Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2010

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

AMF11

WEDNESDAY 20 JANUARY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that
$\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or a scientific calculator in this paper.

1 The circles C_1 and C_2 are given by the following equations

$$C_1: \quad x^2 + y^2 - 2x - 24 = 0$$

$$C_2: \quad x^2 + y^2 - 6x - 8y + 20 = 0$$

Find the points of intersection of the circles C_1 and C_2 [8]

2 **(a)** The matrix $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Describe fully the transformation represented by **P** [3]

(b) The matrix $\mathbf{Q} = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$

(i) Find the determinant of **Q** [1]

(ii) Explain clearly how this value relates to the areas of a triangle T and its image under the transformation represented by **Q** [1]

The matrix **R** represents the combined effect of the transformation represented by **P** followed by the transformation represented by **Q**

(iii) Calculate the matrix **R** [3]

(iv) The point A is mapped to the point (1, –1) by the matrix **R**
Find the coordinates of A. [4]

3 **(i)** Show that the determinant of

$$\begin{pmatrix} 2 & 1 & a+1 \\ 3 & a & 2 \\ -1 & -3 & 3 \end{pmatrix}$$

is

$$a^2 - 2a - 8$$ [3]

Consider the system of linear equations, where x, y and z are real numbers.

$$2x + y + (a + 1)z = a$$

$$3x + ay + 2z = 2$$

$$-x - 3y + 3z = 6$$

(ii) If $a = 3$, find how many solutions the system of equations has. [3]

(iii) Find how many solutions exist when $a = 4$ [4]

4 A child's toy consists of 5 congruent equally spaced shapes as shown in **Fig. 1** below.

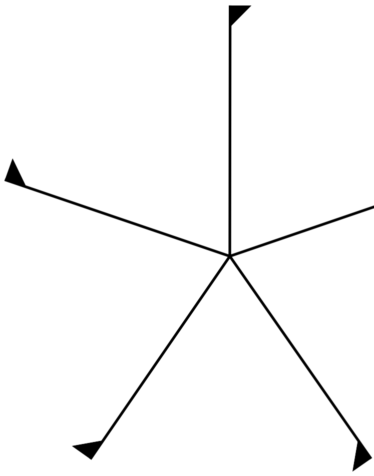

Fig. 1

(i) Define clearly the symmetries of this shape. [3]

(ii) Hence construct the table for the symmetry group G of this shape. [5]

(iii) Copy and complete the table for the group H formed under addition modulo 5

	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2				
3	3				
4	4				

[3]

(iv) Are the groups G and H isomorphic? Justify your answer. [1]

5 The matrix $\mathbf{M} = \begin{pmatrix} 4 & -2 & 0 \\ -2 & 8 & 1 \\ 0 & 1 & 4 \end{pmatrix}$

(i) Find the eigenvalues of **M** [7]

(ii) For the eigenvalue $\lambda = 3$ find a corresponding eigenvector. [4]

(iii) Verify that $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix}$ are eigenvectors of **M** [4]

(iv) If $\mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P} = \mathbf{D}$, where **D** is a diagonal matrix, write down a possible matrix **P** [2]

6 **A solution by scale drawing will not be accepted in this question.**

(a) The complex number p is given by $p = 3 + 2\mathrm{i}$

Calculate $\dfrac{1}{p}$ leaving your answer in the form $a + b\mathrm{i}$, where a and b are rational numbers. [3]

(b) **(i)** Sketch, on an Argand diagram, the locus of those points z which satisfy

$$\arg(z - 3\mathrm{i}) = \frac{\pi}{4}$$ [3]

(ii) On the same diagram, sketch the locus of those points w which satisfy

$$|w - 4 + \mathrm{i}| = |w + 4 - 3\mathrm{i}|$$ [3]

(iii) Find the point of intersection of these loci. [7]

THIS IS THE END OF THE QUESTION PAPER