GCE A2

Mathematics

Summer 2009

Mark Schemes

Issued: October 2009

NORTHERN IRELAND GENERAL CERTIFICATE OF SECONDARY EDUCATION (GCSE) AND NORTHERN IRELAND GENERAL CERTIFICATE OF EDUCATION (GCE)

MARK SCHEMES (2009)

Foreword

Introduction

Mark Schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of 16- and 18-year-old students in schools and colleges. The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes therefore are regarded as a part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

The Council hopes that the mark schemes will be viewed and used in a constructive way as a further support to the teaching and learning processes.

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MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

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A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

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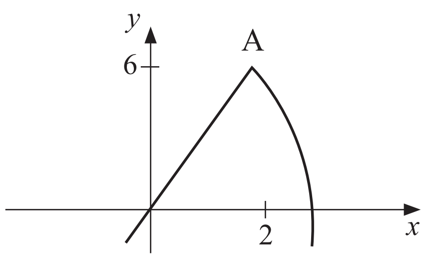
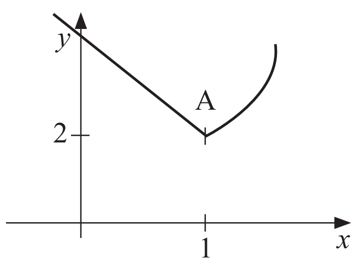
Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

			AVAILABLE MARKS
1 **(i)**	$u = x \qquad v = 4 - x^2$		
	$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = -2x$		
	$\frac{d}{dx}\left(\frac{x}{4 - x^2}\right) = \frac{(4 - x^2) - x(-2x)}{(4 - x^2)^2}$	M1W2	
	$= \frac{4 + x^2}{(4 - x^2)^2}$	MW1	
(ii)	$\frac{d}{dx}[(x^2 + 3)^5] = 5(2x)(x^2 + 3)^4 = 10x\,(x^2 + 3)^4$	M1W2	7
2 **(a)**	$\frac{(-1)(-2)(-3)(2x)^3}{6}$	MW3	
	$= -8x^3$	MW1	
(b)	$\frac{6x - 4}{(2x - 1)^2} = \frac{A}{2x - 1} + \frac{B}{(2x - 1)^2}$	M1W1	
	$6x - 4 = A(2x - 1) + B$	M1	
	coeffs of x $\quad 6 = 2A$	M1	
	$A = 3$	W1	
	$x = \frac{1}{2} \quad -1 = B$	MW1	10
	$\frac{6x - 4}{4x^2 - 4x + 1} = \frac{3}{2x - 1} - \frac{1}{(2x - 1)^2}$		
3 **(a)**	$\int_1^{10} 1 + 20e^{-x}\,dx$	M2W1	
	$= \left[x - 20e^{-x}\right]_1^{10}$	MW2	
	$= 9.99909 - (-6.35758) = 16.357 \approx 16.4$	MW1	
(b)	$3 \ln x - \frac{x^2}{10} + \frac{1}{2} \sec 2x + 7x + c$	MW5	11
4 **(i)**		MW2	
(ii)		MW2	4

				AVAILABLE MARKS
5	(i)	$x^2 + \ln x - 2 = 0$	M1	
		$x = 1 \quad x^2 + \ln x - 2 = -1$	MW1	
		$x = 2 \quad x^2 + \ln x - 2 = 2.693$	MW1	
		Curve is cns between $x = 1$ and $x = 2$ and there is a change of sign therefore there is a root between $x = 1$ and $x = 2$	MW1	
	(ii)	$f(x) = x^2 + \ln x - 2$		
		$f^1(x) = 2x + \frac{1}{x}$	MW2	
		$x_1 = 1 - \frac{-1}{3} = \frac{4}{3}$	M1W1	
		$x_2 = \frac{4}{3} - \frac{0.06545985}{3.41666667} = 1.31417 \approx 1.31$	MW1	9
6	(i)	$t = 0 \rightarrow x = 5$	MW1	
	(ii)	$\frac{dx}{dt} = 2\sqrt{3}\cos 2t - 2\sin 2t$	M1W2	
	(iii)	$\frac{dx}{dt} = 2\sqrt{3}\cos 2t - 2\sin 2t = 0$	M1	
		$2\sqrt{3}\cos 2t = 2\sin 2t$ $\tan 2t = \sqrt{3}$	M1W1	
		$2t = \frac{\pi}{3}, \frac{4\pi}{3}$		
		$t = \frac{\pi}{6}, \frac{2\pi}{3}$	MW1	
		$\frac{d^2x}{dt^2} = -4\sqrt{3}\sin 2t - 4\cos 2t$	M1W1	
		$t = \frac{\pi}{6} \Rightarrow \frac{d^2x}{dt^2} = -6 - 2 = -8 \therefore$ max	MW1	
		$t = \frac{2\pi}{3} \Rightarrow \frac{d^2x}{dt^2} = 6 + 2 = 8 \therefore$ min		11
7	(a)	Let $x = 2\theta - 30$		
		$\sec x = \frac{-2}{\sqrt{3}} \Rightarrow \cos x = \frac{-\sqrt{3}}{2}$	M1W1	
		$x = \pm 150°$ or $x = \pm 210°$	MW2	
		$2\theta - 30° = \pm 150°$ or $2\theta - 30° = \pm 210°$	M1	
		$\theta = 90°, -60°, 120°, -90°$	MW2	
	(b)	$(\mathrm{cosec}^2\theta - 1)(\tan^2\theta + 1)$ $= (\cot^2\theta)(\sec^2\theta)$	M1W2	
		$= \frac{\cos^2\theta}{\sin^2\theta} \times \frac{1}{\cos^2\theta}$	MW2	
		$= \frac{1}{\sin^2\theta}$	MW1	
		$= \mathrm{cosec}^2\,\theta$	MW1	

Alternative Solution

		AVAILABLE MARKS
(b) $(\text{cosec}^2\theta)(\tan^2\theta) + \text{cosec}^2\theta - \tan^2\theta - 1$	MW1	
$\left(\frac{1}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta}\right) + \text{cosec}^2\theta - \tan^2\theta - 1$	MW2	
$\left(\frac{1}{\cos^2\theta}\right) + \text{cosec}^2\theta - \sec^2\theta$	M1W1	
$\sec^2\theta + \text{cosec}^2\theta - \sec^2\theta$	MW1	
$\text{cosec}^2\theta$	W1	14
8 $\frac{dy}{dx} = x^2\left(\frac{3}{3x-2}\right) + 2x\ln(3x-2)$	M2W3	
$x = 1 \quad \frac{dy}{dx} = 3$	MW1	
$m_{\perp} = -\frac{1}{3}$	MW1	
$x = 1 \quad y = 5$	MW1	
$y - 5 = \frac{-1}{3}(x - 1)$		
$3y + x = 16$	MW1	9
	Total	**75**

Rewarding Learning

ADVANCED
General Certificate of Education
2009

Mathematics

Assessment Unit C4
assessing
Module C4: Core Mathematics 4

[AMC41]

WEDNESDAY 20 MAY, AFTERNOON

MARK SCHEME

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			AVAILABLE MARKS
1	$V = \int \pi y^2 \, dx$	M2	
	$= \int_0^a \pi 5 \, x \, dx$	W1W1	
	$= \left[\frac{\pi 5 x^2}{2} \right]_0^a$	W1	
	$= \frac{5\pi a^2}{2}$	W1	6
2	**(i)** distance $= \sqrt{2^2 + 5^2 + 4^2} = \sqrt{45}$ units	M1W1	
	(ii) direction vector $\begin{pmatrix} +2 \\ -5 \\ -4 \end{pmatrix}$	M1W1	
	vector equation of line $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} - 5\mathbf{j} - 4\mathbf{k})$	M1W2	
	(iii) $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} - 5\mathbf{j} - 4\mathbf{k})$		
	if (5, –7, –4) on line then		
	$5\mathbf{i} - 7\mathbf{j} - 4\mathbf{k} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} - 5\mathbf{j} - 4\mathbf{k})$	M1	
	equate coefficients $5 = 1 + 2\lambda \Rightarrow \lambda = 2$		
	$-7 = 3 - 5\lambda \Rightarrow \lambda = 2$		
	$-4 = 4 - 4\lambda \Rightarrow \lambda = 2$	M1	
	$\therefore$ point on line	W2	11

