GCE AS

Mathematics

Summer 2010

Mark Schemes

Issued: October 2010

NORTHERN IRELAND GENERAL CERTIFICATE OF SECONDARY EDUCATION (GCSE) AND NORTHERN IRELAND GENERAL CERTIFICATE OF EDUCATION (GCE)

MARK SCHEMES (2010)

Foreword

Introduction

Mark Schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of 16 and 18-year-old students in schools and colleges. The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes therefore are regarded as a part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

The Council hopes that the mark schemes will be viewed and used in a constructive way as a further support to the teaching and learning processes.

CONTENTS

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2010

Mathematics

Assessment Unit C1

assessing

Module C1: Core Mathematics 1

[AMC11]

WEDNESDAY 9 JUNE, AFTERNOON

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

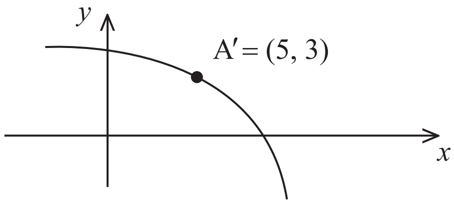
(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

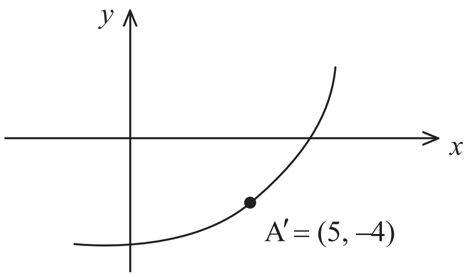
				AVAILABLE MARKS
1	**(i)**	Midpoint P = (1, –2)	MW1	
	(ii)	Grad $m = \frac{-7-3}{4--2} = \frac{-10}{6} = \frac{-5}{3}$	M1W1	
		Perp. grad = $\frac{3}{5}$	MW1	
		$y + 2 = \frac{3}{5}(x - 1)$	M1W1	
		$3x - 5y - 13 = 0$		6
2	**(i)**	$f(-1) = -a - 3 - b + 6 = 12$ $a + b = -9$	M2W1	
	(ii)	$f(3) = 27a - 27 + 3b + 6 = 0$ $9a + b = 7$	M1W1	
	(iii)	$9a + b = 7$ $a + b = -9$ $8a = 16$		
		$a = 2$	MW1	
		$b = -11$	MW1	7

3 **(i)**



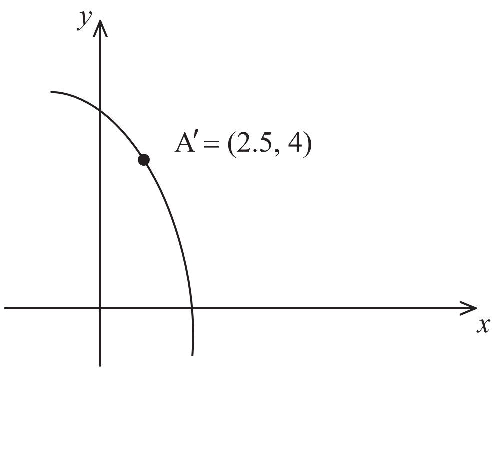
M1W1

(ii)



M1W1

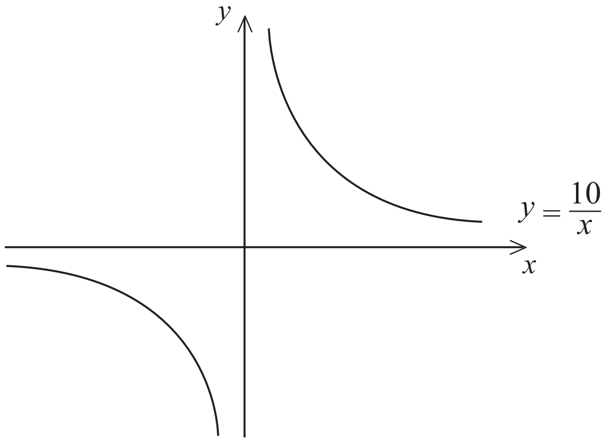
(iii)



