GCE AS

Mathematics

January 2009

Mark Schemes

Issued: April 2009

**NORTHERN IRELAND GENERAL CERTIFICATE OF SECONDARY EDUCATION (GCSE)
AND NORTHERN IRELAND GENERAL CERTIFICATE OF EDUCATION (GCE)**

MARK SCHEMES (2009)

Foreword

Introduction

Mark Schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of 16- and 18-year-old students in schools and colleges. The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes therefore are regarded as a part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

The Council hopes that the mark schemes will be viewed and used in a constructive way as a further support to the teaching and learning processes.

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General Certificate of Education
January 2009

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[AMC11]

WEDNESDAY 7 JANUARY, AFTERNOON

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

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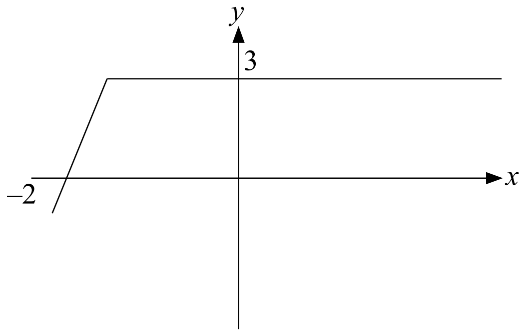
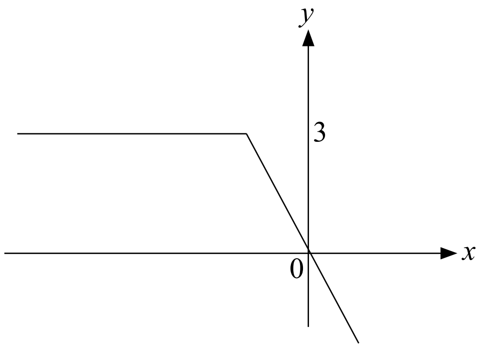
		AVAILABLE MARKS
1 $m_{AB} = \frac{-1-7}{-3-1} = \frac{-8}{-4} = 2$	M1W1	
$m\perp = -\frac{1}{2}$	MW1	
= mid point (–1, 3)	MW1	
$(y-3) = -\frac{1}{2}(x+1)$	M1W1	
$2y + x = 5$		6

2 **(a)** **(i)**



M1W1

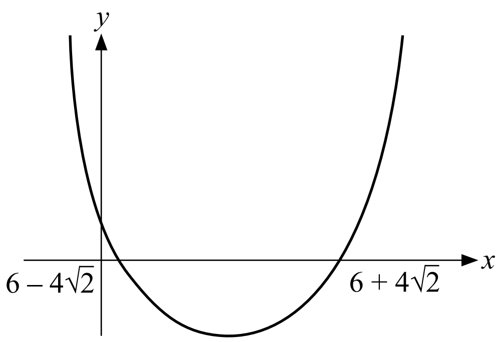
(ii)



M1W1

(b) **(i)** $[(x+3)^2 - 9] - 1$	MW1	
$(x+3)^2 - 10$	MW1	
(ii) Min value = –10 when $x = -3$	MW2	
(iii) $x > -3$	M1W1	10

			AVAILABLE MARKS
3 **(a)**	$\dfrac{\sqrt{7}+1}{3-\sqrt{7}} \times \dfrac{3+\sqrt{7}}{3+\sqrt{7}}$	M1W1	
	$\dfrac{3\sqrt{7}+7+3+\sqrt{7}}{9-7} = \dfrac{10+4\sqrt{7}}{2} = 5+2\sqrt{7}$	MW1	
(b) **(i)**	$f(2) = 16 + 4 - 26 + 6 = 0$	MW1	
(ii)	$\begin{array}{r} 2x^2 + 5x - 3 \\ x-2\,\overline{)\,2x^3 + x^2 - 13x + 6} \\ \underline{2x^2 - 4x^2} \\ 5x^2 - 13x \\ \underline{5x^2 - 10x} \\ -3x + 6 \\ \underline{-3x + 6} \\ 0 \end{array}$	M2W1	
	$(x-2)(2x^2+5x-3) = (x-2)(2x-1)(x+3)$	MW1	
(iii)	$(x-2)(2x-1)(x+3) = 0$	M1	
	$x = 2 \quad x = \frac{1}{2} \; x = -3$	W2	11
4 **(a)**	$\dfrac{dy}{dx} = 5 - 6x^{-3} + 2x^{-\frac{1}{2}}$	MW4	
	$\dfrac{dy}{dx} = 5 - \dfrac{6}{x^3} + \dfrac{2}{\sqrt{x}}$		
(b)	$\dfrac{dy}{dx} = 6x^2 - 8x$	M1W1	
	At $x = 3$ grad $= 54 - 24 = 30$	MW1	
	Normal grad $= \dfrac{-1}{30}$	MW1	
	At $x = 3$ $y = 54 - 36 + 9 = 27$	MW1	
	$(y - 27) = \dfrac{-1}{30}(x - 3)$	M1W1	11
	$x + 30y = 813$		

