{8}{70} \therefore \theta = 6.56^\circ$$

M1W1

AVAILABLE
MARKS

(ii) ${}_P\mathbf{V}_W + {}_W\mathbf{V}_E = {}_P\mathbf{V}_E$



$$\begin{aligned} \frac{\sin \varphi}{80} &= \frac{\sin (180 - \alpha)^\circ}{700} \\ &= \frac{\sin \alpha}{700} = \frac{0.8}{700} \end{aligned}$$

$$\sin \varphi = \frac{64}{700}$$

$$\varphi = 5.246^\circ \rightarrow 5.25^\circ$$

M1

MW1

M1

W1

W1

Bearing is $360^\circ - \varphi = 355^\circ$

MW1

(iii) $\alpha = \delta + \varphi \therefore \delta = \alpha - \varphi = 47.8843^\circ$

$$\frac{v}{\sin \delta} = \frac{700}{\sin \alpha}$$

$$v = \frac{700 \sin \delta}{\sin \alpha} = 649.068 \text{ kh}^{-1}$$

$$t = \frac{2000}{v}$$

$$= 3.08 \text{ hours}$$

MW1

MW1

W1

MW1

Total

12

75



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Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

[AMM41]

FRIDAY 18 JUNE, AFTERNOON

**MARK
SCHEME**

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

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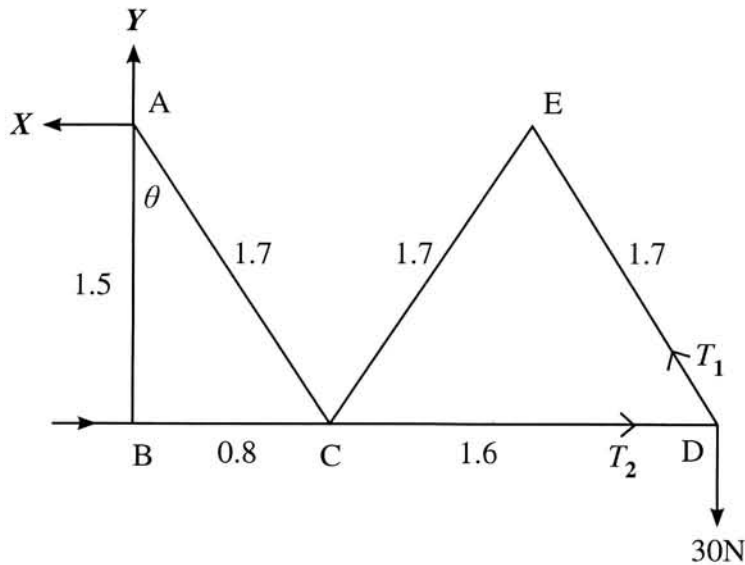
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1 (i)	$\text{mass} = \rho \times \text{area so } m = \rho \int_0^4 y \, dx = \rho \int_0^4 (4-x)^{\frac{1}{2}} \, dx$ $= \frac{2}{3} \rho \left[(-1)(4-x)^{\frac{3}{2}} \right]_0^4$ $= \frac{2}{3} \rho \times 8$ $= \frac{16\rho}{3}$	M2W1	
		W1	
		W1	
(ii)	<p>c.o.m. of strip is $\left(x, \frac{y}{2}\right)$</p> $\therefore M_{0x} = \rho \int_0^4 \frac{y}{2} \times y \, dx = \frac{\rho}{2} \int_0^4 y^2 \, dx$ $= \frac{\rho}{2} \int_0^4 (4-x) \, dx$	MW1	
		MW1	
		W1	
(iii)	$M_{0x} = \frac{\rho}{2} \left[\frac{-(4-x)^2}{2} \right]_0^4$ $= \frac{\rho}{4} [0 - (-16)]$ $= 4\rho$ $\bar{y} = \frac{M_{0x}}{M} = \frac{4\rho \times 3}{16\rho} = \frac{3}{4}$	MW1	
		W1	
		MW1	

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2



- (i) ED must have an upwards vertical component to balance the load. So a tension T_1 . So ED has an inwards horizontal component and so the force in CD must act out, so a thrust T_2

M1
M1

(ii) $\cos \theta = \frac{15}{17}$ MW1

$T_1 \cos \theta = W \quad \frac{15}{17} T_1 = 30 \quad T_1 = 34 \text{ N}$ M1W1

$T_1 \sin \theta = T_2 \quad T_2 = 34 \times \frac{8}{17} = 16 \text{ N}$ MW1

- (iii) The only action at B is horizontal from the rod CB and B is in equilibrium. Only a horizontal force can balance this. M1