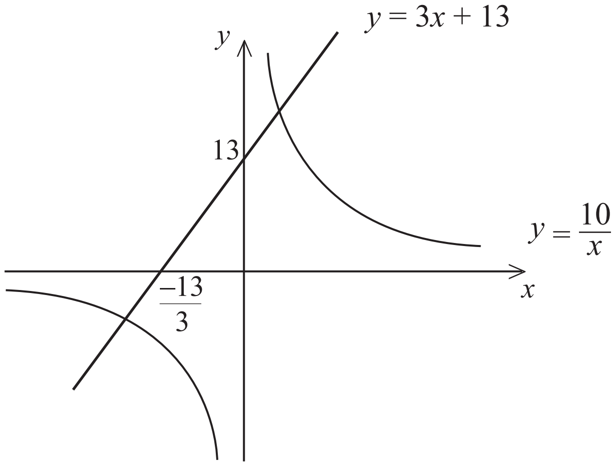
M1W1 — 6

			AVAILABLE MARKS
4 **(a)**	$\dfrac{(3x-2)(3x+2)}{2x+1} \div \dfrac{3x-2}{3(2x+1)}$	M1W1	
	$\dfrac{(3x-2)(3x+2)}{2x+1} \times \dfrac{3(2x+1)}{3x-2}$	MW1	
	$3(3x+2)$	M1W1	
(b)	$\dfrac{(3-\sqrt{7})\times(\sqrt{7}+2)}{(\sqrt{7}-2)\times(\sqrt{7}+2)}$	M1W1	
	$\dfrac{3\sqrt{7}+6-7-2\sqrt{7}}{7-4} = \dfrac{\sqrt{7}-1}{3}$	MW2	
(c)	$3^{x+1} = \dfrac{3^{3x}}{3^2}$	M1W1	
	$3^{x+1} = 3^{3x-2}$	MW1	
	$x+1 = 3x-2$	M1	
	$x = \dfrac{3}{2}$	W1	14
5 **(i)**	$\dfrac{dy}{dx} = 4x^3 - 6x^2$	MW2	
(ii)	$\dfrac{dy}{dx} = 4x^3 - 6x^2 = 0$	M1	
	$x^2(4x-6) = 0$		
	$x = 0$ or $x = \dfrac{3}{2}$	W2	
	$\dfrac{d^2y}{dx^2} = 12x^2 - 12x$	MW1	
	$x = \dfrac{3}{2} \Rightarrow \dfrac{d^2y}{dx^2} = 27 - 18 \Rightarrow +\text{ve} \therefore \text{min at } x = \dfrac{3}{2}$	MW1	
	$x = -1 \quad \dfrac{dy}{dx} = -4 - 6 = -10$	M1	
	$x = +1 \quad \dfrac{dy}{dx} = 4 - 6 = -2 \therefore$ Point of inflection at $x = 0$	W1	9

6 **(i)** M1 MW1



(ii) MW1



(iii) $3x + 13 = \frac{10}{x}$ M1

$3x^2 + 13x - 10 = 0$ W1

$(3x - 2)(x + 5) = 0$ M1

$x = \frac{2}{3}$ or $x = -5$ W2

$y = 15$ or $y = -2$ W1

AVAILABLE MARKS
9

		AVAILABLE MARKS
7 **(a)** **(i)** $2x(x + 10) = 800$	MW1	
$2x^2 + 20x - 800 = 0$	MW1	
$x^2 + 10x - 400 = 0$		
(ii) $x = \dfrac{-10 \pm \sqrt{100 + 1600}}{2}$	M1	
$x = \dfrac{-10 \pm \sqrt{1700}}{2}$	W1	
$\text{Width} = 2x = -10 + 10\sqrt{17}$	MW1	
$\text{Length} = x + 10 = 5 + 5\sqrt{17}$	MW1	
(b) $b^2 - 4ac > 0$		
$9 - 4p(5 - p) > 0$		
$4p^2 - 20p + 9 > 0$	M2	
$(2p - 1)(2p - 9)$	W1	
$p = \dfrac{1}{2}$ or $p = \dfrac{9}{2}$	M1	
	W2	
$p < \dfrac{1}{2}$ or $p > 4\dfrac{1}{2}$	MW1	13
8 **(i)** $P = 2l + 2\pi r$	MW1	
(ii) $400 = 2l + 2\pi r$	M1	
$l = 200 - \pi r$	W1	
(iii) $A = 2lr$	MW1	
$A = 2(200 - \pi r)r$	MW1	
$A = 400r - 2\pi r^2$		
$\dfrac{dA}{dr} = 400 - 4\pi r$	M1W1	
$400 - 4\pi r = 0$	M1	
$r = \dfrac{100}{\pi}$	W1	
$\dfrac{d^2A}{dr^2} = -4\pi \Rightarrow -\text{ve} \quad \therefore \text{ max}$	MW1	
$l = 100$ m	MW1	11
	Total	**75**

Rewarding Learning

ADVANCED
General Certificate of Education
2010

Mathematics

Assessment Unit C2

assessing

Module C2: Core Mathematics 2

[AMC21]

THURSDAY 27 MAY, MORNING

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

				AVAILABLE MARKS
1	(i)	$u_{n+1} = \frac{2}{1+u_n}$		
		$u_1 = 3$		
		$u_2 = \frac{1}{2}$		
		$u_3 = \frac{4}{3}$		
		$u_4 = \frac{6}{7}$		
		$u_5 = \frac{14}{13}$	MW3	
	(ii)	converges and oscillates	MW2	5
2	(i)	centre (0, –2)	MW2	
		radius = $\sqrt{g^2 + f^2 - c}$	M1	
		$= \sqrt{25}$		
		$= 5$	W1	
	(ii)	gradient of radius $\frac{y_2 - y_1}{x_2 - x_1} = \frac{2+2}{3-0} = \frac{4}{3}$	M1W1	
		$\therefore$ gradient of tangent $= \frac{-3}{4}$	MW1	7