				AVAILABLE MARKS
5	**(i)**	$V = x^2h = 500\ \text{m}^3$		
		$h = \dfrac{500}{x^2}$	M1W1	
	(ii)	$A = x^2 + 4xh$	M1	
		$A = x^2 + 4x\left(\dfrac{500}{x^2}\right) = x^2 + \left(\dfrac{2000}{x}\right)$	MW1	
	(iii)	$\dfrac{dA}{dx} = 2x - 2000x^{-2}$	M1W1	
		$2x - 2000x^{-2} = 0$	M1	
		$2x = \dfrac{2000}{x^2}$		
		$x^3 = 1000$		
		$x = 10$	MW1	
		$\dfrac{d^2y}{dx^2} = 2 + 4000x^{-3}$	M1	
		$x = 10\ \dfrac{d^2y}{dx^2} = 6$ +ve so minimum	MW1	
		Dimensions 10 m × 10 m × 5 m	W1	11
6	**(i)**	$b^2 - 4ac = (k-2)^2 - 8k$	M1W1	
	(ii)	$(k-2)^2 - 8k > 0$	M1	
		$k^2 - 4k + 4 - 8k > 0$	MW1	
		If $k^2 - 12k + 4 = 0$		
		$k = \dfrac{12 \pm \sqrt{144 - 16}}{2}$	M1	
		$k = \dfrac{12 \pm \sqrt{128}}{2} = \dfrac{12 \pm 8\sqrt{2}}{2} = 6 \pm 4\sqrt{2}$	W1	
			M1	
		$k < 6 - 4\sqrt{2}$ or $k > 6 + 4\sqrt{2}$	MW1	8

			AVAILABLE MARKS
7	**(i)** $t = \frac{75}{v}$	MW1	
	(ii) $\frac{75}{v} - \frac{5}{4} = \frac{75}{v+5}$	M1W1	
	$(300 - 5v)(v + 5) = 75 \times 4v$		
	$300v + 1500 - 5v^2 - 25v = 300v$		
	$5v^2 + 25v - 1500 = 0$	M1W2	
	$v^2 + 5v - 300 = 0$	MW1	
	(iii) $(v - 15)(v + 20) = 0$	M1	
	$v = 15 \text{ km h}^{-1}$	W1	9
8	$(3^3)^x \times (3^2)^{y+3} = 3 \times 3^{\frac{1}{2}}$	M1W1	
	$3^{3x} \times 3^{2y+6} = 3^{\frac{3}{2}}$	MW1	
	$3^{3x+2y+6} = 3^{\frac{3}{2}}$	MW1	
	$3x + 2y + 6 = \frac{3}{2}$	M1	
	$3x + 2y = \frac{-9}{2}$	W1	
	$6x + 4y = -9$		
	$12x - 9y = 33$		
	$12x + 8y = -18$		
	$-17y = 51$	M1	
	$y = -3$	W1	
	$x = \frac{1}{2}$	W1	9
		Total	**75**

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2009

Mathematics

Assessment Unit C2

assessing

Module C2: Core Mathematics 2

[AMC21]

THURSDAY 15 JANUARY, MORNING

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Mark Schemes

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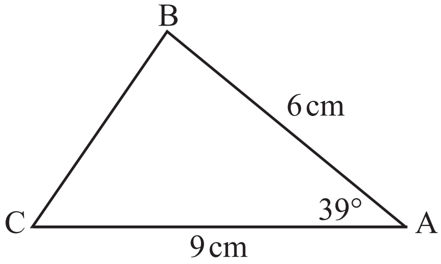
AVAILABLE MARKS