		AVAILABLE MARKS
3 $\int_{-1}^{0} x(1+x)^{\frac{1}{2}}\,dx$		
let $u = 1 + x$	MW1	
$du = dx$		
$x = -1 \quad u = 0$		
$x = 0 \quad u = 1$	MW1	
$= \int_{0}^{1} (u-1)u^{\frac{1}{2}}\,du$	M1W1	
$= \int_{0}^{1} u^{\frac{3}{2}} - u^{\frac{1}{2}}\,du$	W1	
$= \left[\frac{2u^{\frac{5}{2}}}{5} - \frac{2u^{\frac{3}{2}}}{3}\right]_0^1$	W2	
$= \frac{2}{5} - \frac{2}{3}$		
$= \frac{-4}{15}$	MW1	8
4 **(a)** $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$		
$= \frac{2 \times \frac{1}{7}}{1-\frac{1}{49}}$	M1W1	
$= \frac{2}{7} \times \frac{49}{48}$		
$= \frac{7}{24}$	W1	
(b) $3\cos\theta = \sin(\theta + 30°)$	M1W1	
$= \sin\theta\cos 30° + \cos\theta\sin 30°$		
$= \frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta$	MW1	
$\frac{5}{2}\cos\theta = \frac{\sqrt{3}}{2}\sin\theta$		
$\frac{\sqrt{5}}{3} = \tan\theta$	MW2	
$\Rightarrow \theta = 70.9°$ or $251°$	MW2	10

				AVAILABLE MARKS
5	**(i)**	$f(x) > 7$	MW1	
	(ii)	$gf : x \to 3x + 1 \to \frac{1}{3x+1}$	M1W1	
		$gf : x \to \frac{1}{3x+1}$	W1	
		domain $x > 2 \quad x \in \mathbb{R}$	MW1	
		range $0 < gf(x) < \frac{1}{7}$	MW1	6
6	**(i)**	$\frac{d}{dx}\left(\frac{x}{1+x}\right) = \frac{(1+x)-x}{(1+x)^2}$	M1W2	
		$= \frac{1}{(1+x)^2}$	W1	
	(ii)	$\frac{x}{1+x} - x^2 + \frac{y}{1+y} = 0$		
		$\frac{1}{(1+x)^2} - 2x + \frac{1}{(1+y)^2}\frac{dy}{dx} = 0$	M1W3	
		at (1, 1) $\frac{1}{4} - 2 + \frac{1}{4}\frac{dy}{dx} = 0$	M1	
		$\frac{dy}{dx} = 7$	W1	10
7		$\frac{dy}{dx} = \frac{3y}{x+1}$		
		$\int \frac{dy}{y} = \int \frac{3\,dx}{x+1}$	M2W1	
		$\ln y = 3\ln\lvert x+1\rvert + c$	W2	
		when $x = 1,\ y = 16$		
		$\ln 16 = 3\ln 2 + c$		
		$\ln 16 - \ln 8 = c$		
		$c = \ln 2$	M1W1	
		$\ln y = 3\ln\lvert x+1\rvert + \ln 2$		
		$\ln y = \ln 2(x+1)^3$	M2	
		$y = 2(x+1)^3$	MW1	10

			AVAILABLE MARKS
8	**(i)**	$\int_0^2 x e^{-x} dx$	
		$= \left[-xe^{-x}\right]_0^2 + \int_0^2 e^{-x} dx$ M1W2	
		$= \left[-xe^{-x} - e^{-x}\right]_0^2$ W1	
		$= (-2e^{-2} - e^{-2}) - (0 - 1)$ MW2	
		$= 1 - \frac{3}{e^2}$ W1	
	(ii)	$\int \sin^3 x \, dx$	
		$= \int \sin x \sin^2 x \, dx$ M1	
		$\int \sin x \, (1 - \cos^2 x) \, dx$ M1	
		$= \int \sin x - \sin x \cos^2 x \, dx$ W1	
		$= - \cos x + \frac{\cos^3 x}{3} + c$ M1W3	14
		Total	**75**

Rewarding Learning

ADVANCED
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2009

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

FRIDAY 19 JUNE, AFTERNOON

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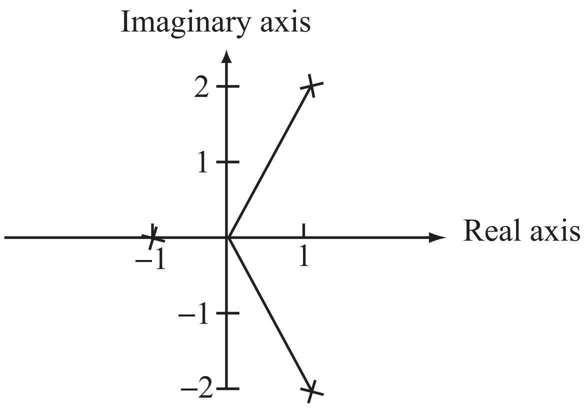
1	$\dfrac{1}{(2x^2+3)(x-1)} = \dfrac{Ax+B}{2x^2+3} + \dfrac{C}{x-1}$	M1W1	
	$1 = (Ax+B)(x-1) + (2x^2+3)C$	MW1	
	$x = 1 \quad 1 = 5C \Rightarrow C = \frac{1}{5}$	M1	
	x^2 coefficient $0 = A + 2C \Rightarrow A = -\frac{2}{5}$		
	x^0 coefficient $1 = -B + 3C \Rightarrow B \quad \frac{3}{5} - 1 = -\frac{2}{5}$	W1	5
	$= \dfrac{-2-2x}{5(2x^2+3)} + \dfrac{1}{5(x-1)}$		
2	$\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$		
	$2\left(\dfrac{\sqrt{3}}{2}\sin\theta - \dfrac{1}{2}\cos\theta\right) = \sqrt{2}$		
	$\sin\theta\cos\alpha - \cos\theta\sin\alpha = \dfrac{1}{\sqrt{2}}$	M1W1	
	$\left.\begin{aligned}\cos\alpha &= \frac{\sqrt{3}}{2} \\ \sin\alpha &= \frac{1}{2}\end{aligned}\right\} \alpha = \dfrac{\pi}{6}$	W1	
	$\sin\left(\theta - \dfrac{\pi}{6}\right) = \dfrac{1}{\sqrt{2}}$	MW1	
	$\theta - \dfrac{\pi}{6} = \begin{cases} 2n\pi + \dfrac{\pi}{4} \\ (2n+1)\pi - \dfrac{\pi}{4} \end{cases}$		
	$\theta = \begin{cases} 2n\pi + \dfrac{5\pi}{12} \\ 2n\pi + \dfrac{11\pi}{12} \end{cases}$	M1W1 W1	7