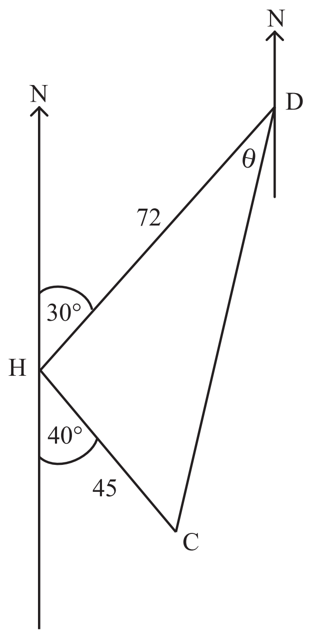
		AVAILABLE MARKS
3 **(i)** Area of sector $\frac{1}{2}r^2\theta$	M1	
$=\frac{1}{2}\times 9\times\frac{\pi}{4}$		
$=\frac{9\pi}{8}$ cm^2	W1	
(ii) Area of triangle $\frac{1}{2}ab\sin c$	M1	
Area of triangle $=\frac{1}{2}\times 1\times 1\times\sin 60°$	W1	
$= 0.433$ cm^2	W1	
$\therefore$ Volume of gold $=\left(\frac{9\pi}{8}-0.433\right)\times 0.1$	M1	
$= 0.310$ cm^3	MW1	7
4 **(i)** $a = 5$		
$ar = 3$ $\qquad \frac{x}{3}=\frac{3}{5}$	M1	
$\therefore r=\frac{3}{5}$ $\qquad x = 3\times\frac{3}{5}=\frac{9}{5}$	W1	
$x=\frac{9}{5}$	MW1	
(ii) $\lvert r\rvert < 1$	MW1	
(iii) $S_\infty=\frac{a}{1-r}$	M1	
$=5\div\frac{2}{5}$		
$=12\frac{1}{2}$	W1	6

			AVAILABLE MARKS
5	**(i)** $(1 + 3x)^4$		
	$= 1 + 4 \times 3x + \dfrac{4 \times 3}{1 \times 2}(3x)^2 + \dfrac{4 \times 3 \times 2}{1 \times 2 \times 3}(3x)^3$	MW3	
	$= 1 + 12x + 54x^2 + 108x^3$	W1	
	(ii) $1 + 12x + 66x^2 = 1 + 12x + 54x^2 + 108x^3$	M1	
	$0 = 108x^3 - 12x^2$	W1	
	$0 = 12x^2(9x - 1)$		
	$\Rightarrow x = \dfrac{1}{9}$	W1	7

		AVAILABLE MARKS
6 **(a)** $\int 3 - x^{-3}\,dx$		
$= 3x + \dfrac{x^{-2}}{2} + c$	MW3	
(b) Area $= \int_0^a 4x^2 - x^3\,dx$	M2W1	
$= \left[\dfrac{4x^3}{3} - \dfrac{x^4}{4}\right]_0^a$	MW2	
$= \dfrac{4a^3}{3} - \dfrac{a^4}{4}$	W1	
if 2 areas equal then $\dfrac{4a^3}{3} - \dfrac{a^4}{4} = 0$	M1	
$a^3\left[\dfrac{4}{3} - \dfrac{a}{4}\right] = 0$		
$a \neq 0$ or $a = \dfrac{16}{3}$	MW1	
or Area $= \int_0^4 4x^2 - x^3\,dx$	M2W1	
$= \left[\dfrac{4x^3}{3} - \dfrac{x^4}{4}\right]_0^4$	MW2	
$= \dfrac{256}{3} - 64 = 21\dfrac{1}{3}$	W1	
Area $= -\int_4^a 4x^2 - x^3\,dx$	M1	
$= -\left[\dfrac{4x^3}{3} - \dfrac{x^4}{4}\right]_4^a$		
$= -\left(\dfrac{4a^3}{3} - \dfrac{a^4}{4} - 21\dfrac{1}{3}\right)$		
if 2 areas equal then $21\dfrac{1}{3} = -\dfrac{4a^3}{3} + \dfrac{a^4}{4} + 21\dfrac{1}{3}$		
$\Rightarrow a = \dfrac{16}{3}$	MW1	11

			AVAILABLE MARKS
7	**(a)** $3\sin^2 x + 8\cos x = 0$		
	$3(1 - \cos^2 x) + 8\cos x = 0$	M1	
	$3 - 3\cos^2 x + 8\cos x = 0$	W1	
	$3\cos^2 x - 8\cos x - 3 = 0$		
	$(3\cos x + 1)(\cos x - 3) = 0$		
	$\cos x = -\frac{1}{3}$ or $\cos x = 3$	MW2	
	$x = \pm 1.91^c$ no sol$^{n:}$	MW3	

(b)



MW3

Distance $d^2 = 72^2 + 45^2 - 2 \times 72 \times 45 \times \cos 110°$ — M1W1

$d = 97.1$ nm — W1

Bearing $\frac{\sin\theta}{45} = \frac{\sin 110°}{97.1}$ — M1W1

$\theta = 25.8°$ — W1

bearing $180° + 30° - 25.8° = 184°$ — MW1 — 17

			AVAILABLE MARKS
8 **(a)**	$3^{2x} = 7$		
	$\log 3^{2x} = \log 7$	M1	
	$2x \log 3 = \log 7$	M1W1	
	$2x = \frac{\log 7}{\log 3}$		
	$= 0.886$	W1	
(b)	$\log x + \log x^2 + 2 \log x^3 = 1$		
	$\log x + 2 \log x + 6 \log x = 1$	M1W1	
	$\log x = \frac{1}{9}$	MW1	
	$x = 1.29$	M1W1	
	or $\log x + \log x^2 + \log x^6 = 1$	M1W1	
	$\log x^9 = 1$	MW1	
	$x^9 = 10$		
	$x = 1.29$	M1W1	
(c)	$\log_2 x - \log_2 y = 6$		
	$\log_2 \frac{x}{y} = 6$	M1W1	
	$2^6 = \frac{x}{y}$	MW1	
	$2^3 \times 2^3 = \frac{x}{y}$	M1	
	$\frac{1}{z^2} = \frac{x}{y}$	MW1	
	$y = xz^2$	W1	15
		Total	**75**