1



(i) $BC^2 = 9^2 + 6^2 - 2(9)(6)\cos 39°$ M1W1

$BC^2 = 33.0682 \ldots$

$BC = 5.75$ cm MW1

(ii) $\frac{\sin 39°}{5.75} = \frac{\sin C}{6}$ M1W1

$\sin C = \frac{6 \sin 39°}{5.75}$

Angle C = 41.0° W1 6

2 **(a)** $\int 3 - 2\sqrt{x} + 3x^{-2} dx$

$= 3x - \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3x^{-1}}{-1} + c$

$= 3x - \frac{4}{3}\sqrt{x^3} - 3x^{-1} + c$ MW4

AVAILABLE MARKS

(b)

x	y
0	1.000
$\frac{1}{2}$	1.414
1	2.000
$1\frac{1}{2}$	2.828
2	4.000

x values, y values — MW1MW2

$\int = \frac{1}{2}h(y_0 + 2(y_1 + y_2 + y_3) + y_4)$

$\int = \frac{1}{2} \times \frac{1}{2}(1.000 + 2(1.414 + 2.000 + 2.828) + 4.000)$ — MW1M1

$\int = 4.37(4.371)$ — W1 — 10

3 **(i)** $x^2 + y^2 + 8x - 2y - 9 = 0$

Centre $(-4, 1)$ — W1

Radius = $\sqrt{4^2 + 1^2 + 9} = \sqrt{26} = 5.10$ — M1W1

(ii) $(-5)^2 + (-4)^2 + 8(-5) - 2(-4) - 9 = 0$ — M1

$25 + 16 - 40 + 8 - 9 = 0$ — W1

(iii) Gradient of radius = $\dfrac{1 - (-4)}{-4 - (-5)} = \dfrac{5}{1} = 5$ — M1W1

Gradient of tangent at $(-5, -4) = \dfrac{-1}{5}$ — MW1

$y - (-4) = \dfrac{-1}{5}(x - (-5))$ — M1

$5y + 20 = -x - 5$

$5y + x + 25 = 0$ — W1 — 10

4 **(a)** **(i)** $(1 + \frac{1}{2}x)^{10} = 1 + 10(\frac{1}{2}x) + \dfrac{10.9}{2.1}(\frac{1}{2}x)^2 + \dfrac{10.9.8}{3.2.1}(\frac{1}{2}x)^3$ — M1W2

$= 1 + 5x + 11.25x^2 + 15x^3$ — W1

(ii) $(1 + \frac{1}{2}x)^{10} = 1 + 5(0.01) + 11.25(0.01)^2 + 15(0.01)^3$ — MW1M1

$= 1 + 0.05 + 0.001125 + 0.000015$

$= 1.05114$ — W1

		AVAILABLE MARKS
(b) (i) $u_{10} = 225 + 9(50) = £675$	M1W1	
(ii) $S_{20} = \frac{1}{2}(20)\{2(225) + 19(50)\}$	M1W1	
$= £14\,000$	W1	
(iii) Brad's savings $S = \frac{1}{2}(20)\{2P + 19(60)\}$		
$10(2P + 1140) = 14\,000$	M1W1	
$2P + 1140 = 1400$		
$2P = 260$		
$P = 130$	W1	15
5 (i) $5 - 2\cos\theta - 8\sin^2\theta =$		
$5 - 2\cos\theta - 8(1 - \cos^2\theta)$	M1W1	
$= 5 - 2\cos\theta - 8 + 8\cos^2\theta$		
$= 8\cos^2\theta - 2\cos\theta - 3$	MW1	
(ii) $8\cos^2\theta - 2\cos\theta - 3 = 0$	M1	
$(4\cos\theta - 3)(2\cos\theta + 1) = 0$	M1	
$4\cos\theta = 3$ or $2\cos\theta = -1$		
$\cos\theta = 0.75$ or $\cos\theta = -0.5$	W1	
$\theta = 41.4°$ or $\theta = 120°$	MW2	8
6 (i) Length of major arc CD $= r\theta = 4 \times \frac{5\pi}{3} = \frac{20\pi}{3} = 20.9$ m	M1W1	
Total perimeter $= 25 + 21 + 21 + 20.94 = 87.9$ m	M1W1	
(ii) Area of major sector $= \frac{1}{2}r^2\theta = \frac{1}{2}(4)^2 \times \frac{5\pi}{3}$	M1	
$= \frac{40\pi}{3} = 41.9$	W1	
Area of a triangle $= \frac{1}{2}ab\sin C = \frac{1}{2}(25)(25)\sin\frac{\pi}{3}$	M1	
$= 271$	W1	
Total area $= 41.9 + 271 = 313$ m^2	MW1	9