			AVAILABLE MARKS
3	$= \sum_{r=1}^{n} (2r-1)^3$	M1	
	$= \sum_{r=1}^{n} (8r^3 - 12r^2 + 6r - 1)$	MW1	
	$= 8\left[\frac{1}{4}n^2(n+1)^2\right] - 12\left[\frac{1}{6}n(n+1)(2n+1)\right] + 6\left[\frac{1}{2}n(n+1)\right] - n$	MW4	
	$= n[2n(n^2+2n+1) - 2(2n^2+3n+1) + 3n + 3 - 1]$		
	$= n[2n^3 + 4n^2 + 2n - 4n^2 - 6n - 2 + 3n + 2]$		
	$= n(2n^3 - n) = n^2(2n^2 - 1)$	W1	7
4	Complex conjugate is a root $z = 1 + 2i$	MW1	
	Factors $(z - 1 - 2i)(z - 1 + 2i)$	M1	
	$= (z-1)^2 - (2i)^2 = z^2 - 2z + 5$	W1	
	$z^3 - z^2 + 3z + 5 = (z^2 - 2z + 5)(az + b)$		
	By inspection 3rd factor is $(z + 1)$		
	Roots $z = 1 \pm 2i$ and -1	M1W1	
		MW1	6

			AVAILABLE MARKS
5	$n = 1 \quad u_1 = 2 \times 3^1 + 1 = 7$	MW1	
	Assume $u_k = 2(3^k) + 1$	M1	
	Then $u_{k+1} = 3u_k - 2$	M1	
	$= 3\{2(3^k) + 1\} - 2$	MW1	
	$= 2(3^{k+1}) + 3 - 2$		
	$= 2(3^{k+1}) + 1$	MW1	
	u_1 is correctly given by $u_n = 2(3^n) + 1$ and if u_k is correct, then u_{k+1} is correct so u_n is true for $n \in \mathbb{Z}^+$.	M1	6
6	$m^2 - 6m + 9 = 0$ repeated root $m = 3$	M1W1	
	$y_{CF} = (Ax + B)e^{3x}$	M1W1	
	$y_{PI} = ce^{-3x}$	M1	
	$y' = -3ce^{-3x} \quad y'' = 9ce^{-3x}$	W1	
	$y'' - 6y' + 9y = 9ce^{-3x} + 18ce^{-3x} + 9ce^{-3x}$		
	$\therefore c = 1$	M1W1	
	$y_{GS} = (Ax + B)e^{3x} + e^{-3x}$		
	Using conditions $x = 0, y = 2$		
	$2 = B + 1 \therefore B = 1$	MW1	
	$y' = Ae^{3x} + (Ax + B)3e^{3x} - 3e^{-3x}$	MW1	
	$x = 0 \quad y' = 5$		
	$5 = A + 3 - 3 \quad A = 5$	W1	11
	$y_{PS} = (5x + 1)e^{3x} + e^{-3x}$		

		AVAILABLE MARKS
7 **(i)** $f(\theta) = \sin\theta \quad f(0) = 0$ $f'(\theta) = \cos\theta \quad f'(0) = 1$ $f''(\theta) = -\sin\theta \quad f''(0) = 0$ $f'''(\theta) = -\cos\theta \quad f'''(0) = -1$ $f^{iv}(\theta) = \sin\theta \quad f^{iv}(0) = 0$ $f^{v}(\theta) = \cos\theta \quad f^{v}(0) = 1$	M1W1 MW1	
$f(\theta) = f(0) + \theta\ f'(0) + \dfrac{\theta^2}{2!} f''(0) + \ldots$	M1	
$\sin(\theta) = \theta - \dfrac{\theta^3}{3!} + \dfrac{\theta^5}{5!} + \ldots$	W1	
(ii) $\cos 3\theta + i \sin 3\theta$	M1	
$= (\cos\theta + i\sin\theta)^3$	M1	
$= \cos^3\theta + 3\cos^2\theta i \sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta$	W1	
Compare imaginary parts	M1	
$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$ $= 3(1 - \sin^2\theta)\sin\theta - \sin^3\theta$ $= 3\sin\theta - 4\sin^3\theta$	W1	
(iii) $\sin^3\theta = \frac{1}{4}\{3\sin\theta - \sin 3\theta\}$	MW1	
$= \frac{1}{4}\left\{3\theta - \dfrac{3\theta^3}{3!} + \dfrac{3\theta^5}{5!} - \left(3\theta - \dfrac{27\theta^3}{3!} + \dfrac{243\theta^5}{5!}\right)\right\}$ $= \frac{1}{4}\left\{4\theta - \dfrac{243}{5!}\theta^5 \ldots\right\} = \theta^3 - \frac{1}{2}\theta^5$	M1W2	14

		AVAILABLE MARKS
8 **(i)** $F(a,0) = (2,0)$	MW1	
(ii) $y^2 = 16t^2 = 8 \times 2t^2 = 8x$	M1W1	
(iii) gradient of tangent $= m = \dfrac{dy}{dx}$		
$= \dfrac{dy}{dt} \times \dfrac{dy}{dt}$	M1	
$= \dfrac{4}{4t} = \dfrac{1}{t}$	W1	
gradient of normal $= -\dfrac{1}{m} = -t$	MW1	
equation of normal $y - 4t = -t(x - 2t^2)$	M1W1	
$y + tx = 2t^3 + 4t$	W1	
(iv) For G $tx = 2t^3 + 4t$ ($y = 0$ on normal)	M1	
$x = 2t^2 + 4$	W1	
so $FG = 2t^2 + 4 - 2 = 2(t^2 + 1)$	M1W1	
$FP = \sqrt{\{2t^2 - 2)^2 + (4t - 0)^2\}}$	M1	
$= \sqrt{\{4t^4 - 8t^2 + 4 + 16t^2\}}$		
$= \sqrt{\{4t^4 + 8t^2 + 4\}} = 2t^2 + 2 = FG$	W2	
(v) $F\hat{P}G = F\hat{G}P$ as Δ FPG is isosceles	M1	
$F\hat{G}P = G\hat{P}P'$ alternate angles	M1	
$\therefore\ F\hat{P}G = G\hat{P}P'$	MW1	19
	Total	**75**

Rewarding Learning

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General Certificate of Education
2009

Mathematics

Assessment Unit F3
assessing
Module FP3: Further Pure Mathematics 3

[AMF31]

FRIDAY 22 MAY, MORNING

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

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		Marks	AVAILABLE MARKS
1	$x = \frac{5}{2}\sin u$		
	$\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{5}{2}\cos u$	M1W1	
	$\sqrt{25(1 - \sin^2 u)}$	W1	
	$= 5\cos u$	MW1	
	$\int \frac{\mathrm{d}x}{25 - 4x^2} = \frac{5}{2}\int \frac{\cos u\,\mathrm{d}u}{5\cos u}$		
	$= \frac{u}{2} + c$	W1	
	$= \frac{1}{2}\sin^{-1}\left(\frac{2x}{5}\right) + c$	W1	6
2	Point of intersection		
	$l_1 \sim (3 + 2\lambda, p + 3\lambda, 1 - \lambda)$	MW1	
	$l_2 \sim (3 + \mu, -1 - 2\mu, 4 + \mu)$	MW1	
	Compare **i** and **k** coefficients:	M1	
	$3 + 2\lambda = 3 + \mu$		
	$1 - \lambda = 4 + \mu$	W1	
	Subtract $2 + 3\lambda = -1 \qquad \lambda = -1$	W1	
	$\mu = -2$	W1	
	$p + 3\lambda = -1 - 2\mu$		
	$\therefore\ p = 6$	MW1	
	Substitute $\lambda = -1$ giving coordinates of A(1, 3, 2)	W1	8
3	**(i)** $\frac{\mathrm{d}}{\mathrm{d}x}\{\ \} = \frac{1}{2}\left(\frac{1}{\sqrt{1 - x^2}} + \sqrt{1 - x^2} - x^2(1 - x^2)^{-\frac{1}{2}}\right)$	MW1 M1 W1	
	$= \sqrt{1 - x^2}$	W1	
	(ii) $4x - x^2 - 3 = 1 - (x - 2)^2$	MW1	
	(iii) $\int_2^3 \sqrt{4x - x^2 - 3}\,\mathrm{d}x = \int_2^3 \sqrt{1 - (x - 2)^2}\,\mathrm{d}x$	M1	
	$= \frac{1}{2}\left[\sin^{-1}(x - 2) + (x - 2)\sqrt{1 - (x - 2)^2}\right]_2^3$	W1W1	
	$= \frac{1}{2}(\sin^{-1} 1 + 0) - \frac{1}{2}(\sin^{-1} 0 + 0)$	W1	
	$= \frac{\pi}{4}$	W1	10
4	**(i)** $\cosh^2 2x + \sinh^2 2x \equiv \left(\frac{e^{2x} + e^{-2x}}{2}\right)^2 + \left(\frac{e^{2x} - e^{-2x}}{2}\right)^2$	M1W1	
	$\equiv 2\left(\frac{e^{4x} + e^{-4x}}{4}\right)$	W1	
	$\equiv \cosh 4x$	MW1	