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2010

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

THURSDAY 24 JUNE, MORNING

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

			AVAILABLE MARKS
1	**(i)** $\lvert \mathbf{A} - \lambda\mathbf{I} \rvert = 0$	M1	
	$\Rightarrow \begin{vmatrix} 2-\lambda & 7 \\ 3 & -2-\lambda \end{vmatrix} = 0$	MW1	
	$\Rightarrow (2-\lambda)(-2-\lambda) - 21 = 0$	M1	
	$\Rightarrow -4 + 2\lambda - 2\lambda + \lambda^2 - 21 = 0$		
	$\Rightarrow \lambda^2 - 25 = 0$	W1	
	$\Rightarrow (\lambda - 5)(\lambda + 5) = 0$		
	$\Rightarrow \lambda = \pm 5$	W1	
	(ii) $\begin{pmatrix} 2 & 7 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 5\begin{pmatrix} x \\ y \end{pmatrix}$	M1	
	which gives $2x + 7y = 5x \Rightarrow 7y = 3x$	M1	
	$3x - 2y = 5y \Rightarrow 3x = 7y$		
	Hence an eigenvector is $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$	W1	
	Therefore a corresponding unit eigenvector is $\frac{1}{\sqrt{58}}\begin{pmatrix} 7 \\ 3 \end{pmatrix}$	MW1	9
2	**(i)** $\begin{vmatrix} 9 & 2 & a \\ 1 & -1 & -3 \\ a-1 & 1 & 3 \end{vmatrix} = 0$	M1W1	
	$\Rightarrow 9(-3+3) - 2(3 + 3a - 3) + a(1 + a - 1) = 0$	M1	
	$\Rightarrow -6a + a^2 = 0$	W1	
	$\Rightarrow a(-6 + a) = 0$		
	$\Rightarrow a = 0, 6$	W2	
	(ii) $9x + 2y + 6z = 0$ ①	M1	
	$x - y - 3z = 0$ ②		
	$5x + y + 3z = 0$ ③		
	② + ③ gives $6x + 0 + 0 = 0$	M1	
	Hence $x = 0$	W1	
	Therefore using either ② or ③ gives $y = -3z$	MW1	
	Check equation ① which gives $0 - 6z + 6z = 0$		
	The general solution is $(0, -3t, t)$	W1	11

						AVAILABLE MARKS
3	**(i)**					

∘	p	q	r	s	t	u
p	p	q	r	s	t	u
q	q	p	s	r	u	t
r	r	t	p	u	q	s
s	s	u	q	t	p	r
t	t	r	u	p	s	q
u	u	s	t	q	r	p

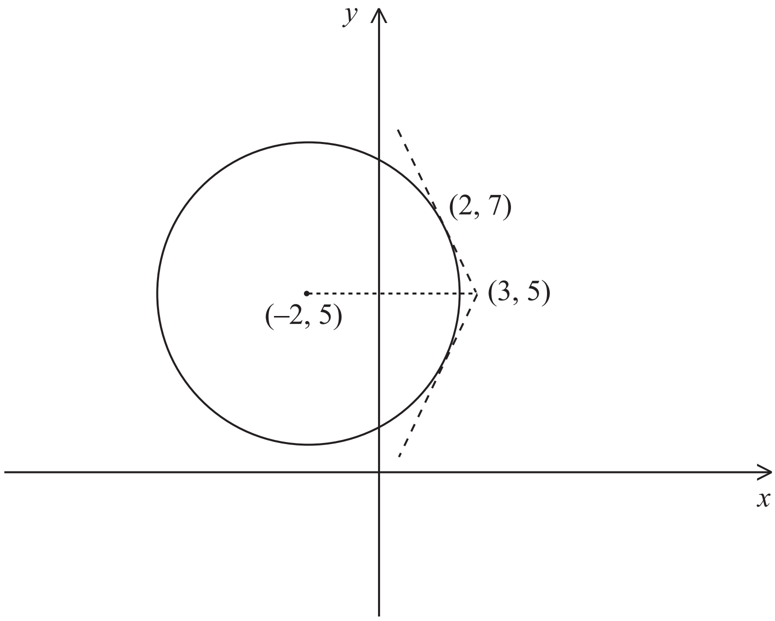
MW6

(ii) Identity is p — M1
$s^3 = p$ and hence s has period 3 — MW1

(iii) Inverse of r is r since $r^2 = p$ — M1W1

(iv) {p, s, t} — M1W1 — 12

			AVAILABLE MARKS
4	**(i)** Centre is (–2, 5)	MW1	
	Gradient of radius joining centre and the point (2, 7)		
	$= \frac{7-5}{2+2} = \frac{1}{2}$	MW1	
	The tangent is perpendicular to the radius.		
	Hence gradient of tangent is –2	MW1	
	Equation of tangent is $y - 7 = -2(x - 2)$	M1	
	which gives $y = -2x + 11$	W1	
	(ii) Substituting the point (3, 5) into the equation of the tangent gives		
	$5 = -2(3) + 11$		
	$\Rightarrow 5 = -6 + 11$	MW1	
	Hence (3, 5) lies on the tangent.		
	(iii)		



MW1

	By symmetry the gradient of the other tangent is 2.	M1W1	
	Using the point (3, 5) we obtain		
	$y - 5 = 2(x - 3)$	M1	
	$\Rightarrow y = 2x - 6 + 5$		
	$\Rightarrow y = 2x - 1$	W1	11

AVAILABLE MARKS

5 **(a)** **(i)** $\mathbf{M} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ MW2

(ii) Image $= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 7 \\ -3 \end{pmatrix}$ M1