		AVAILABLE MARKS
7 **(i)** $3x - x^2 = x^2$	M1	
$3x - 2x^2 = 0$		
$x(3 - 2x) = 0$	MW1	
$x = 0$ or $x = 1\frac{1}{2}$		
A has x coordinate $1\frac{1}{2}$	MW1	
(ii) Area $= \int_0^{1\frac{1}{2}} ((3x - x^2) - (x^2))\,dx$	M1W2	
Area $= \int_0^{1\frac{1}{2}} (3x - 2x^2)\,dx$		
$= \left[\frac{3x^2}{2} - \frac{2x^3}{3}\right]_0^{1\frac{1}{2}}$	MW2	
$= \left[\frac{27}{8} - \frac{9}{4}\right] - [0 - 0]$	W1	9
$= \frac{9}{8} = 1.125 = 1.13$		
8 $\log_x 9 = \frac{\log_3 9}{\log_3 x}$	M1MW1	
$\log_x 9 = \frac{2}{\log_3 x}$	MW1	
$\frac{2}{\log_3 x} = 2\log_3 x + 3$		
Let $y = \log_3 x$		
$\frac{2}{y} = 2y + 3$		
$2 = 2y^2 + 3y$	M1MW1	
$0 = 2y^2 + 3y - 2$		
$(2y - 1)(y + 2) = 0$		
$y = \frac{1}{2}$ or $y = -2$	MW1	
$\log_3 x = \frac{1}{2}$ or $\log_3 x = -2$		
$x = \sqrt{3}$ or $x = \frac{1}{9}$	MW2	8
	Total	**75**

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2009

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

TUESDAY 13 JANUARY, MORNING

MARK SCHEME

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				AVAILABLE MARKS
1	(i)	$\det(\mathbf{A} - \lambda\mathbf{I}) = 0$		
		$\Rightarrow \begin{vmatrix} 10-\lambda & 6 \\ 3 & 3-\lambda \end{vmatrix} = 0$	M1	
		$\Rightarrow (10-\lambda)(3-\lambda) - 18 = 0$	M1	
		$\Rightarrow 30 - 13\lambda + \lambda^2 - 18 = 0$		
		$\Rightarrow \lambda^2 - 13\lambda + 12 = 0$	W1	
		$\Rightarrow (\lambda - 12)(\lambda - 1) = 0$		
		$\Rightarrow \lambda = 1, 12$	W2	
	(ii)	$\begin{pmatrix} 10 & 6 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 1\begin{pmatrix} x \\ y \end{pmatrix}$	M1	
		$\Rightarrow 10x + 6y = x \qquad \Rightarrow 9x + 6y = 0$		
		and $3x + 3y = y \qquad \Rightarrow 3x + 2y = 0$	M1	
		Hence $y = -\frac{3}{2}x$		
		Therefore an eigenvector is $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$	W1	8

2 **(i)** Reflection in the x-axis MW2

AVAILABLE MARKS

(ii) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & 4 \\ -3 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 4 \\ -5 & -4 \end{pmatrix}$