			AVAILABLE MARKS
(ii)	$\cosh^2 2x + \sinh^2 2x = 2$		
	$\Rightarrow \cosh 4x = 2$	M1	
	$x = \pm \frac{1}{4} \cosh^{-1} 2$	W1	
	$= \pm \frac{1}{4} \ln (2 + \sqrt{3})$	M1W1	
(ii)	Alternative solution		
	$\cosh 4x = \frac{e^{4x} + e^{-4x}}{2} = 2$	M1	
	$(e^{4x})^2 - 4e^{4x} + 1 = 0$	W1	
	$e^{4x} = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$	MW1	
	$x = \frac{1}{4} \ln (2 \pm \sqrt{3})$	W1	8
5 (i)	$\overrightarrow{AC} = -3\mathbf{i} + \mathbf{k}$	MW1	
	$\overrightarrow{BC} = 5\mathbf{i} + \mathbf{j} - \mathbf{k}$	W1	
	$\overrightarrow{AC} \times BC = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 0 & 1 \\ 5 & 1 & -1 \end{vmatrix}$	MW1	
	$= -\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$	W1	
(ii)	Plane has explanation $-x + 2y - 3z = d$	M1	
	A (5, 3, 1) on plane		
	$-5 + 6 - 3 = d \quad d = -2$	M1W1	
	Equation $-x + 2y - 3z = -2$ or $x - 2y + 3z = 2$		
(iii)	Equation of perpendicular $\mathbf{r} = 6\mathbf{i} - 6\mathbf{j} + 4\mathbf{k} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$	MW1	
	$\mathbf{r} = (6 - \lambda)\,\mathbf{i} + (-6 + 2\lambda)\mathbf{j} + (4 - 3\lambda)\mathbf{k}$		
	Substitute in $x - 2y + 3z = 2$	M1	
	$(6 - \lambda) - 2(-6 + 2\lambda) + 3(4 - 3\lambda) = 2$	W1	
	$\lambda = 2$	W1	
	Coordinates of P (4, –2, –2)	W1	
(iv)	Vector from (6, –6, 4) to (4, –2, –2)	M1	
	Distance $= \sqrt{4 + 16 + 36} = \sqrt{56} = 2\sqrt{14}$	MW1	14

				AVAILABLE MARKS
6	**(a)**	$y = x - 2\sinh^{-1} x$		
		$\dfrac{dy}{dx} = 1 - \dfrac{2}{\sqrt{x^2+1}}$	M1W1	
		Let $\dfrac{dy}{dx} = 0 \quad \therefore \sqrt{x^2+1} = 2$	MW1	
		$x = \pm\sqrt{3}$	W1	
		Points $(\sqrt{3}, -0.902) \quad (-\sqrt{3}, 0.902)$	W1	
		$(1.73, -0.902) \;\; -(1.73, 0.902)$		
		$\dfrac{d^2y}{dx^2} = 2x(x^2+1)^{-\frac{3}{2}}$	MW1	
		$(\sqrt{3}, -0.902)$ minimum $\quad (-\sqrt{3}, 0.902)$ maximum	W1	
	(b)	$\int \sinh^{-1} x \, dx = \int 1 \sinh^{-1} x \, dx$	M1	
		$v = x \quad \dfrac{dv}{dx} = \dfrac{1}{x^2+1}$	W1	
		$\int \sinh^{-1} x \, dx = x \sinh^{-1} x - \int \dfrac{x}{\sqrt{x^2+1}} \, dx$	W1	
		$= x\sinh^{-1} x - \sqrt{x^2+1}$	W1	
		$\int_{-2}^{0} x - 2\sinh^{-1} x \, dx = \left[\dfrac{x^2}{2} - 2(x\sinh^{-1} x - \sqrt{x^2+1})\right]_{-2}^{0}$	MW1	
		$= 2 - 2 - 4\ln(\sqrt{5} - 2) - 2\sqrt{5})$	W1	
		$= 1.30$	W1	14
7	**(i)**	$y = \dfrac{x^5}{5}(\ln x)^n$		
		$\dfrac{dy}{dx} = x^4(\ln x)^n + \dfrac{x^5}{5} n(\ln x)^{n-1} \dfrac{1}{x}$	M1W1	
		$= x^4(\ln x)^n + \dfrac{n}{5} x^4(\ln x)^{n-1}$	M1	
	(ii)	$\dfrac{d}{dx}\left[\dfrac{1}{5} x^5(\ln x)^n\right] = x^4(\ln x)^n + \dfrac{n}{5} x^4(\ln x)^{n-1}$	M1	
		Integrate between $x = 1$ and $x = e$	M1	
		$\left[\dfrac{1}{5} x^5(\ln x)^n\right]_1^e = \int_1^e x^4(\ln x)^n \, dx + \dfrac{n}{5}\int_1^e x^4(\ln x)n^{-1} \, dx$	W1W1	
		$\dfrac{e^5}{5} = I_n + \dfrac{n}{5} I_{n-1}$		
		$\therefore I_n = \dfrac{e^5}{5} - \dfrac{n}{5} I_{n-1}$	W1	

		AVAILABLE MARKS
(iii) Vol $= \pi \int y^2 \, dx$		
$= \pi \int_1^e x^4 (\ln x)^2 \, dx$	MW1	
$= \pi I_2$	M1	
$= \pi \left[\frac{e^5}{5} - \frac{2}{5} I_1 \right]$	W1	
$= \pi \left[\frac{e^5}{5} - \frac{2}{5} \left(\frac{e^5}{5} - \frac{1}{5} I_0 \right) \right]$	W1	
$I_0 = \int_1^e x^4 \, dx = \frac{e^5}{5} - \frac{1}{5}$	MW1	
$= \pi \left[\frac{e^5}{5} - \frac{2e^5}{25} + \frac{2}{25} \left(\frac{e^5}{5} - \frac{1}{5} \right) \right]$	W1	
$= \pi \left[\frac{17e^5}{125} - \frac{2}{125} \right]$	W1	15
	Total	**75**

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Mathematics

Assessment Unit M2

assessing

Module M2: Mechanics 2

[AMM21]

