Rewarding Learning

ADVANCED
General Certificate of Education
January 2010

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

AMF21

WEDNESDAY 3 FEBRUARY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that
$\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find, in radians, the general solution of the equation

$$4 \sin x \cos x + 2 \cos x - 2 \sin x - 1 = 0$$ [7]

2 Use mathematical induction to prove that

$$1^3 + 2^3 + \ \dots \ + n^3 = \tfrac{1}{4}n^2(n+1)^2$$

for all positive integers n. [7]

3 **(i)** Express in partial fractions

$$\frac{1+x}{(x^2+1)(1-x)}$$ [6]

(ii) Hence, or otherwise, find the general solution to the differential equation

$$\frac{dy}{dx} - \frac{2x}{x^2+1}y = \frac{1+x}{1-x}$$ [8]

(iii) Given that $y = 3$ when $x = 0$, find the particular solution. [2]

4 **(a)** Illustrate on an Argand diagram the roots of the equation

$$z^8 - (3e^{i\frac{\pi}{10}})^8 = 0$$ [3]

(b) Let $z = \cos\theta + i\sin\theta$ be a complex number.

(i) Show that

$$\frac{1}{2}(z + z^{-1}) = \cos\theta$$ [2]

(ii) Hence, find an expression for $\cos^6\theta$ in the form

$$\cos^6\theta = a\cos 6\theta + b\cos 4\theta + c\cos 2\theta + d$$

where a, b, c and d are rational numbers which are to be determined. [6]

(iii) Hence find

$$\int \cos^6\theta \, d\theta$$ [2]

5 **(i)** Use Maclaurin's theorem to derive the series expansion for $\cos x$ up to and including the term in x^4 [5]

(ii) Hence show that the series expansion of

$$(1 - x^2)^{-\frac{1}{2}} \cos kx$$

is $1 + (\frac{1}{2} - \frac{1}{2}k^2)x^2 + (\frac{1}{24}k^4 - \frac{1}{4}k^2 + \frac{3}{8})x^4 + \ldots$

where k is a real number. [8]

(iii) If this series expansion takes the form $1 + px^4 + \ldots$, find the values of k and p. [3]

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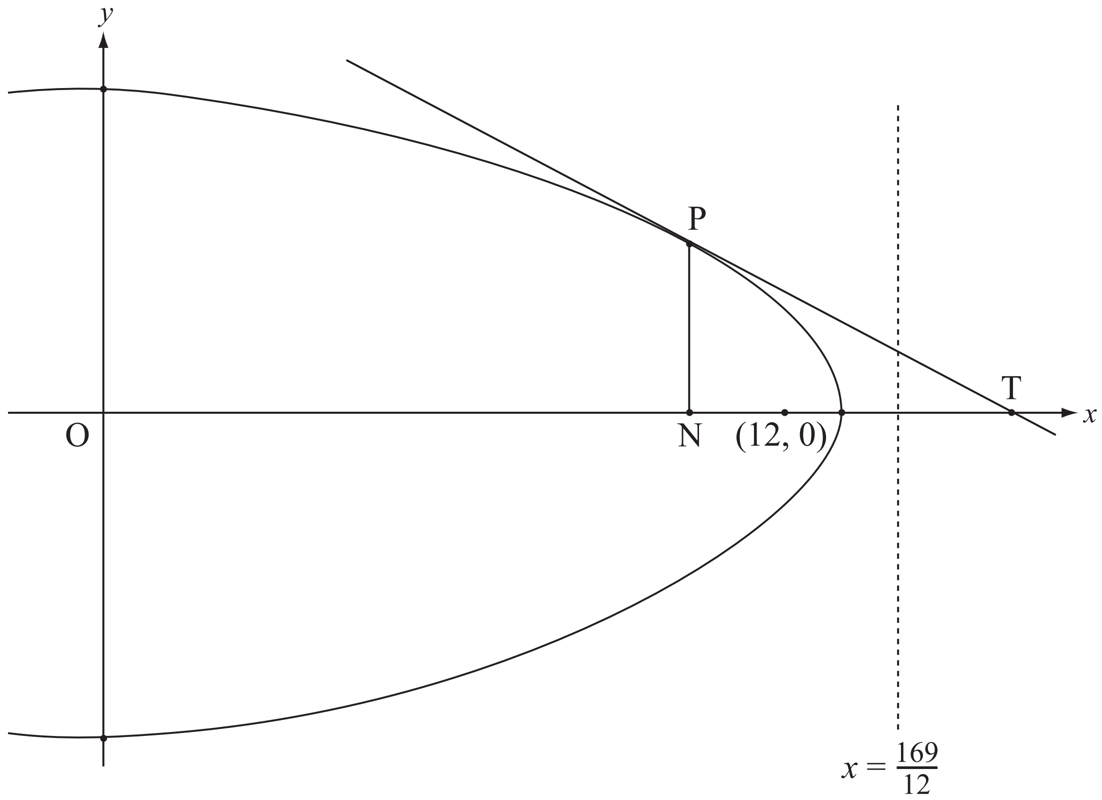

Fig. 1

Fig. 1 above shows part of an ellipse centred at the origin O.

The point (12, 0) is a focus, and the line $x = \frac{169}{12}$ is a directrix of the ellipse.

(i) Show that the equation of the ellipse is

$$\frac{x^2}{169} + \frac{y^2}{25} = 1$$ [5]

P is a point on the ellipse with y coordinate 3

(ii) Show that the equation of the tangent to the ellipse at P is

$$39y + 20x = 325$$ [8]

T is the point where the tangent at P cuts the x-axis and N is the foot of the perpendicular from P to the x-axis.

(iii) Show that OT × ON equals the square of the length of the semi-major axis. [3]