$= \begin{pmatrix} -7 \\ 3 \end{pmatrix}$

Hence image is the point (–7, 3) W1

(b) **(i)** Area of T = det **N** × Area of S M1

det **N** = –7 – 8 = –15 MW1

Hence area of T = 15 × 3 = 45 cm^2 W1

(ii) $\begin{pmatrix} 1 & 2 \\ 4 & -7 \end{pmatrix}\begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} y \\ -my \end{pmatrix}$ M1M1

which gives $x + 2mx = y$

$4x - 7mx = -my$ MW1

Divide to give M1

$\dfrac{1 + 2m}{4 - 7m} = \dfrac{1}{-m}$

$\Rightarrow -m - 2m^2 = 4 - 7m$ MW1

$\Rightarrow 2m^2 - 6m + 4 = 0$

$\Rightarrow m^2 - 3m + 2 = 0$ W1

Factorise to give $(m - 2)(m - 1) = 0$

Hence $m = 2, 1$ W2 15

6 **(i)** $\left|\sqrt{2} + \sqrt{2}\,i\right| = \sqrt{2+2}$ M1

Modulus = 2 W1

$\left|1 + \sqrt{3}\,i\right| = \sqrt{1+3}$

Modulus = 2 MW1

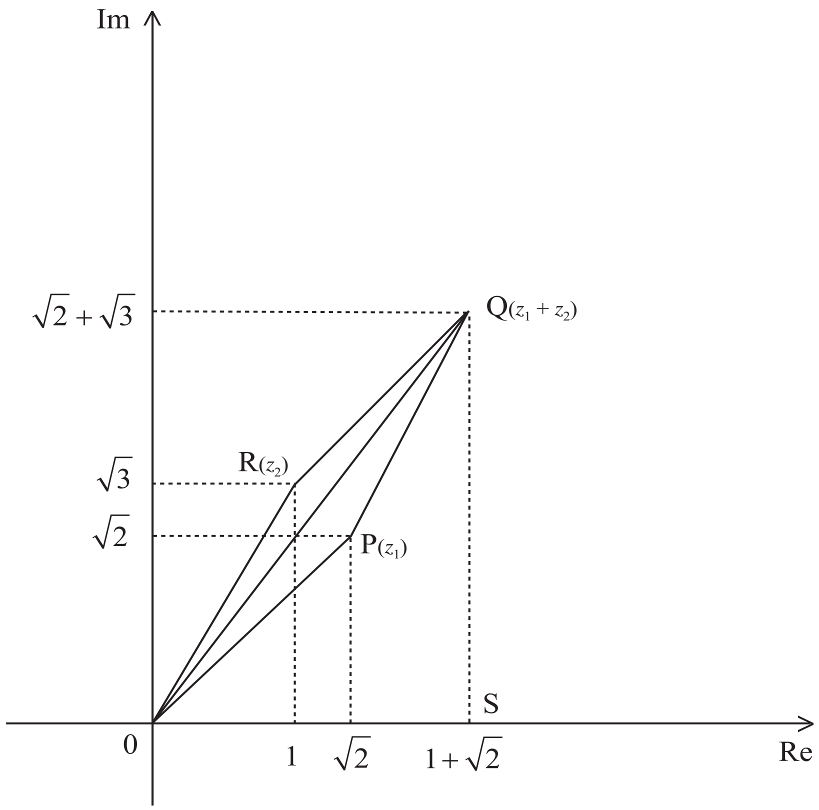
$\arg\left(\sqrt{2} + \sqrt{2}\,i\right) = \tan^{-1}\dfrac{\sqrt{2}}{\sqrt{2}}$ M1

Hence argument $= \dfrac{\pi}{4}$ W1

$\arg\left(1 + \sqrt{3}\,i\right) = \tan^{-1}\dfrac{\sqrt{3}}{1}$

Hence argument $= \dfrac{\pi}{3}$ MW1

(ii)



MW1

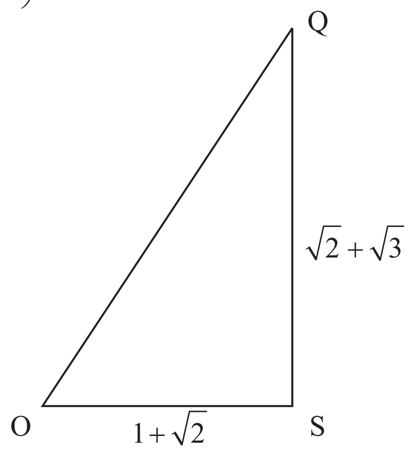
MW1

M1W1

MW1

AVAILABLE MARKS

		AVAILABLE MARKS
(iii) Angle POR $= \frac{\pi}{3} - \frac{\pi}{4}$	M1	
$= \frac{\pi}{12}$	W1	
Angle QOS $= \frac{\pi}{4} + \frac{1}{2}\left(\frac{\pi}{12}\right)$	M1	
$= \frac{7\pi}{24}$	W1	



Hence $\tan\frac{7\pi}{24} = \frac{\sqrt{2}+\sqrt{3}}{1+\sqrt{2}}$	M1W1	17
	Total	**75**

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2010

Mathematics

Assessment Unit M1

assessing

Module M1: Mechanics 1

[AMM11]