THURSDAY 11 JUNE, MORNING

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

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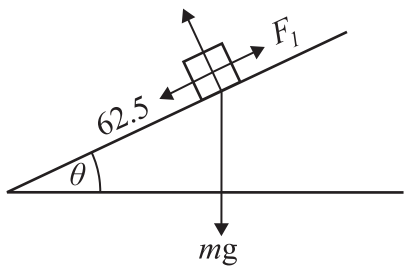
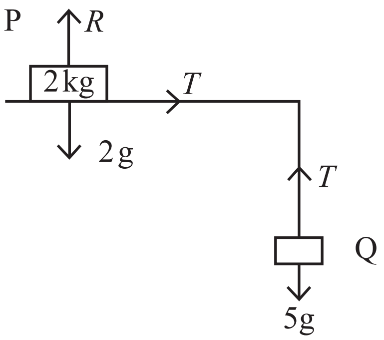
				AVAILABLE MARKS
1	(i)	$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$	M1	
		$\mathbf{F} = (4 + 5 + p)\mathbf{i} + (2 + 2 + q)\mathbf{j} + (1 + 2 - 3)\mathbf{k}$		
		$\mathbf{F} = (9 + p)\mathbf{i} + (4 + q)\mathbf{j}$	W1	
		$\mathbf{F} = m\mathbf{a}$	M1	
		$(9 + p)\mathbf{i} + (4 + q)\mathbf{j} = 5(\mathbf{i} + \mathbf{j})$	W1	
		$9 + p = 5 \quad p = -4$		
		$4 + q = 5 \quad q = 1$	M1, W2	
	(ii)	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$	M1	
		$\mathbf{v} = \mathbf{i} + 2\mathbf{k} + (\mathbf{i} + \mathbf{j})3$	W1	
		$\mathbf{v} = 4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$	W1	
	(iii)	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	
		$\mathbf{s} = (\mathbf{i} + 2\mathbf{k})6 + \frac{1}{2}(\mathbf{i} + \mathbf{j})(36)$	W1	
		$\mathbf{s} = 6\mathbf{i} + 12\mathbf{k} + 18\mathbf{i} + 18\mathbf{j}$		
		$\mathbf{s} = 24\mathbf{i} + 18\mathbf{j} + 12\mathbf{k}$	W1	13
2	(i)	Increase in KE $= \frac{1}{2}mv^2 - \frac{1}{2}mu$	M1	
		$= \frac{1}{2}(80)(15)^2 - 0$	M1	
		$= 9000$ J	W1	
	(ii)	Work done by gravity on skier $= mgh$	M1	
		$= 80(9.8)(300)$		
		$= 235\,200$ J	W1	
	(iii)	Work done by resultant force = change in KE	M1	
		235 200 – work done by R = 9000	W3	
		work done by R = 235 200 – 9000		
		R = 226 200 J	W1	
	(iv)	Skier modelled as a particle		
		Skis ignored in mass, etc.	M1	11

				AVAILABLE MARKS
3	(i)	$\mathbf{v} = 6\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$	MW1	
		$\lvert\mathbf{v}\rvert = \sqrt{(36 + 36 + 9)}$	M1	
		$\lvert\mathbf{v}\rvert = 9\,\text{m}\,\text{s}^{-1}$	W1	
	(ii)	$\mathbf{s} = \int 3t\mathbf{i} - 3t\mathbf{j} + 3\mathbf{k}\,\mathrm{d}t$	M1	
		$\mathbf{s} = \frac{3}{2}t^2\,\mathbf{i} - \frac{3}{2}t^2\,\mathbf{j} + 3t\mathbf{k} + \mathbf{c}$	W1	
		$\mathbf{s} = \frac{3}{2}t^2\,\mathbf{i} - \frac{3}{2}t^2\,\mathbf{j} + 3t\mathbf{k} + \mathbf{i} + 3\mathbf{j}$	M1W1	
		$\mathbf{s} = 24\mathbf{i} - 24\mathbf{j} + 12\mathbf{k} + \mathbf{i} + 3\mathbf{j}$		
		$\mathbf{s} = 25\mathbf{i} - 21\mathbf{j} + 12\mathbf{k}$	W1	8
4	(i)	$P = Fv$		
		$F = \dfrac{P}{v}$	M1W1	
		$F = \dfrac{500}{8}$		
		$F = 62.5\,\text{N}$	W1	
		Moving at constant speed so no acceleration		
		Equate forces $F = S$		
		$S = 62.5\,\text{N}$	MW1	



	(ii)	Force up plane $= F_1 - 62.5 - mg\sin\theta$	M1	
		$= F_1 - 62.5 - 60(9.8)(\frac{1}{7})$	W1	
		$= F_1 - 146.5$	W1	
		but $F_1 = \dfrac{500}{2} = 250$	MW1	
		so $250 - 146.5 = 103.5 = ma$	MW1	
		$a = \dfrac{103.5}{60} = 1.725\,\text{m}\,\text{s}^{-2}$		
		$= 1.73\,\text{ms}^{-2}$ (3 s.f.)	W1	10

			AVAILABLE MARKS
5 **(i)**		MW2	
(ii)	$T = 5\text{g}$ N	M1W1	
(iii)	Using $mr\omega^2$	M1	
	$T = 5\text{g}$ $F = ma$		
	$5\text{g} = mr\omega^2$	MW1	
	$5\text{g} = 2(10)^2 r$	W1	
	$r = \dfrac{5\text{g}}{2(100)}$	M1	
	$r = 0.245$ m	W1	9
6 **(i)**	$F = ma$	M1	
	$-0.005v^2 = 0.2a$ $\dfrac{\mathrm{d}v}{\mathrm{d}t} = -0.025v^2$	MW1W1	
(ii)	$\displaystyle\int_{25}^{v} \frac{\mathrm{d}v}{v^2} = -0.025\int_{0}^{2} \mathrm{d}t$	M2W1	
	$-\left\lvert \dfrac{1}{v} \right\rvert_{25}^{v} = -0.025\, \lvert t \rvert_0^2$	W2	
	$\dfrac{1}{v} - \dfrac{1}{25} = 0.05$	W1	
	$\dfrac{1}{v} = 0.09$	W1	
	$v = 11.1\ \text{m s}^{-1}$	W1	
	Alternative solution without limits		
	$-\dfrac{1}{v} = -0.025t + c$	W2	
	$t = 0,\ v = 25$ so $c = -0.04$	MW1	
	$\dfrac{1}{v} = 0.025t + 0.04$	W1	
	$\dfrac{1}{v} = 0.05 + 0.04$	W1	
	$v = 11.1\ \text{m s}^{-1}$	W1	11

			AVAILABLE MARKS
7	**(i)** Horizontal velocity = $u \cos \theta$	MW1	
	$s = ut + \frac{1}{2} at^2$	M1	
	$t = \dfrac{x}{u \cos \theta}$	W1	
	(ii) $s = ut + \frac{1}{2}at^2$	M1	
	$y = u \sin\theta(t) - \frac{1}{2} g t^2$		
	$y = u \sin\theta\left(\dfrac{x}{u\cos\theta}\right) - \frac{1}{2} g\left(\dfrac{x}{u\cos\theta}\right)^2$	W1MW1	
	$y = x \tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$	W1	
	(iii) $2.5 = 50\,(\tan 30°) - 9.8\left(\dfrac{50^2}{2u^2\cos^2 30°}\right)$	M1W1	
	$u^2 = 619.44$		
	$u = 24.9\ \text{ms}^{-1}$	W1	
	(iv) $v^2 = u^2 + 2as$	M1	
	$0 = (u \sin \theta)^2 - 2gs$	MW1	
	$s = 7.91$ m	W1	13
		Total	**75**

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General Certificate of Education
Summer 2009

Mathematics

Assessment Unit M3

assessing

Module M3: Mechanics 3

[AMM31]

MONDAY 15 JUNE, AFTERNOON

MARK
SCHEME

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AVAILABLE MARKS

1 **(i)**

mass	x		
1	1	$6X = 1 + 8 + 9 = 18$	M2
2	4	$X = 3$	W2
3	3		
6	X		

(ii)

mass	y		
1	a^2	$6Y = a^2 - 2a - 3$	M1
2	$-a$	$Y = \frac{1}{6}(a^2 - 2a - 3)$	W1
3	-1		
6	Y		

