



Rewarding Learning

ADVANCED
General Certificate of Education
2010

Mathematics

Assessment Unit M4
assessing
Module M4: Mechanics 4

[AMM41]



FRIDAY 18 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8 \text{ m s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that
 $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

- 1** A lamina, density $\rho \text{ kg m}^{-2}$, is bounded by the x - and y -axes and the curve

$$y = (4 - x)^{\frac{1}{2}}$$

The curve crosses the axes at A (0, 2) and B (4, 0) as shown in **Fig. 1** below.

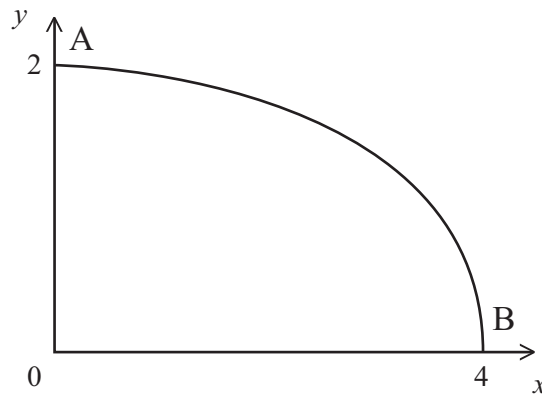


Fig. 1

- (i)** Show that the mass of the lamina is $\frac{16\rho}{3}$ kg. [5]

- (ii)** Show that the moment of the mass of the lamina about the x -axis is

$$\frac{\rho}{2} \int_0^4 (4 - x) dx \text{ kg m} \quad [3]$$

The centre of mass of the lamina is at G.

- (iii)** Find the distance of G from the x -axis. [3]

- 2 The framework ABCDE of six light pin-jointed rods is freely hinged to a rigid vertical wall at A and B. It carries a load of 30 newtons at D as shown in **Fig. 2** below.

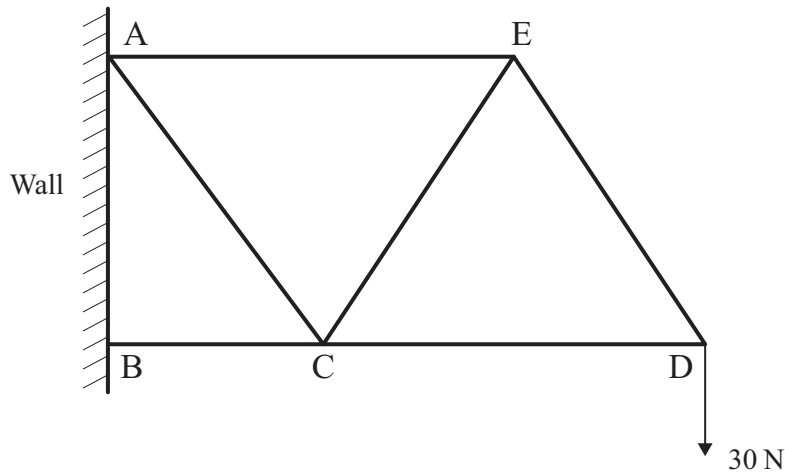


Fig. 2

AE, BC and CD are horizontal.
AE = CD = 1.6 m, BC = 0.8 m.
AC = CE = ED = 1.7 m.

- (i) By considering the forces acting at D, explain briefly why the rod DE is in a state of tension and rod CD is in a state of thrust. [2]
- (ii) Hence find these forces in DE and CD. [4]
- (iii) Explain briefly why the reaction of the wall at B is a horizontal force. [1]
- (iv) Find the magnitude of the reaction of the wall at A. [5]

3 Investigators have arrived at the scene of an accident. Witnesses have reported that the lorry involved simultaneously skidded and toppled outwards.

- (i) Explain briefly why the normal reaction of the road on the inner set of wheels was zero at that instant. [1]

The lorry had been moving at $v \text{ m s}^{-1}$ in a horizontal circle of radius r metres around a bend banked at α to the horizontal.

The centre of mass G of the loaded lorry is d metres from either side of the lorry and h metres above the road surface as shown in **Fig. 3** below.