TUESDAY 18 MAY, MORNING

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

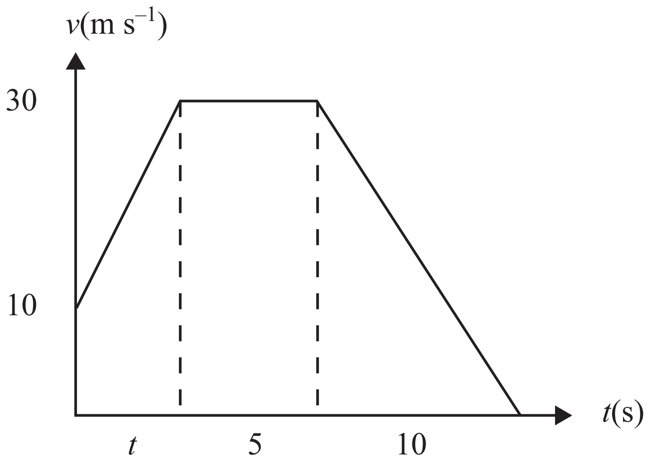
It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

			AVAILABLE MARKS
1	↕ $6 \sin 30° = P \cos 10°$	M1W1	
	$P = 3.05\,N$	M1	
		W1	
	⟷ $Q = P \sin 10° + 6 \cos 30°$	M1W1	
	$Q = 5.73\,N$	W1	7
2	**(i)** 	MW2	
	(ii) at = change in velocity	M1	
	$5 \times t = 30 - 10$		
	$t = 4\,s$	W1	
	(iii) Total distance = area under graph	M1	
	$\frac{10 + 30}{2} \times 4 + \frac{5 + 15}{2} \times 30$	MW2	
	$= 380\,m$	W1	8

AVAILABLE MARKS

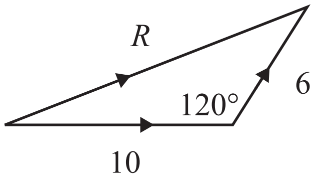
3 **(i)** change in momentum = 0.05 (200–450) M1

$= -12.5\,\text{Ns}$ W1

(ii) $I = -F \times 0.002 = -12.5$ M1

$F = 6250\,\text{N}$ W1 — 4

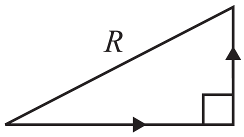
4 **(i)**



$R^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \times \cos 120°$ M1W1

$R = 14\,\text{N}$ W1

or



6 sin 60° M1

10 + 6 cos 60°

$R^2 = (10 + 6\cos 60°)^2 + (6 \sin 60°)^2$ W1

$R = 14\,\text{N}$ W1

(ii) $F = ma$ M1

$14 = 4 \times a$

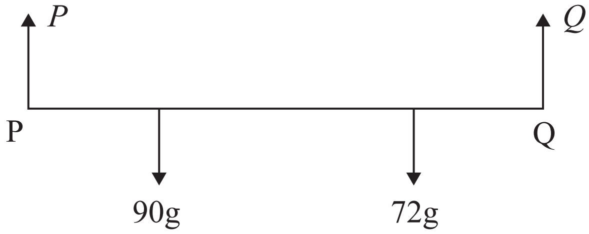
$a = 3.5\,\text{m}\,\text{s}^{-2}$ W1

(iii) $u = 0$

$a = 3.5$ $\quad s = ut + \frac{1}{2}at^2$ M1

$t = 3$ $\quad s = \frac{1}{2} \times 3.5 \times 9$

$s = ?$ $\quad = 15.8\,\text{m}$ W1 — 7

				AVAILABLE MARKS
5	**(i)**		MW2	
	(ii)	$P + Q = 90g + 72g$	M1W1	
		$P + 2P = 162g$		
		$P = 54g\,N$ (529)	MW1	
		$Q = 108g\,N$ (1058)	MW1	
	(iii)	Moments about P ↙ $72g \times 4 + 90g \times x = 108g \times 6$	M2W2	
		$x = 4\,m$	W1	11
6	**(i)**	$a = 4t - 10t^3$		
		$v = \int 4t - 10t^3 \ dt$	M1	
		$= 2t^2 - \frac{5}{2}t^4 + c$	W1	
		at $t = 0 \quad v = \frac{5}{2} \quad \therefore c = \frac{5}{2}$	M1W1	
		$v = 2t^2 - \frac{5}{2}t^4 + \frac{5}{2}$		
	(ii)	$s = \int 2t^2 - \frac{5}{2}t^4 + \frac{5}{2} \ dt$	M1	
		$= \frac{2t^3}{3} - \frac{t^5}{2} + \frac{5}{2}t + d$	W1	
		at $t = 0 \quad s = 0 \quad \therefore d = 0$	MW1	
		$s = \frac{2t^3}{3} - \frac{t^5}{2} + \frac{5}{2}t$		

(iii) when returns to start $s = 0$ M1

$$\frac{2t^3}{3} - \frac{t^5}{2} + \frac{5t}{2} = 0$$

$$4t^3 - 3t^5 + 15t = 0$$

$$3t^5 - 4t^3 - 15t = 0$$

$$t\left(3t^4 - 4t^2 - 15\right) = 0$$ W1

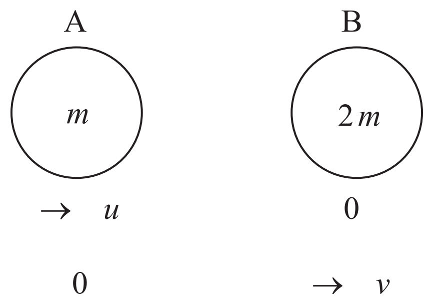
$$t\left(3t^2 + 5\right)\left(t^2 - 3\right) = 0$$ W1