$Y = \frac{1}{6}(a^2 - 2a - 3) \ = 0$

$\frac{1}{6}(a + 1)(a - 3) = 0, \quad a > 0$ M1

$a = 3$ W1

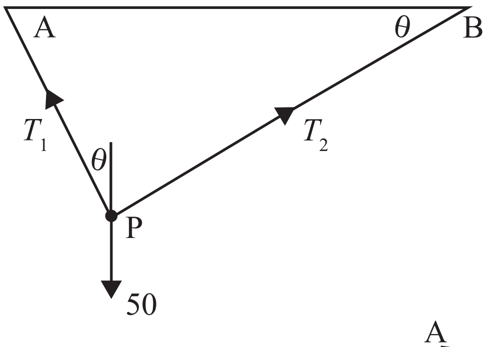
(iii)

mass	z		
		sub $a = 3$	M1
1	3	$6Z = 3 + 6 + 3 = 12$	MW1
2	3	$Z = 2$	W1
3	1		
6	Z		

11

AVAILABLE MARKS

2 **(i)**



$A\hat{P}B = 90°$ M1

$\sin\theta = 0.6$

$\cos\theta = 0.8$ W1

Res (A ↔ P) $T_1 = 50\cos\theta = 50 \times 0.8$ M1

$= 40$ W1

(ii) Hooke

$T_1 = 40 = \dfrac{\lambda}{l}x = \dfrac{\lambda}{0.5}\,0.1$ M1

$\lambda = 200\,\text{N}$ W1

(iii) Re (B ↔ P) $T_2 = 50\sin\theta = 50 \times 0.6 = 30$ M1

Hooke $30 = \dfrac{50(0.8 - l)}{l}$ MW1 MW1

$30l = 40 - 50l$

$80l = 40$

$l = 0.5$ W1

Alternative Solution

(i) M1, W1 for trig as before M1W1

Re ↕ $T_1 0.8 + T_2 0.6 = 50$ MW1

Re ↔ $0.6T_1 = 0.8T_2$ MW1

$0.8T_1 + 0.6\,.\,0.75T_1 = 1.25T_1 = 50$

$T_1 = 40$ MW1

T_1 and T_2 $\therefore\ T_2 = 30$ W1

(ii) Hooke $\dfrac{\lambda 0.1}{0.5} = 40$ M1

$\therefore\ \lambda = 200$ N W1

(iii) Hooke $\dfrac{50(0.8 - l)}{l} = 30$ MW1

$40 - 50l = 30l$

$40 = 80l$

$l = 0.5$ W1 10

			AVAILABLE MARKS
3	**(i)** $v^2 = \omega^2(a^2 - x^2)$	M1	
	$64 = \omega^2(a^2 - 9) = a^2\omega^2 - 9\omega^2$ ①	MW1	
	$36 = \omega^2(a^2 - 16) = a^2\omega^2 - 16\omega^2$ ②	MW1	
	① − ② $28 = 7\omega^2$	M1	
	$\omega^2 = 4, \omega > 0$		
	$\therefore \omega = 2$	W1	
	$\therefore 36 = 4(a^2 - 16)$	M1	
	$9 = a^2 - 16$		
	$a^2 = 25, a > 0$		
	$\therefore a = 5$	W1	
	(ii) $a\omega = 10$	MW1	
	$a\omega^2 = 50$	MW1	
	$\therefore \omega = 5$	W1	
	$a = 2$	W1	11
4	**(i)** $\text{WD} = \begin{pmatrix} 4 \\ -3 \\ 12 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} = 12 - 18 - 24$	M1	
	$= -30\text{ J}$	W1	
	$\lvert \mathbf{F_1} \rvert = \sqrt{16 + 9 + 144} = 13$	W1	
	$\lvert \overrightarrow{\text{AB}} \rvert = \sqrt{9 + 36 + 4} = 7$	W1	
	$13 \times 7 = 91 \neq -30$	W1	
	(ii) $\mathbf{R} = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3}$		
	$= \begin{pmatrix} 4 \\ -3 \\ 12 \end{pmatrix} + \begin{pmatrix} 0 \\ 8 \\ -6 \end{pmatrix} + \begin{pmatrix} 2a \\ -a \\ -2a \end{pmatrix} = \begin{pmatrix} 2a+4 \\ -a+5 \\ -2a+6 \end{pmatrix}$	MW1	
	(iii) $\mathbf{R} . \overrightarrow{\text{AB}} = \begin{pmatrix} 4+2a \\ 5-a \\ 6-2a \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} = 12 + 6a + 30 - 6a - 12 + 4a$	M1	
	$= 30 + 4a$	W1	
	$\frac{1}{2} \cdot 1.26^2 - \frac{1}{2} \cdot 1.24^2 = 50 = 30 + 4a$	M2W1	
	$4a = 20$		
	$a = 5$	W1	12

			AVAILABLE MARKS
5	**(i)** *at* 90°	M1	
	(ii) $t = \dfrac{100}{\frac{1}{3}} = 300\,\text{s} = 5$ mins	M1W1	
	(iii) In 300 s current carries her		
	$300 \times \frac{2}{5} = 120\,\text{m}$ downstream	M1W1	
	(iv) Components of $\frac{1}{3}$		
	$\leftarrow \frac{1}{3}\cos\theta, \uparrow \frac{1}{3}\sin\theta$	M1W1	
	time to cross $\dfrac{100}{\frac{1}{3}\sin\theta} = \dfrac{300}{\sin\theta}$	M1W1	
	Vel downstream $= \frac{2}{5} - \frac{1}{3}\cos\theta$	M1	
	$= \dfrac{1}{15}(6 - 5\cos\theta)$	W1	
	Dist downstream $= \dfrac{300}{\sin\theta} \cdot \dfrac{1}{15}(6 - 5\cos\theta)$	M1	
	$= \dfrac{20(6 - 5\cos\theta)}{\sin\theta}$	W1	
	(v) $d = \dfrac{20\left(6 - 5 \times \frac{5}{6}\right)}{\sin\cos^{-1}\left(\frac{5}{6}\right)}$	M1	
	$= 66.33 \to 66.3\,\text{m}$	W1	
	(vi) Differentiate $\left(\dfrac{\text{d}}{\text{d}\theta}\right)$ and check using 2nd derivative	M1	
	or		
	any other appropriate method.		16

		AVAILABLE MARKS
6 **(i)** $R = 18 - 6x^{\frac{1}{2}}$	MW1	
A = (0, 18)	MW1	
B = ? $x^{\frac{1}{2}} = 3$ $\therefore x = 9$		
B = (9, 0)	MW1	
(ii) $WD = \int_0^{16} 18 - 6x^{\frac{1}{2}}\,dx$	M1W2	
$= \left[18x - 4x^{\frac{3}{2}}\right]_0^{16}$	W1	
$= 18.16 - 4.64$	W1	
$= 32$		
(iii) $\frac{1}{2}\cdot\frac{1}{4}v^2 - \frac{1}{2}\cdot\frac{1}{4}\,.\,12^2 = 32$	M1W1	
$v^2 = 8.32 + 12^2$		
$= 400$		
$v = 20\,\text{ms}^{-1}$	W1	
(iv) $v_{\max}$ at $R = 0$ i.e. $x = 9$	M1	
$W = [18x - 4x^{\frac{3}{2}}]_0^9$		
$\therefore W = 18(9) - 4(27) = 54$	W1	
$\frac{1}{8}v_{\max}^2 - \frac{1}{8}(12^2) = 54$	MW1	
$v_{\max}^2 = 8(54) + 144$		
$= 576$		
$v_{\max} = 24\,\text{ms}^{-1}$	W1	15
	Total	**75**

Rewarding Learning

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General Certificate of Education
Summer 2009

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

[AMM41]

