



Rewarding Learning

ADVANCED
General Certificate of Education
2009

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]



WEDNESDAY 20 MAY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all eight** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

- 1 A bowl is formed by rotating through 2π radians about the x -axis, the arc of the curve

$$y = \sqrt{5x}$$

between $x = 0$ and $x = a$, where a is a positive constant.

The bowl is full of water.

Find the volume of water in the bowl.

[6]

- 2 Two points A and B have coordinates (1, 3, 4) and (3, -2, 0) respectively.

(i) Find the distance between A and B.

[2]

(ii) Find the vector equation of the line that passes through A and B.

[5]

(iii) Show that the point (5, -7, -4) lies on this line.

[4]

- 3 Using the substitution $u = 1 + x$, find the **exact** value of

$$\int_{-1}^0 x(1+x)^{\frac{1}{2}} dx$$

[8]

- 4 (a) Without using your calculator, find the **exact** value of $\tan 2A$ given that $\tan A = \frac{1}{7}$ and that A is acute.

[3]

(b) Solve the equation

$$3 \cos \theta = \sin (\theta + 30^\circ)$$

where $0^\circ \leq \theta \leq 360^\circ$

[7]

5 The functions f and g are defined as:

$$f : x \rightarrow 3x + 1 \qquad x \in \mathbb{R} \ x > 2$$

$$g : x \rightarrow \frac{1}{x} \qquad x \in \mathbb{R} \ x > 0$$

(i) State the range of f [1]

(ii) Find the composite function gf and state its domain and range. [5]

6 (i) Show that

$$\frac{d}{dx} \left(\frac{x}{1+x} \right) = \frac{1}{(1+x)^2} \qquad [4]$$

(ii) A curve has the equation

$$\frac{x}{1+x} - x^2 + \frac{y}{1+y} = 0$$

Find the gradient of the curve at the point $(1,1)$ [6]

7 Given the differential equation

$$\frac{dy}{dx} = \frac{3y}{x+1}$$

and that $x = 1$ when $y = 16$, express y in terms of x [10]

8 Find

(i) $\int_0^2 x e^{-x} dx$ [7]

(ii) $\int \sin^3 x dx$ [7]