$\Rightarrow 2a - 3b = -1$ $\quad 2c - 3d = -5$ M1

and $4a = 4$ $\quad 4c = -4$ M1

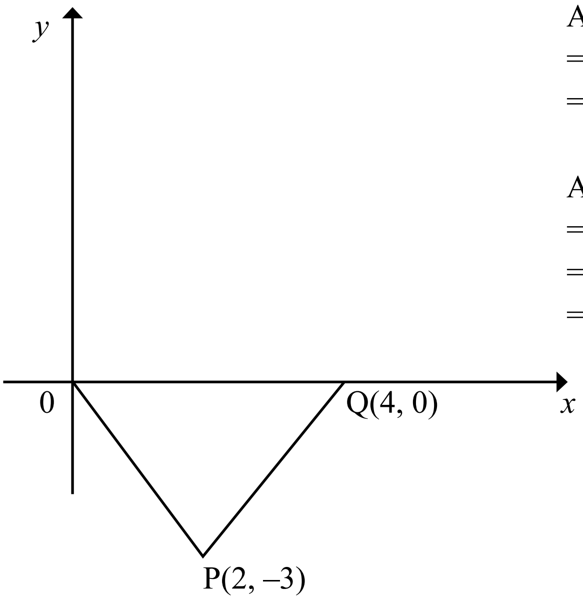
Hence $a = 1$ $\quad$ Hence $c = -1$

and $2 - 3b = -1$ $\quad$ and $-2 - 3d = -5$ W1

$\Rightarrow b = 1$ $\quad d = 1$ W1

Hence $\mathbf{M} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

(iii)



Area of triangle OPQ

$= \frac{1}{2} \times 4 \times 3$ M1

$= 6$ W1

Area of OP′Q′

$= \det M \times$ Area of OPQ M1

$= 2 \times 6$

$= 12$ W1

(iv) $\mathbf{S} = \mathbf{NM}$ M1M1

$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ W1 13

				AVAILABLE MARKS
3	(i)	$\begin{vmatrix} 2 & 4 & 2 \\ \lambda & 12 & 5 \\ 1 & 8 & \lambda \end{vmatrix} = 2(12\lambda - 40) - 4(\lambda^2 - 5) + 2(8\lambda - 12)$	M1	
		$= 24\lambda - 80 - 4\lambda^2 + 20 + 16\lambda - 24$	W1	
		$= -4\lambda^2 + 40\lambda - 84$	W1	
	(ii)	det $\mathbf{T} \neq 0$	M1	
		Hence		
		$4\lambda^2 - 40\lambda + 84 = 0$	M1	
		$\Rightarrow \lambda^2 - 10\lambda + 21 = 0$		
		$\Rightarrow (\lambda - 7)(\lambda - 3) = 0$		
		$\lambda = 3, 7$	W2	
		Inverse will exist if $\lambda \neq 3, \lambda \neq 7$	W1	
	(iii)	If $\lambda = 2$, then $\mathbf{T} = \begin{pmatrix} 2 & 4 & 2 \\ 2 & 12 & 5 \\ 1 & 8 & 2 \end{pmatrix}$		
		Matrix of minors $= \begin{pmatrix} -16 & -1 & 4 \\ -8 & 2 & 12 \\ -4 & 6 & 16 \end{pmatrix}$	MW3	
		Matrix of cofactors $= \begin{pmatrix} -16 & 1 & 4 \\ 8 & 2 & -12 \\ -4 & -6 & 16 \end{pmatrix}$	MW1	
		Determinant $= -16 + 80 - 84 = -20$	MW1	
		Hence inverse $= -\frac{1}{20}\begin{pmatrix} -16 & 8 & -4 \\ 1 & 2 & -6 \\ 4 & -12 & 16 \end{pmatrix}$	MW1	
	(iv)	If $\lambda = 3$, the equations become		
		$2x + 4y + 2z = \mu$		
		$3x + 12y + 5z = 7$		
		$x + 8y + 3z = 6$	MW1	
		② – ① gives $x + 8y + 3z = 7 - \mu$	M1	
		This must be the same as ③ for solutions to exist.		
		Hence $7 - \mu = 6$ which gives $\mu = 1$	W1	17

			AVAILABLE MARKS
4	**(i)** $\begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ y & 1 \end{pmatrix}$	M1M1	
	$= \begin{pmatrix} 1 & 0 \\ x+y & 1 \end{pmatrix}$	W1	
	which is of the same form as the original matrix and therefore multiplication is closed for **S**	MW1	
	(ii) The matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the identity for matrix multiplication and is a member of **S** where $x = 0$	M1W1	
	(iii) Inverse $= \frac{1}{1}\begin{pmatrix} 1 & 0 \\ -x & 1 \end{pmatrix}$	M1	
	$= \begin{pmatrix} 1 & 0 \\ (-x) & 1 \end{pmatrix}$	W1	
	(iv) Since we can assume the associative law and we have proved closure, identity and inverse conditions, then **S** forms a group.	MW1	9