$t = 0$ no sol^n $t^2 = 3$

$$\therefore t = \sqrt{3} \text{ s}$$

returns to start after $\sqrt{3}$ s W1 | 11

7 **(i)** $\rightarrow +$



momentum before = momentum after M1

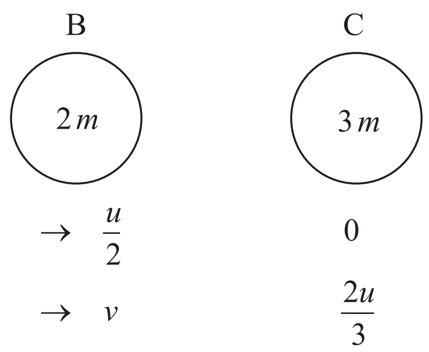
$$mu + 0 = 0 + 2mV$$ MW2

$$V = \frac{u}{2}$$ W1

AVAILABLE MARKS
11

AVAILABLE MARKS

(ii) $\rightarrow +$



momentum before = momentum after

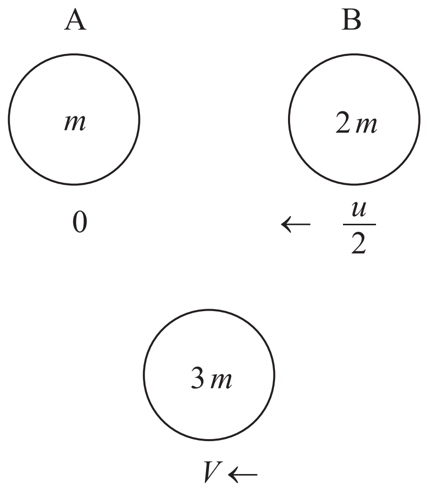
$$2m \times \frac{u}{2} + 0 = 2m \times V + 3m \times \frac{2u}{3}$$ MW2

$$u = 2V + 2u$$ W1

$$V = \frac{-u}{2}$$

$\therefore$ B will collide with A again as B now moving towards A which is at rest. W1

(iii) $+ \leftarrow$

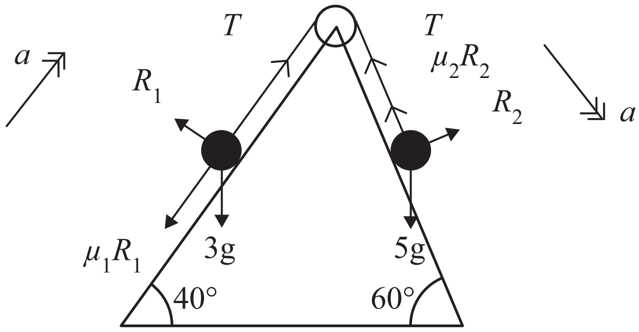


momentum before = momentum after

$$2m \times \frac{u}{2} + 0 = 3m \times V$$ MW1

$$V = \frac{u}{3}$$ W1

Speed $\frac{u}{3}$ in opposite direction to initial direction of A W1 11

			AVAILABLE MARKS
8	**(i)** 	MW3	
	(ii) P $F = ma$	M1	
	$T - \mu_1 R_1 - 3g \sin 40° = 3a$	M1W2	
	Q $F = ma$		
	$5g \sin 60° - T - \mu_2 R_2 = 5a$	MW2	
	$R_1 = 3g \cos 40°$ $R_2 = 5g \cos 60°$	M1W1 MW1	
	$T - 0.3 \times 3g \cos 40° - 3g \sin 40° = 3a$ $5g \sin 60° - T - 0.15g \cos 60° = 5a$	M1	
	$5g \sin 60° - 0.9g \cos 40° - 3g \sin 40° - 0.5g \cos 60° = 8a$		
	$a = 1.79\,\text{m s}^{-2}$	W2	
	$T - 0.9g \cos 40° - 3g \sin 40° = 3 \times 1.79$ $T = 31.0\,\text{N}$	MW1	16
		Total	**75**

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
Summer 2010

Mathematics

Assessment Unit 1

assessing

Module S1: Statistics 1

[AMS11]

FRIDAY 4 JUNE, MORNING

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

				AVAILABLE MARKS
1	Midvalues: 24.5, 74.5, 124.5, 174.5, 224.5, (275)		M1	
	From calculator $n = 200$ $\sum fx = 27300$ $\sum fx^2 = 4412650$			
	$\bar{x} = 136.5$		M1 W1	
	$\sigma_n = 58.574\ldots = 58.6$ (3sf)		M1 W1	5
2	**(i)**	The outcome of one trial does not influence another	M1	
	(ii)	• 2 possible outcomes	M1	
		• probability of success is constant	M1	
	(iii)	Let X be r.v. "the number of people carrying a donor card"		
		$X \sim \mathrm{B}(6, 0.35)$		
		$\mathrm{P}(X = 3) = \binom{6}{3}(0.35)^3(0.65)^3 = 0.235$ (3sf)	M1 W1	
	(iv)	$\mathrm{P}(X \geq 3) = 1 - \mathrm{P}(X < 3)$	M1	
		$= 1 - \mathrm{P}(X = 0, 1, 2)$		
		$= 1 - [\binom{6}{0}(0.35)^0(0.65)^6 + \binom{6}{1}(0.35)^1(0.65)^5$		
		$+ \binom{6}{2}(0.35)^2(0.65)^4]$	MW3	
		$= 1 - [0.0754\ldots + 0.24366\ldots + 0.32800\ldots]$		
		$= 0.3529...$		
		$= 0.353$ (3s.f.)	W1	10