(iv) $\begin{matrix} \uparrow Y \\ \leftarrow X \end{matrix}$ Consider the system

Res $Y = 30 \text{ N}$ MW1

$\mathcal{M}_B \quad 1.5X = 2.4 \times 30$ M1W1

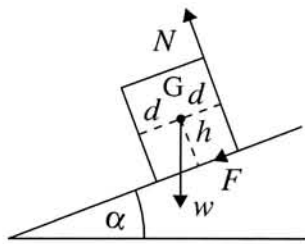
$X = 48 \text{ N}$ W1

$R_A = \sqrt{30^2 + 48^2} = 56.6 \text{ N}$ MW1

12

3 (i) The wheels have just lifted off the road so *normal reaction* = 0 M1

(ii)



$$\begin{aligned} \mathcal{M}_G \quad Nd &= Fh \\ F &= \mu N \\ \mu &= \frac{d}{h} \end{aligned}$$

M2W1

MW1

MW1

(iii) $\mu = \frac{d}{1.25d} = 0.8$ MW1

(iv) $v^2 = rg \left(\frac{\tan \alpha + \mu}{1 - \mu \tan \alpha} \right) = 0.990066 \times 196$ M1W1

$v = 13.93 \rightarrow 13.9 \text{ m s}^{-1}$ W1

10

4 (i) $\begin{matrix} 20 \\ \rightarrow \end{matrix} \quad \begin{matrix} 0 \\ \rightarrow \end{matrix}$

$\begin{matrix} \textcircled{1} \\ \rightarrow \\ -v \end{matrix} \quad \begin{matrix} \textcircled{m} \\ \rightarrow \\ 2v \end{matrix}$

Conservation of Momentum $2mv - v = 20$ M1W1

$2mv = 20 + v$

$m = \frac{20 + v}{2v}$ W1

(ii) P's loss $\frac{1}{2} \times 1 \times 20^2 - \frac{1}{2} \times 1 \times v^2$ M1W1

$\therefore \frac{1}{4} (20^2 - v^2) = \frac{1}{2} m \times 4v^2 = 2mv^2$ M1W1
 $= v(20 + v)$

$400 - v^2 = 80v + 4v^2$

$5v^2 + 80v - 400 = 0$ W1

$v^2 + 16v - 80 = 0$

$(v - 4)(v + 20) = 0$

$v = 4$ or -20 , but $v > 0$

$\therefore v = 4$ W1

(iii) Newton's Law of Restitution M1

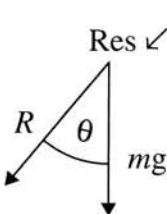
$2v - (-v) = -e(0 - 20)$ W1

$3v = 20e$

$20e = 12$

$e = 0.6$ MW1

12

- 5 (i) Take G.P.E. = 0 at B
- ∴ at A Total Energy = $\frac{1}{2}m \times 2gr + mg \times 2r = 3mgr$ M1W1
- ∴ at P Total Energy = $\frac{1}{2}mv^2 + mgr(1 + \cos\theta)$ MW2
- ∴ equating energies $\frac{1}{2}mv^2 = 2mgr - mgr \cos\theta$ M1
- $$mv^2 = 2mgr(2 - \cos\theta)$$
- W1
- 
- Res \swarrow $\frac{mv^2}{r} = R + mg \cos\theta$ M2W1
- $$R = \frac{mv^2}{r} - mg \cos\theta$$
- $$= 4mg - 3mg \cos\theta$$
- $$= mg(4 - 3 \cos\theta)$$
- W1
- (ii) $-1 \leq \cos\theta \leq 1 \therefore 1 \leq (4 - 3 \cos\theta) \leq 7$ MW1
i.e. $R > 0$ so R always points to C MW1
- (iii) $R_{\min} = 1mg, R_{\max} = 7mg$ MW2
∴ $R_{\max} = 7R_{\min}$

14

Alternative solution

- (i) By W.E.P. Work done = change in K.E. M1
- $$W = (2r - r - r \cos\theta)mg$$
- MW1
-
- $$= rmg(1 - \cos\theta)$$
- W1
- Change in K.E. = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$ M1
- $$= \frac{1}{2}mv^2 - mgr$$
- W1
- $$\frac{1}{2}mv^2 - mgr = mgr - mgr \cos\theta$$
- $$\frac{1}{2}mv^2 = 2mgr - mgr \cos\theta$$
- $$mv^2 = 4mgr - 2mgr \cos\theta$$
- W1
- Res \swarrow $R + mg \cos\theta = \frac{mv^2}{r}$ M1M1W1
- $$R = 4mg - 3mg \cos\theta$$
- $$= mg(4 - 3 \cos\theta)$$
- W1

