Rewarding Learning

ADVANCED
General Certificate of Education
2009

Mathematics

Assessment Unit F3

assessing

Module FP3: Further Pure Mathematics 3

[AMF31]

AMF31

FRIDAY 22 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Use the substitution $x = \frac{5}{2}\sin u$ to find

$$\int \frac{\mathrm{d}x}{\sqrt{25 - 4x^2}}$$ [6]

2 Straight lines l_1 and l_2 have equations

$$l_1 \qquad \frac{x-3}{2} = \frac{y-p}{3} = \frac{z-1}{-1}$$

$$l_2 \qquad \frac{x-3}{1} = \frac{y+1}{-2} = \frac{z-4}{1}$$

where p is a scalar constant.
The lines intersect at the point A.
Find the value of p and the coordinates of the point A. [8]

3 **(i)** Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{\frac{1}{2}\left(\sin^{-1} x + x\sqrt{1-x^2}\right)\right\} = \sqrt{1-x^2}$$ [4]

(ii) Write $4x - x^2 - 3$ in the form $a - (x - b)^2$ [1]

(iii) Hence find the exact value of

$$\int_2^3 \sqrt{4x - x^2 - 3}\,\mathrm{d}x$$ [5]

4 **(i)** Using the definition of the hyperbolic functions in terms of the exponential function, prove that

$$\cosh^2 2x + \sinh^2 2x \equiv \cosh 4x \qquad [4]$$

(ii) Hence solve the equation

$$\cosh^2 2x + \sinh^2 2x = 2$$

leaving your answers in logarithmic form. [4]

5 A plane Π passes through the points A (5, 3, 1), B (–3, 2, 3) and C (2, 3, 2).

(i) Find $\overrightarrow{AC} \times \overrightarrow{BC}$. [4]

(ii) Hence or otherwise find in Cartesian form an equation for Π. [3]

The perpendicular from the point Q(6, –6, 4) to Π meets the plane at the point P.

(iii) Find the coordinates of P. [5]

(iv) Show that the perpendicular distance from Q(6, –6, 4) to the plane is $2\sqrt{14}$ [2]

6 **(a)** Find the coordinates of the stationary points on the curve with equation

$$y = x - 2\sinh^{-1} x$$

and determine their nature. [7]

(b) Evaluate

$$\int_{-2}^{0} x - 2\sinh^{-1} x \, dx$$

correct to 2 decimal places. [7]

7 **(i)** Differentiate with respect to x

$$\frac{x^5}{5}(\ln x)^n \qquad [3]$$

For each non-negative integer n, let

$$\mathrm{I}_n = \int_1^{\mathrm{e}} x^4 (\ln x)^n \, \mathrm{d}x$$

(ii) Using your answer to **(i)** or otherwise, show that if $n \geqslant 1$, then

$$\mathrm{I}_n = \frac{1}{5}\mathrm{e}^5 - \frac{n}{5}\mathrm{I}_{n-1} \qquad [5]$$

The shaded region in **Fig. 1** below is bounded by the curve with equation $y = x^2 \ln x$, the line $x = \mathrm{e}$ and the x-axis.

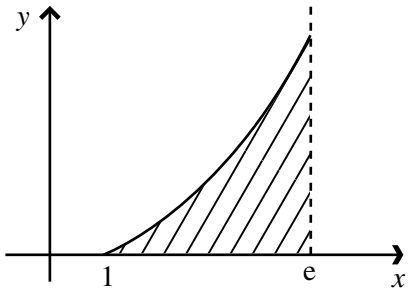

Fig. 1

The region is rotated through 2π about the x-axis.

(iii) Show that the volume of the solid formed is $\frac{\pi}{125}[17\mathrm{e}^5 - 2]$. [7]

THIS IS THE END OF THE QUESTION PAPER