AVAILABLE MARKS

3 **(i)** Let X be r.v. "No. of misprints per page"

$X \sim \text{Po}(2.6)$ M1

$P(X = 2) = \dfrac{e^{-2.6} \times 2.6^2}{2!} = 0.251$ (3s.f.) M1 W1

(ii) $P(X \geq 3) = 1 - P(X < 3)$ M1

$= 1 - P(X = 0, 1, 2)$

$= 1 - e^{-2.6}\left[\dfrac{2.6^0}{0!} + \dfrac{2.6^1}{1!} + \dfrac{2.6^2}{2!}\right]$ MW2

$= 1 - 6.98e^{-2.6}$

$= 0.482$ (3sf) W1

(iii) $P(X = 5 \mid X \geq 3) = \dfrac{P(X = 5 \cap X \geq 3)}{P(X \geq 3)}$ M1

$= \dfrac{P(X = 5)}{P(X \geq 3)}$ MW1

$P(X = 5) = \dfrac{e^{-2.6} \times 2.6^5}{5!} = 0.0735$ MW1

$P(X = 5 \mid X \geq 3) = \dfrac{0.0735}{0.482} = 0.153$ (3s.f.) W1 11

4 **(i)**

x	3	5	7	9	11	13	15
$P(X = x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

MW4

(ii) Distribution is symmetrical. 9 is central value of X M2

(iii) $E(X^2) = 3^2 \times \dfrac{1}{16} + 5^2 \times \dfrac{2}{16} + 7^2 \times \dfrac{3}{16} + 9^2 \times \dfrac{4}{16} + 11^2 \times \dfrac{3}{16}$ M1

$+ 13^2 \times \dfrac{2}{16} + 15^2 \times \dfrac{1}{16} = 91$ W1

$\text{Var}(X) = E(X^2) - [E(X)]^2$ M1

$= 91 - 9^2 = 10$ W1 10

		AVAILABLE MARKS
5 **(i)** $\int_0^1 k(x - x^3)\,dx = 1$	M2	
$k\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 1$	W1	
$k\left(\frac{1}{2} - \frac{1}{4}\right) = 1 \longrightarrow \quad k = 4$	W1	
(ii) $P(0 \le X \le \frac{1}{2}) = \int_0^{\frac{1}{2}} (4x - 4x^3)\,dx$	M1	
$= [2x^2 - x^4]_0^{\frac{1}{2}}$	MW1	
$= \frac{1}{2} - \frac{1}{16} = \frac{7}{16}$	W1	
(iii) $E(X^2) = \int_0^1 x^2(4x - 4x^3)\,dx = \int_0^1 (4x^3 - 4x^5)\,dx$	M1	
$= \left[x^4 - \frac{2}{3}x^6\right]_0^1 = \left(1 - \frac{2}{3}\right) - (0)$	W1	
$= \frac{1}{3}$	W1	
$\text{Var}(X) = E(X^2) - [E(X)]^2$	M1	
$= \frac{1}{3} - (\frac{8}{15})^2 = \frac{11}{225}$ (0.048)	W1	12

	AVAILABLE MARKS

6 **(i)** $X \sim N(4000, \sigma^2)$

$P(X > 4035) = 0.2420$

$\longrightarrow P(X < 4035) = 0.7580$ MW1

$\longrightarrow P\left(Z < \frac{4035 - 4000}{\sigma}\right) = 0.7580$ M1

$\frac{35}{\sigma} = \Phi^{-1}(0.7580)$

$\frac{35}{\sigma} = 0.7$ MW1 W1

$\sigma = 50$ W1

(ii) $P(X > 4110) = P\left(Z > \frac{4110 - 4000}{50}\right) = P(Z > 2.2)$ MW1

$= 1 - \Phi(2.2)$

$= 1 - 0.9861$ MW1

$= 0.0139$ W1

$P(X < 3888) = P\left(Z < \frac{3888 - 4000}{50}\right) = P(Z < -2.24)$ MW1

$= 1 - \Phi(2.24)$

$= 1 - 0.9875$ MW1

$= 0.0125$ W1

$P(X > 4110 \text{ or } X < 3888) = 0.0139 + 0.0125 = 0.0264$ M1

$= 2.64\%$ W1 13

		AVAILABLE MARKS
7 **(a)** **(i)** The events can't both happen	M1	
$P(A \cap B) = 0$	M1	
(ii) It is certain that at least one event occurs	M1	
$P(A \cup B) = 1$	M1	
(b) $P(G) = 0.4$ $P(F) = 0.35$ $P(G \cup F) = 0.61$		
$P(F \cap G) = P(G) + P(F) - P(G \cup F)$	M1	
$= 0.4 + 0.35 - 0.61 = 0.14$	W1	
$P(G) \times P(F) = 0.4 \times 0.35 = 0.14 = P(F \cap G)$	M1	
Hence **Yes** the events are independent	M1	
(c) **(i)** $P(D) = x$ $P(W \mid D) = 0.75$ $P(W \cup D) = 0.36$		
$P(W \cap D) = P(D) \times P(W \mid D) = 0.75x$	M1 W1	
$P(W \cup D) = P(W) + P(D) - P(W \cap D)$	M1	
$0.36 = P(W) + x - 0.75x$		
$\longrightarrow P(W) = 0.36 - 0.25x$	W1	
(ii) $P(W) = 1.25x$		
$1.25x = 0.36 - 0.25x$	MW1	
$1.5x = 0.36$		
$x = 0.24$	W1	14
	Total	**75**