		AVAILABLE MARKS
5 $x^2 + y^2 + 6y - 16 = 0$		
$x^2 + y^2 - 24x - 12y + 80 = 0$		
Subtract to give $24x + 18y - 96 = 0$	M1	
$\Rightarrow 4x + 3y = 16$	W1	
$\Rightarrow x = \dfrac{16 - 3y}{4}$	W1	
Substitute into equation ① to give	M1	
$\left(\dfrac{16 - 3y}{4}\right)^2 + y^2 + 6y - 16 = 0$	W1	
$\Rightarrow (16 - 3y)^2 + 16y^2 + 96y - 256 = 0$		
$\Rightarrow 256 - 96y + 9y^2 + 16y^2 + 96y - 256 = 0$		
$\Rightarrow 25y^2 = 0$		
$\Rightarrow y = 0$	W1	
$\Rightarrow x = 4$	W1	
Therefore the point of intersection is (4, 0)		
Since there is only one point of intersection the circles touch	M1	
The centres of the circles are (0, –3) and (12, 6)	MW2	
The point of intersection (4, 0) lies between these two centres and hence the circles must touch externally	MW1	11

AVAILABLE MARKS

6 **(a)** **(i)** $\frac{z_1}{z_2} = \frac{10 + 5\text{i}}{2 - \text{i}} \times \frac{2 + \text{i}}{2 + \text{i}}$ M1

$= \frac{20 + 20\text{i} - 5}{4 + 1}$ W2

$= \frac{15 + 20\text{i}}{5}$ W1

$= 3 + 4\text{i}$ W1

(ii) $|3 + 4\text{i}| = \sqrt{3^2 + 4^2}$ M1

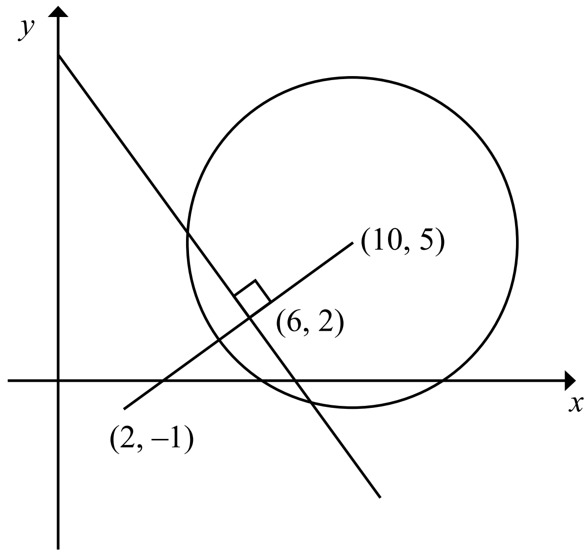
Hence modulus = 5 W1

$\arg(3 + 4\text{i}) = \tan^{-1}\left(\frac{4}{3}\right)$ M1

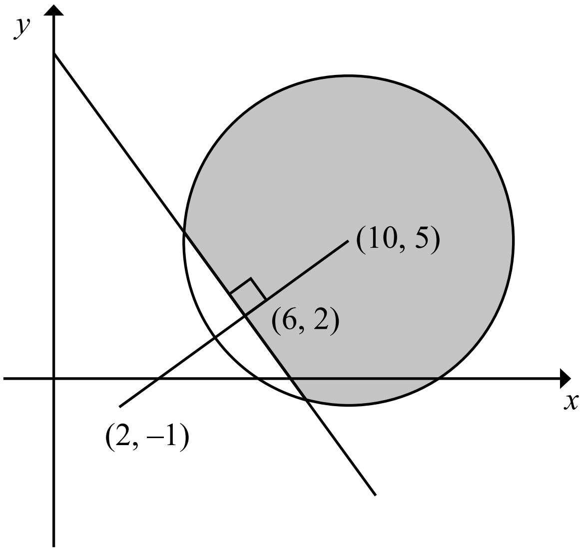
Hence argument = 53.1° W1

(b) **(i)** Perpendicular bisector of the line joining (10, 5) and (2, –1) MW3

(ii) Circle, centre (10, 5) and of radius 6 MW3



(iii)



	AVAILABLE MARKS
MW2	17
Total	**75**

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2009

Mathematics

Assessment Unit M1

assessing

Module M1: Mechanics 1

[AMM11]