WEDNESDAY 17 JUNE, MORNING

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

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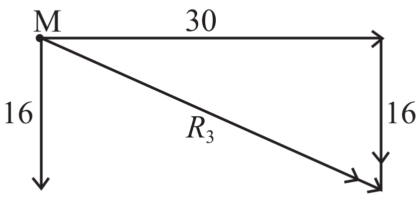
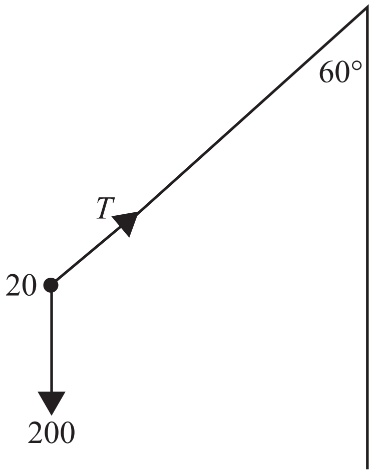
				AVAILABLE MARKS
1	(i)	$f = kP^{\alpha} l^{\beta} \rho^{\gamma}$		
		$[T]^{-1} = [MLT^{-2}]^{\alpha} [L]^{\beta} [ML^{-1}]^{\gamma}$	M1W2	
		$[M] \quad \alpha + \gamma = 0$	M1	
		$[L] \quad \alpha + \beta - \gamma = 0$		
		$[T] \quad -2\alpha = -1$		
		$\therefore \alpha = \frac{1}{2}$	MW1	
		$\therefore \gamma = -\frac{1}{2}$	MW1	
		and $\beta = -1$	W1	
	(ii)	$\dfrac{f_1}{f_6} = 4 = \dfrac{kP^{\frac{1}{2}} l^{-1} \rho_1^{-\frac{1}{2}}}{kP^{\frac{1}{2}} l^{-1} \rho_6^{-\frac{1}{2}}}$	M1 M1	
		$\dfrac{\rho_6^{\frac{1}{2}}}{\rho_1^{\frac{1}{2}}} = 4$		
		$\rho_6 = 16\rho_1$	W1	10
2	(i)	$R_2 = 2 \times 10 \cos\theta = 2 \times 10 \times 0 \times 8 = 16\,\text{N}$	M1W1	
		A and M on line of action	MW1	
	(ii)		M1W1	
		R_3 passes through M as the 16 N and 30 N do.	MW1	
	(iii)	M B (moments) $\quad 34d = 10 \times 0.85 \sin 2\theta$	M2W1	
		$8.5 \times 2 \times 0.6 \times 0.8$	MW1	
		$d = 0.24\,\text{m}$	W1	11

3 (i)



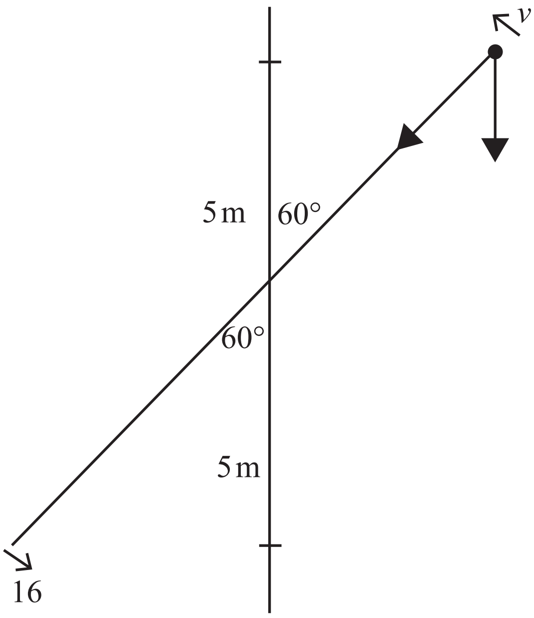
$m = 2a$

R ↗ $\frac{mv^2}{r} = T - 200\cos 60°$ M2W2

$\frac{20.16^2}{10} = T - 100$

$T = 612\,\text{N}$ W1

(ii)



$V_G = 20.(10)(5 + 5) = 2000$

$\text{KE} = 10v^2$ M1W1

$V_G = 0 \quad \text{KE} = \frac{1}{2}\,20.16^2$

$= 2560$ W1

Cons EN $2000 + 10v^2 = 2560$ M1

$v^2 = 56$ W1

R ↙ $\frac{mv^2}{r} = T + 200\cos 60°$ MW2

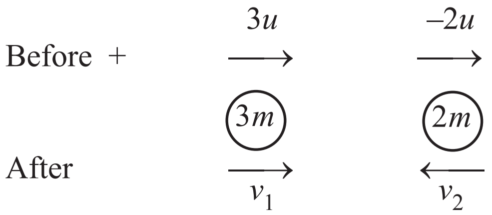
$\frac{2\not{0}.56}{\not{10}} = T + 100$

$T = 12\,\text{N}$ W1

AVAILABLE MARKS

13

4 **(i)**



① Cons Mom. $3mv_1 - 2mv_2 = 9mu - 4mu = 5mu$ M1W2

② Rest $-v_2 - v_1 = -e(-2u - 3u) = 5eu$ M1W1

① $-2v_2 + 3v_1 = 5u$

$3 \times$ ② $-3v_2 - 3v_1 = 15eu$

$-5v_2 = 5u(1 + 3e)$

$v_2 = -(1 + 3e)\,u$ MW1

$\therefore\ v_1 = -v_2 - 5eu = (1 - 2e)\,u$ MW1

(ii) $\Delta = \frac{1}{2}.3m\,.\,au^2 + \frac{1}{2}.2m\,.4u^2 - \frac{1}{2}\,3m(1 - 2e)^2\,u^2 - \frac{1}{2}.2m(1 + 3e)^2\,u^2$ MW2

$= mu^2\left(17\tfrac{1}{2} - \left(\frac{-3}{2} - 6e + 6e^2 + 1 + 6e + 9e^2\right)\right)$ MW1

$= mu^2(15 - 15e^2)$

$= 15mu^2(1 - e^2)$ W1

(iii) $\dfrac{15\cancel{mu^2}(1 - e^2)}{\frac{35}{2}\cancel{mu^2}} = \dfrac{37.5}{100} = \dfrac{3}{8}$ MW2

$120(1 - e^2) = 52.5$

$120e^2 = 67.5$

$e^2 = 0.5625$

$e = 0.75$ W1

AVAILABLE MARKS: 14

			AVAILABLE MARKS
5	**(i)** $M = \frac{2}{3}\pi a^3 \rho$	MW1	
	(ii) $M = \int_0^a \pi \rho x y^2 \, dx$	M1	
	$= \pi\rho \int_0^a x(a^2 - x^2)\, dx$	MW1W1	
	$= \pi\rho \int_0^a (a^2 x - x^3)\, dx$		
	$= \pi\rho \left[\frac{a^2x^2}{2} - \frac{x^4}{4}\right]_0^a$	MW1	
	$= \pi\rho \frac{a^4}{4}$	MW1	
	$\bar{x} = \frac{\pi\rho a^4 .3}{4.2\pi\rho a^3} = \frac{3a}{8}$	M1W1	
	(iii) item — mass — moment	M1	
	20 cm — $\frac{2}{3}\pi\rho\, 20^3$ — $\frac{1}{4}\pi\rho\, 20^4$	W1	
	15 cm — $\frac{2}{3}\pi\rho\, 15^3$ — $\frac{1}{4}\pi\rho\, 15^4$	MW1	
	bowl — $\frac{2}{3}\pi\rho\, (20^3 - 15^3)$ — $\frac{1}{4}\pi\rho\, (20^4 - 15^4)$	W1	
	$\bar{x} = \frac{\frac{1}{4}\pi\rho .5^4 (4^4 - 3^4)}{\frac{2}{3}\pi\rho\, 5^3 (4^3 - 3^3)}$	M1	
	$= \frac{15 . 175}{8 . 37}$		
	$= 8.868$ cm below rim		
	$= 8.87$ cm (3 s.f.)	W1	14