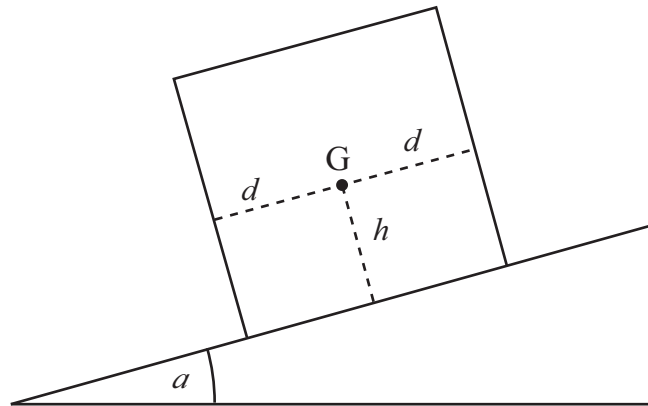


Fig. 3

The coefficient of friction between the sets of wheels and the road is μ .

- (ii) Find μ in terms of d and h . [5]

The lorry had a high load and $h = 1.25d$.

- (iii) Find the value of μ . [1]

The investigators know that

$$v^2 = \frac{rg(\tan \alpha + \mu)}{1 - \mu \tan \alpha}$$

and that $\tan \alpha = \frac{7}{66}$ and $r = 20 \text{ m}$.

- (iv) Find the maximum speed at which the lorry could have negotiated the bend. [3]

- 4 A particle P of mass 1 kg is moving in a straight line at 20 m s^{-1} when it directly impacts on a particle Q of mass m which is at rest. P rebounds at $v \text{ m s}^{-1}$ and Q moves forward at $2v \text{ m s}^{-1}$ in P's original direction of motion.

(i) Find m in terms of v . [3]

Half of the kinetic energy lost by P is transferred to Q.

(ii) Find v . [6]

The coefficient of restitution between the particles is e .

(iii) Find e . [3]

- 5 A small ring of mass m is threaded on to a fixed smooth vertical circular hoop ABP centre C and radius r . The diameter AB is vertical and the angle ACP is θ as shown in Fig. 4 below.

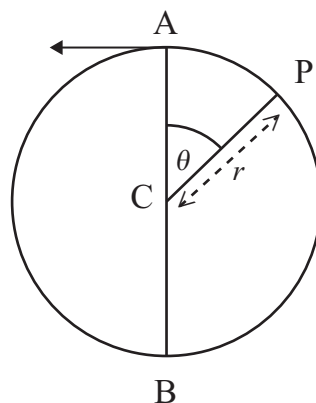


Fig. 4

The ring is projected horizontally from A with speed $\sqrt{2gr}$

The ring moves round the hoop through B eventually passing through A again.

The reaction between the hoop and the ring at P is R .

(i) Show that the magnitude of R is $mg(4 - 3 \cos \theta)$. [10]

(ii) Explain briefly why R always points towards C. [2]

(iii) Show that $R_{\max} = 7R_{\min}$ [2]

- 6 In her laboratory Irina is investigating the period of oscillation P of torsion pendulums. P is believed to be related to the moment of inertia of the disc I , the modulus of rigidity of the wire n , its length l and radius r as follows:

$$P = kn^s I^t l^u r^v$$

where k is a dimensionless constant and the indices have to be determined.

The dimensions of n are $[ML^{-1}T^{-2}]$ and those of I are $[ML^2]$

Use the method of dimensions to:

(i) find s and t ; [6]

(ii) find v in terms of u . [2]

(iii) Show that a possible formula for P is

$$P = k \sqrt{\frac{I}{nr^3}} \left(\frac{l}{r}\right)^u \quad [2]$$

Irina was using copper wire of a particular radius and finding how the period varied as she changed only the length of the wire.

She discovered that when l was 0.4 m, P was 1.7725 s and when l was 1.6 m, P was 3.545 s.

(iv) Using the formula in (iii) find u . [4]

For the disc $I = 2.2 \times 10^{-5} \text{ kg m}^2$

For the copper wire $n = 4.4 \times 10^{10} \text{ kg m}^{-1} \text{ s}^{-2}$ and $r = 2 \times 10^{-4} \text{ m}$.

(v) Find k . [2]

THIS IS THE END OF THE QUESTION PAPER

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