TUESDAY 13 JANUARY, MORNING

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

Introduction

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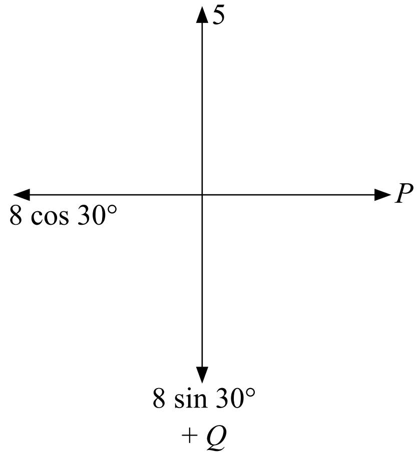
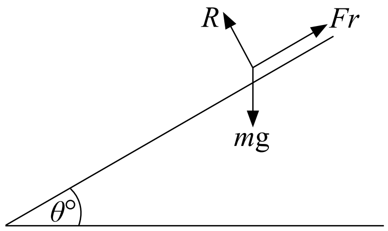
				AVAILABLE MARKS
1				
		$P = 8 \cos 30°$	M1W1	
		$= 6.93\,\text{N}$	MW1	
		$Q + 8 \sin 30° = 5$	M1W1	
		$Q = 1\,\text{N}$	W1	6
2	**(i)**	$u = 10$ $\quad s = ut + \frac{1}{2}at^2$	M1	
		$a = 0.5$ $\quad s = 10 \times 60 + \frac{1}{2} \times 0.5 \times 3600$	MW1	
		$t = 60$		
		$s = ?$ $\quad s = 1500\text{ m}$	W1	
	(ii)	$F = ma$	M1	
		$5500 - R = 1000 \times 0.5$	W2	
		$R = 5000\,\text{N}$	W1	7
3	**(i)**	$I = mv - mu$	M1	
		$= 0.2 \times -6 - 0.2 \times 8$	M2	
		$= -2.8\text{ Ns}$	W1	
	(ii)	$I = Ft$	M1	
		$-2.8 = F \times 0.01$		
		$F = 280\,\text{N}$ (upwards)	W1	6