AVAILABLE MARKS

6 **(i)** let the mass of Xeo be m

$$\therefore \frac{GM_E m}{f^2 d^2} = \frac{GM_M m}{(1-f)^2 d^2}$$ M2W2

$$\text{So } \frac{M_E}{M_M} = \frac{f^2}{(1-f)^2}$$ W1

(ii) $$\frac{f^2}{(1-f)^2} = \frac{81}{1}$$ M1

$$\frac{f}{1-f} = 9 \text{ as } f,\ 1-f > 0$$ MW1

$$f = 9 - 9f$$

$$10f = 9$$

$$f = \frac{9}{10}$$ W1

(iii) $$R = 6{\cdot}67\,.10^{-11} \times 500\left(\frac{1{\cdot}99\times10^{30}}{1{\cdot}49^2\times10^{22}} + \frac{16\times7.35\times10^{22}}{3{\cdot}84^2\times10^{16}} - \frac{16\times5{\cdot}98\times10^{24}}{9\times3\cdot84^2\times10^{16}}\right)$$

M1MW2

$= 0.851$ N W1

(iv) S and M pull together against E's pull and $R > 0$ in **(iii)**

$\therefore R = 0$ must be for $f < 0.75$

and $f = 0.9$ in **(ii)** M1 | 13

Total | **75**

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Module S2: Statistics 2

[AMS41]

WEDNESDAY 17 JUNE, MORNING

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GCE Advanced/Advanced Subsidiary (AS) Mathematics

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	AVAILABLE MARKS

1 **(i)** $y = a + bx$

where $b = \dfrac{\sum xy - \dfrac{\sum x \sum y}{n}}{\sum x^2 - \dfrac{(\sum x)^2}{n}} = \dfrac{S_{xy}}{S_{xx}}$ M1

$= \dfrac{11\,063 - \dfrac{(210)(280.7)}{6}}{9100 - \dfrac{210^2}{6}} = \dfrac{1238.5}{1750}$ W1

$= 0.707714 \ldots = 0.708$ (3 s.f.) W1

$a = \bar{y} - b\bar{x}$ M1

$= \dfrac{280.7}{6} - 0.708\left(\dfrac{210}{6}\right)$ W1

$= 22.013 = 22.0$ (3 s.f.) W1

$y = 22.0 + 0.708x$

(ii) $x = 35$ $\hat{y} = 22.0 + 0.708 \times 35$ M1

$= 46.78 = 46.8$ (3 s.f.) W1 — 8

2 **(i)** $n = 20 - 2 = 18$

$\sum x = 158.5 - 6.9 - 9.1 = 142.5$ MW1

$\sum y = 53.7 - 2.7 - 2.8 = 48.2$ MW1

$\sum x^2 = 1266.01 - 6.9^2 - 9.1^2 = 1135.59$ MW1

$\sum y^2 = 146.95 - 2.7^2 - 2.8^2 = 131.82$ MW1

$\sum xy = 422.24 - 6.9 \times 2.7 - 9.1 \times 2.8 = 378.13$ MW1

(ii) $r = \dfrac{\sum xy - \dfrac{\sum x . \sum y}{n}}{\sqrt{\left(\sum x^2 - \dfrac{(\sum x)^2}{n}\right)\left(\sum y^2 - \dfrac{(\sum y)^2}{n}\right)}} = \dfrac{378.13 - \dfrac{142.5 \times 48.2}{18}}{\sqrt{\left(1135.59 - \dfrac{142.5^2}{18}\right)\left(131.82 - \dfrac{48.2^2}{18}\right)}}$ M1 W2

$= -0.7620253432 = -0.762$ (3 s.f.) W1

(iii) Moderate negative correlation between sleep time and reaction time M1
but other factors may be involved. M1 — 11

alternative answer: The outliers have distorted the value of the correlation as it has become more negative when they were removed.

				AVAILABLE MARKS
3	**(i)**	Where the value of a population parameter is estimated by a single value calculated from a sample.	M2	
	(ii)	$\hat{\mu} = \bar{x} = \frac{\sum x}{n} = \frac{690}{50} = 13.8$	M1W1	
		$\hat{\sigma}^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{(\sum x)^2}{n}\right) = \frac{1}{49}\left(10120 - \frac{690^2}{50}\right)$	M1	
		$= 12.2 \text{ (3 s.f.)}$	W1	
	(iii)	$\text{CI} = \bar{x} \pm 1.96\frac{\hat{\sigma}}{\sqrt{n}}$	M1	
		$= 13.8 \pm 1.96\sqrt{\frac{12.2}{50}}$	W2	
		CI = (12.8, 14.8) (3 s.f.)	W2	11
4		From calculator $\bar{x} = 9.78\dot{3} = 9.78$ (3 s.f.)	MW1	
		$\hat{\sigma} = 0.527$ (3 s.f.)	M1W1	
		$H_0: \mu = 10$	M1	
		$H_1: \mu < 10$	M1	
		1-tailed test – t_{test}		
		$v = 12 - 1 = 11$	M1	
		$t_{crit} = -1.796 = t_{11,\,0.95}$	W2	
		$t_{test} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{9.78\dot{3} - 10}{\frac{0.527}{\sqrt{12}}}$	W1 W1	
		$= -1.42$	W1	
		Since $\lvert t_{test} \rvert < 1.796$ we do not reject H_0 and conclude that there is insufficient	M1	
		evidence at 5% level to suggest that the time that a battery works for is less than 10 hours.	M1	13

			AVAILABLE MARKS
5	**(i)** mean = 75g	MW1	
	variance $= \frac{6}{6} = 1\text{g}^2$	MW1	
	(ii) $\bar{X}_6 \sim N(75, 1)$		
	$P(\bar{X}_6 > 76) = P\left(Z > \frac{76-75}{1}\right)$		
	$= P(Z > 1)$	M1	
	$= 1 - \Phi(1)$	M1	
	$= 1 - 0.8413$	W1	
	$= 0.1587$	W1	6
6	$H_0: \mu = 110$	M1	
	$H_1: \mu \neq 110$	M1	
	Two-tailed test at 5% level	M1	
	$Z_{crit} = \pm 1.96$	MW2	
	$Z_{test} = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$		
	$= \dfrac{111.6 - 110}{\frac{5.8}{\sqrt{34}}} = 1.61 \text{ (3 s.f.)}$	W2 W1	
	As $\lvert Z_{test}\rvert < 1.96$ we do not reject H_0 and conclude that at 5% level there is insuffcent evidence to suggest that the pupils' IQ differs from the national average	M1 M1	10

		AVAILABLE MARKS
7 **(i)** $E(C) = 212 + 25 = 237$ (ml)	M1W1	
$Var(C) = 2.2 + 1.7 = 3.9$ (ml^2)	M1W1	
(ii) $C \sim N(237, 3.9)$		
$P(C > 240) = P\left(Z > \dfrac{240 - 237}{\sqrt{3.9}}\right) = P(Z > 1.519)$	MW1	
$= 1 - \Phi(1.519) = 1 - 0.9356$	M1W1	
$= 0.0644$	W1	
With sweetener $S \sim N(237 + 1, 3.9 + 0.1)$		
$S \sim N(238, 4)$	MW2	
$P(S > 240) = P\left(Z > \dfrac{240 - 238}{2}\right) = P(Z > 1)$	MW1	
$= 1 - \Phi(1) = 1 - 0.8413 = 0.1587$	W1W1	
P(overflowing) $= 0.62 \times 0.0644$		
$+ 0.38 \times 0.1587$	M1W1	
$= 0.100234$		
$\approx 10\%$	W1	16
	Total	**75**