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General Certificate of Education  
2009

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## Mathematics

Assessment Unit M3

*assessing*

Module M3: Mechanics 3

[AMM31]



MONDAY 15 JUNE, AFTERNOON

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### TIME

1 hour 30 minutes.

### INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all six** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

### INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take  $g = 9.8 \text{ m s}^{-2}$ , unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that

$\ln z \equiv \log_e z$

**Answer all six questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

- 1** Three particles of mass 1 kg, 2 kg and 3 kg are situated at fixed points whose coordinates, where  $a > 0$ , are shown in **Table 1** below.

**Table 1**

<b>Mass (kg)</b>	<b>Point</b>
1	$(1, a^2, 3)$
2	$(4, -a, a)$
3	$(3, -1, 1)$

The centre of mass of the system is  $(X, Y, Z)$ .

- (i) Find  $X$ . [4]
- (ii) Given that  $Y = 0$ , find  $a$ . [4]
- (iii) Hence find  $Z$ . [3]

- 2 A and B are two points on a fixed horizontal beam one metre apart. A particle P of weight 50 N is attached to two light elastic strings. The other ends of the strings are attached to A and B. P hangs in equilibrium with AP stretched to 0.6 m and BP stretched to 0.8 m as shown in Fig. 1 below.

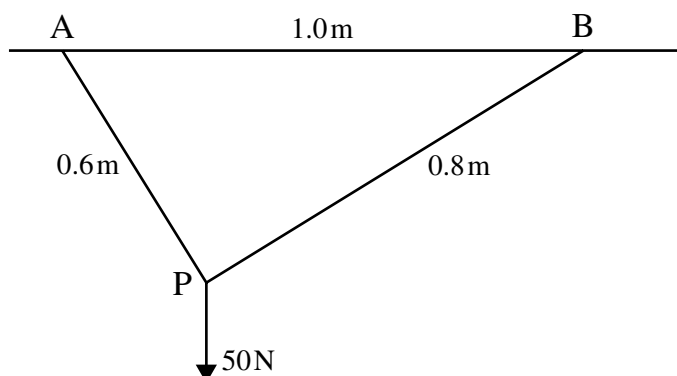


Fig. 1

- (i) By resolving in the direction of AP, or otherwise, show that the tension in AP is 40 N. [4]

The natural length of the string AP is 0.5 m.

- (ii) Find the modulus of elasticity of AP. [2]

The modulus of elasticity of the string BP is 50 N.

- (iii) Find the natural length of BP. [4]

- 3 The equation of motion of a particle moving in a straight line with S.H.M. of amplitude  $a$  is given by

$$\ddot{x} = -\omega^2 x$$

where  $x$  is the displacement from the centre of oscillation.

A particle P moving with S.H.M. has velocity  $8 \text{ m s}^{-1}$  when  $x$  is 3 m and has velocity  $6 \text{ m s}^{-1}$  when  $x$  is 4 m.

- (i) Find  $a$  and  $\omega$  for the motion of P. [7]

A second particle Q also moving with S.H.M. has a maximum speed of  $10 \text{ m s}^{-1}$  and an acceleration of maximum magnitude  $50 \text{ m s}^{-2}$

- (ii) Find  $a$  and  $\omega$  for the motion of Q. [4]

- 4 Three forces  $\mathbf{F}_1$ ,  $\mathbf{F}_2$  and  $\mathbf{F}_3$  newtons move a particle of mass 1 kg from A to B, where AB is measured in metres and

$$\vec{AB} = \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix}$$

- (i) Given that

$$\mathbf{F}_1 = \begin{pmatrix} 4 \\ -3 \\ 12 \end{pmatrix}$$

show that the work done by  $\mathbf{F}_1$  is **not**  $|\mathbf{F}_1| \times |\vec{AB}|$  [5]

The other two forces  $\mathbf{F}_2$  and  $\mathbf{F}_3$  are:

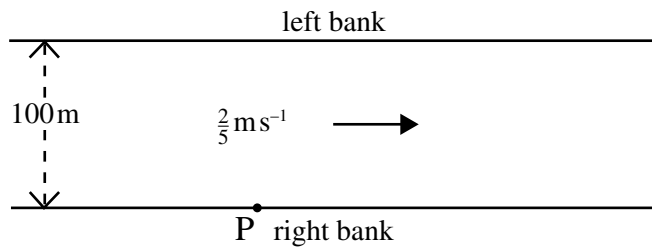
$$\mathbf{F}_2 = \begin{pmatrix} 0 \\ 8 \\ -6 \end{pmatrix} \qquad \mathbf{F}_3 = \begin{pmatrix} 2a \\ -a \\ -2a \end{pmatrix}$$

- (ii) Find, in terms of  $a$ , the resultant of the three forces. [1]

The particle has speeds of  $24 \text{ m s}^{-1}$  at A and  $26 \text{ m s}^{-1}$  at B.

