Rewarding Learning

ADVANCED
General Certificate of Education
2010

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

AMF21

TUESDAY 22 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that
$\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Use de Moivre's theorem to find an expression for $\sin 3\theta$ in the form

$$\sin 3\theta \equiv a \sin^3 \theta + b \sin \theta$$

where a and b are integers to be determined. [5]

2 Show that

$$\sum_{k=n+1}^{2n} (2k-1)k(2k+1) \equiv 15n^4 + 14n^3 + \frac{3}{2}n^2 - \frac{1}{2}n$$ [6]

3 **(i)** Use Maclaurin's theorem to derive the series expansion for $(1+x)^n$ up to and including the term in x^3 [5]

(ii) Express in partial fractions

$$\frac{1+x}{(1+2x^2)(1-2x)}$$ [6]

(iii) Hence find the series expansion of

$$\frac{1+x}{(1+2x^2)(1-2x)}$$

up to and including the term in x^3 [5]

4 Consider the differential equation

$$\frac{d^2y}{dx^2} + p\frac{dy}{dx} + 16y = f(x)$$

where p is a constant.

(i) If $f(x) \equiv e^{3x}$ and $p = -10$, find the general solution of the differential equation. [7]

(ii) If instead $f(x) \equiv 0$ and the general solution is of the form $y = (Ax + B)e^{kx}$, write down the possible values of p and k. [4]

5 **(i)** Use mathematical induction to prove

$$2^{n+1} \sin x \cos x \cos(2x) \cos(4x) \ldots \cos(2^n x) \equiv \sin(2^{n+1}x)$$

where n is a non-negative integer. [7]

(ii) Hence find in radians the general solution to the equation

$$\sin x \cos x \cos 2x \cos 4x = \frac{\sqrt{2}}{2^4}$$ [7]

6 A parabola has the y-axis as directrix and focus at S(6, 0) as shown in **Fig. 1** below.

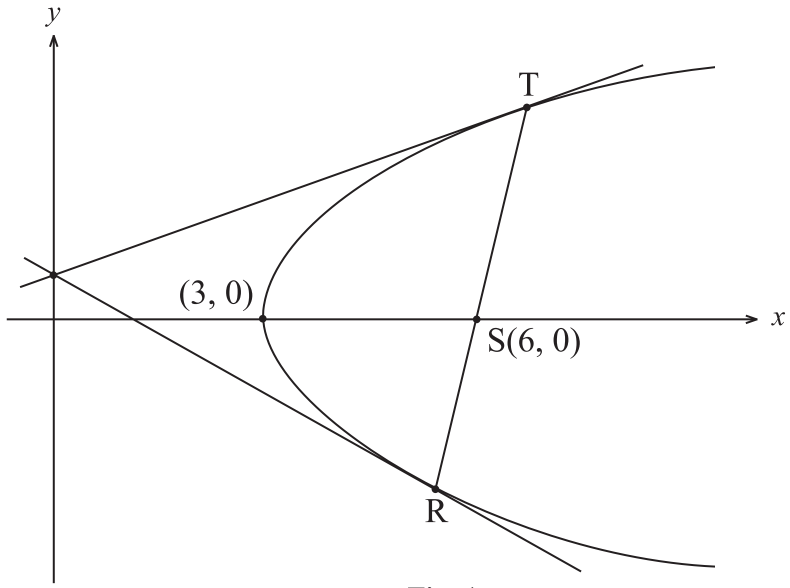

Fig. 1

(i) Show that the equation of the parabola is

$$y^2 = 12(x - 3)$$ [3]

(ii) Verify that any point T with parametric coordinates $(3t^2 + 3, 6t)$ lies on the parabola. [2]

(iii) Show that the equation of the tangent at T can be written as

$$ty - x = 3t^2 - 3$$ [6]

The point R, at the opposite end of the focal chord through T, has parameter $-\frac{1}{t}$

(iv) Show that the tangents at T and R meet on the y-axis. [4]

7 **(i)** Illustrate on an Argand diagram the roots of the equation $z^4 - 1 = 0$ [2]

The roots of the equation $z^8 - 1 = 0$ are illustrated in the Argand diagram in **Fig. 2** below.

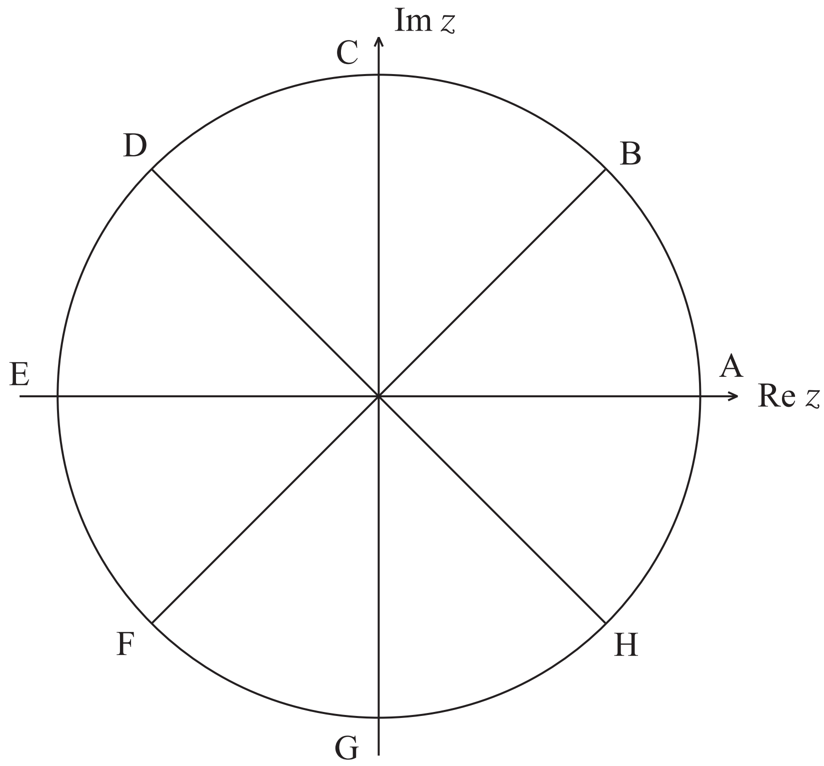

Fig. 2

(ii) Find the root represented by the point B in **Fig. 2** above in the form $r\mathrm{e}^{\mathrm{i}k\pi}$, where r and k are positive numbers. [2]

(iii) Find a complex equation whose roots are B, D, F and H. [4]

THIS IS THE END OF THE QUESTION PAPER