6 (i)	$P = kn^s I^t l^u r^v$	
	$[T] = [ML^{-1}T^{-2}]^s [ML^2]^t [L]^u [L]^v$	M1W1
	equating indices [T], $1 = -2s \therefore s = -\frac{1}{2}$	M1W1
	equating indices [M], $0 = s + t$	MW1
	$t = -s = \frac{1}{2}$	W1
(ii)	equating indices [L], $0 = -s + 2t + u + v$	MW1
	$0 = \frac{1}{2} + 1 + u + v$	
	$v = -u - \frac{3}{2}$	MW1
(iii)	$P = kn^{\frac{1}{2}} I^{\frac{1}{2}} l^u r^{-\frac{3}{2}-u}$	M1
	so $P = k \sqrt{\frac{I}{nr^3}} \left(\frac{l}{r}\right)^u$	W1
(iv)	$1.7725 = k \sqrt{\frac{I}{nr^3}} \left(\frac{0.4^u}{r^u}\right)$	MW1
	$3.545 = k \sqrt{\frac{I}{nr^3}} \left(\frac{1.6^u}{r^u}\right)$	MW1
	$2 = \frac{1.6^u}{0.4^u} = 4^u$	MW1
	$\therefore u = \frac{1}{2}$	MW1
(v)	$1.7725 = k \sqrt{\frac{2.2 \times 10^{-5} \times 0.4}{4.4 \times 10^{10} \times 16 \times 10^{-16}}}$	MW1
	$k = 1.7725 \times \sqrt{8}$	
	$= 5.013$	
	$\rightarrow 5.01$	W1
		16
	Total	75



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assessing

Module S2: Statistics 2

[AMS41]

FRIDAY 18 JUNE, AFTERNOON

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1 (i) $r = \frac{\sum xy - \frac{(\sum x)(\sum y)}{n}}{\sqrt{\sum x^2 - \frac{(\sum x)^2}{n}}} \sqrt{\sum y^2 - \frac{(\sum y)^2}{n}}$ M1

$$= \frac{233000 - \frac{1364 \times 1361}{8}}{\sqrt{233936 - \frac{1364^2}{8}}} \sqrt{232343 - \frac{1361^2}{8}}$$
 MW1
 MW2

= 0.90401916... = 0.904 (3sf) W1

(ii) Strong positive correlation between the heights of twenty-year-old sons and the heights of their father. M1 6

2 (i) The response variable is to be measured against set values and is subject to variation. M1
 In Hooke's Law experiment the extension is the variable measured in response to the mass. M1

(ii) Other factors remain unchanged e.g. all readings at the same temperature M1
 M1

(iii) $S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n}$

$$= 34750000 - \frac{(13500)^2}{6} = 4375000$$
 MW1

$$S_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n}$$

$$= 45680 - \frac{13500 \times 17.73}{6} = 5787.5$$
 MW1

$$b = \frac{S_{xy}}{S_{xx}} = \frac{5787.5}{4375000} = 1.32 \times 10^{-3} \text{ (3sf)}$$
 MW1

$$a = \bar{y} - b\bar{x} = \frac{17.73}{6} - (1.32 \times 10^{-3}) \times \frac{13500}{6}$$
 MW1
$$= -0.021428571... = -0.0214 \text{ (3sf)}$$
 W1

$$y = (1.32 \times 10^{-3})x - 0.0214$$
 W1

(iv) If $x = 2200$, then

$$y = 1.32 \times 10^{-3} \times 2200 - 0.0214 \quad \text{M1}$$

$$y = 2.88\% \text{ (3sf)} \quad \text{W1}$$

3 (i) $\bar{x} = \frac{\sum x}{n} = \frac{1721}{60} = 28.7 \text{ (3sf)} \quad \text{MW1}$

$$\hat{\sigma}^2 = \frac{1}{n-1} \left(\sum x^2 - \frac{(\sum x)^2}{n} \right) \quad \text{M1}$$