- (iii) Use the Work–Energy Principle to find  $a$ . [6]

- 5 **Fig. 2** below shows part of the River Nagal where the banks are parallel and 100 m apart. The water in the river is flowing at  $\frac{2}{5} \text{ m s}^{-1}$



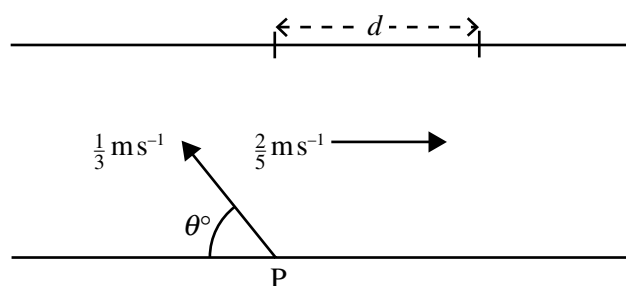
**Fig. 2**

Jean swims only on backstroke at  $\frac{1}{3} \text{ m s}^{-1}$   
She is at the point P on the right bank.

Jean wishes to cross the river as quickly as possible.

- (i) At what angle to the bank should she swim? [1]
- (ii) Find the time taken to cross the river. [2]
- (iii) Find how far downstream she will be when she reaches the left bank. [2]

Jean has returned to P and now wants to cross so that she lands on the left bank a shorter distance,  $d$  metres, downstream. She now swims at an angle  $\theta^\circ$  to the bank as shown in **Fig. 3** below.



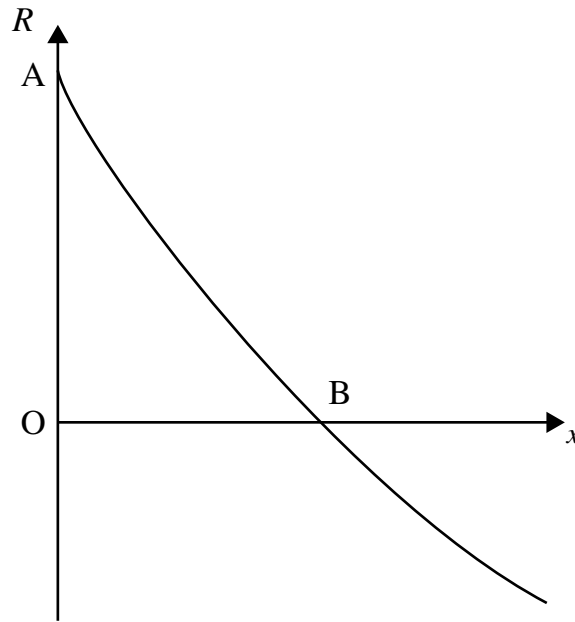
**Fig. 3**

- (iv) Show that  $d = \frac{20(6 - 5 \cos \theta)}{\sin \theta}$  [8]
- (v) Find  $d$  when  $\cos \theta = \frac{5}{6}$  [2]
- (vi) How could you check that this is a minimum value of  $d$ ? [1]

- 6 A particle P of mass 0.25 kg is moving towards the right along a straight horizontal track. The distance of P from a fixed point O on the track is  $x$  metres. Two forces act on P:  
 a constant frictional force of 2 N and  
 a variable force  $F$  newtons, towards the right, where

$$F = 20 - 6\sqrt{x} \quad 0 \leq x \leq 16$$

The graph of the resultant  $R$  of the two forces is shown in **Fig. 4** below.



**Fig. 4**

The intercepts on the axes are A and B.

- (i) State the coordinates of the points A and B. [3]
- (ii) Find the work done by the resultant force  $R$  for  $0 \leq x \leq 16$  [5]

The speed of P at O is  $12 \text{ m s}^{-1}$

- (iii) Use the Work–Energy Principle to find the speed of the particle when  $x = 16$  [3]
- (iv) Find the maximum speed reached by the particle during this motion. [4]

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**THIS IS THE END OF THE QUESTION PAPER**

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