$$= \frac{1}{59} \left(49441 - \frac{1721^2}{60} \right)$$

$$= 1.30 \text{ (3sf)} \quad \text{W1}$$

(ii) $H_0 : \mu = 30 \quad \text{M1}$

$H_1 : \mu \neq 30 \quad \text{M1}$

2 tailed test at 5%
 reject H_0 if $|Z \text{ test}| > 1.96 \quad \text{M1}$
 MW1

$$z_{\text{test}} = \frac{\bar{x} - \mu}{\sqrt{\frac{\hat{\sigma}^2}{n}}} \quad \text{M1}$$

$$= \frac{28.7 - 30}{\sqrt{\frac{1.30}{60}}} \quad \text{MW1}$$

$$\text{MW1}$$

$$= -8.83 \text{ (-8.9 if use exact value)} \quad \text{W1}$$

Since $|z_{\text{test}}| > 1.96$ we reject H_0 and conclude that there is sufficient
 evidence at 5% level to suggest that the train does not take the 30 minutes
 that it should. M1
 M1

13

13

- 4 (i) There is a probability of 0.95 (95%) that the interval contains the true value of the population mean. M2
- (ii) The parent distribution is normally distributed M1
- (iii) $\bar{x} = \frac{\sum x}{n} = \frac{339}{45} = 7.53$ (3sf) MW1
- $$\sigma^2 = \frac{1}{n-1} \left(\sum x^2 - \frac{(\sum x)^2}{n} \right)$$
- M1
- $$= \frac{1}{44} \left(2799 - \frac{339^2}{45} \right) = 5.57$$
- (3sf) W1
- (iv) $CI = \bar{x} \pm 1.96 \sqrt{\frac{\sigma^2}{n}}$ M1
- $$CI = \left(\frac{339}{45} - 1.96 \sqrt{\frac{5.57}{45}}, \frac{339}{45} + 1.96 \sqrt{\frac{5.57}{45}} \right)$$
- MW2
- = (6.84376..., 8.2230...)
- CI = (6.84, 8.22) minutes W2

11

5

Student	A	B	C	D	E	F	G	H	I	J
Initial Score	56	67	48	70	38	66	54	70	45	51
Final Score	63	75	61	74	51	75	67	73	53	62
d	7	8	1	34	1	39	1	33	8	11

M1W1

From calculator

$$\bar{d} = 8.9 \quad \sigma_{n-1} = 3.63 \text{ (3sf)}$$

MW2

$$H_0 : \mu_d = 10$$

M1

$$H_1 : \mu_d < 10$$

M1

One tailed test at 5%

M1

Using t – test at 9 degrees of freedom

M1

Reject H_0 if $t_{\text{test}} < -1.833$

MW1

$$t_{\text{test}} = \frac{\bar{d} - \mu_d}{\hat{\sigma}_d / \sqrt{n}}$$

M1

$$= \frac{8.9 - 10}{3.63 / \sqrt{10}}$$

MW2

$$= -0.958 \text{ (3sf)}$$

W1

Since $t_{\text{test}} > -1.833$ we do not reject H_0 and conclude that there is insufficient evidence at 5% level to suggest that the program maker's claim is incorrect.

M1

M1

15

6 $\bar{X}_{60} \sim N\left(35, \frac{12}{60}\right) = N(35, 0.2)$

MW2

$P(\bar{X}_{60} < 34.7) = P\left(Z < \frac{34.7 - 35}{\sqrt{0.2}}\right) = P(Z < -0.671)$

MW1

$= 1 - \Phi(0.671)$

M1

$= 1 - 0.7489$

W1

$= 0.2511$

$= 0.251$ (3sf)

W1

6

7 (i) $X + Y \sim N(20 + 25, 6 + 4) = N(45, 10)$

M1W1

$P(X + Y > 43) = P\left(Z > \frac{43 - 45}{\sqrt{10}}\right) = P(Z > -0.632)$

MW1

$= \Phi(0.632)$

$= 0.7363$

$= 0.736$ (3sf)

W1

(ii) $P(3X < 2Y) = P(3X - 2Y < 0)$

M1

$3X - 2Y \sim N(3 \times 20 - 2 \times 25, 9 \times 6 + 4 \times 4)$

MW1

$= N(10, 70)$

MW2

$P(3X - 2Y < 0) = P\left(Z < \frac{0 - 10}{\sqrt{70}}\right) = P(Z < -1.195)$

MW1

$= 1 - \Phi(1.195)$

M1

$= 1 - 0.8840$

$= 0.116$

$= 0.116$ (3sf)

W1

11

Total

75