AVAILABLE MARKS

4 **(i)**



MW2

(ii) along plane $Fr = mg \sin \theta$ — M1W1

$\perp$ to plane $R = mg \cos \theta$ — MW1

$Fr = \mu R$ — M1

$Fr = \mu mg \cos \theta$

$\mu mg \cos \theta = mg \sin \theta$ — W1

$\mu = \frac{3}{4}$ — W1 — **8**

5 **(i)** $S = t^3 - 6t^2 + 9t$

$v = 3t^2 - 12t + 9$ — M1W1

(ii) $a = 6t - 12$ — M1W1

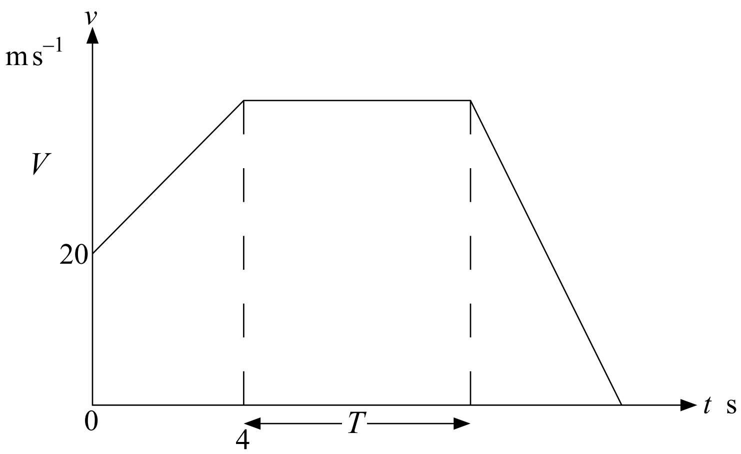
(iii) for max/min $a = 0$ — M1

$6t - 12 = 0$

$t = 2$ s — W1

$\frac{\mathrm{d}a}{\mathrm{d}t} = 6$ +ive:min — MW1 — **7**

6 **(i)**



MW3

(ii) $a = \frac{v - u}{t}$ — M1

$2.5 = \frac{V - 20}{4}$

$30 \text{ m s}^{-1} = V$ — W1

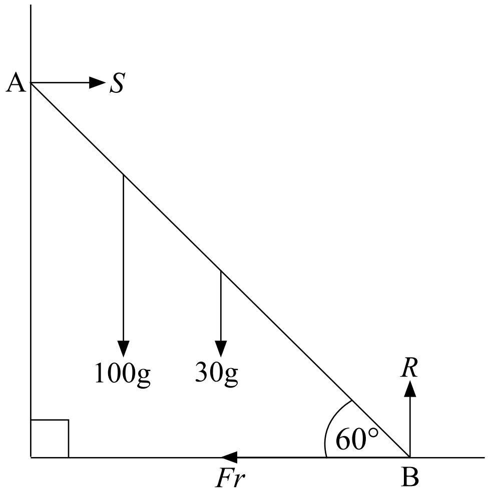
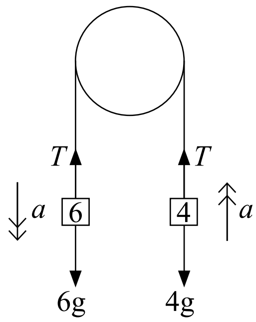
(iii) total distance travelled = area under graph — M1

$1090 = \frac{1}{2}(20 + 30) \times 4 + 30T + \frac{1}{2}(40 - (T + 4)) \times 30$ — M1W4

$1090 = 100 + 30T + 540 - 15T$

$1090 = 640 + 15T$

$T = 30$ s — W1 — **12**

			AVAILABLE MARKS
7	**(i)** 	MW2	
	(ii) ↕ $R = 130\text{g}$	M1W1	
	$Fr = \mu R$		
	$Fr = 65\text{g}$	MW1	
	↔ $S = Fr = 65\text{g}$	MW1	
	B↻ $6S \sin 60° = 3 \times 30\text{g} \cos 60° + x\, 100\text{g} \cos 60°$	M3W2	
	$\therefore 6 \times 65\text{g} \sin 60° = 90\text{g} \cos 60° + x\, 100\text{g} \cos 60°$		
	$x = 5.85$ m	MW1	12
8	**(i)** 		
	$F = ma$	M1	
	$6\text{g} - T = 6a$	W1	
	$T - 4\text{g} = 4a$	W1	
	$2\text{g} = 10a$	M1	
	$1.96\,\text{m s}^{-2} = a$	W1	
	(ii) $u = 0$		
	$s = 2$ $\quad v^2 = u^2 + 2as$	M1	
	$a = 1.96$ $\quad v^2 = 2 \times 1.96 \times 2$		
	$v = ?$ $\quad v = 2.80\,\text{m s}^{-1}$	W1	
	(iii) $u = 0$		
	$s = 2$ $\quad s = ut + \frac{1}{2}at^2$	M1	
	$a = 1.96$ $\quad 2 = \frac{1}{2} \times 1.96 \times t^2$	W1	
	$t = ?$ $\quad t = 1.43$ s	W1	

		AVAILABLE MARKS
(iv) $v = 0$ $\quad v = u + at$	M1M1	
$u = 2.80$ $\quad 0 = 2.80 - 9.8t$	W1	
$a = -9.8$ $\quad t = 0.280$ s	W1	
$t = ?$		
$\therefore$ becomes taut when		
$t = 1.42 + 2 \times 0.289$	M2	
$= 2.00$ s	W1	
Alternative solution:		
4 kg mass now moves under gravity		
$s = 0$ $\quad s = ut + \frac{1}{2}at^2$	M2	
$u = 2.8$		
$a = -9.8$ $\quad 0 = 2.8t + \frac{1}{2}(-9.8)t^2$	W1	
$t =$		
$0 = 2.8t - 4.9t^2$		
$0 = t(2.8 - 4.9t)$		
$t = 0$ **or** $t = \frac{4}{7}$	W1	
So $t = \frac{4}{7}$	W1	
time to become taut		
$1.43 + \frac{4}{7} = 2.005$	M1W1	17
	Total	**75**

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2009

Mathematics

Assessment Unit S1

assessing

Module S1: Statistics 1

[AMS11]

MONDAY 19 JANUARY, AFTERNOON

MARK SCHEME

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				AVAILABLE MARKS
1	(i)	$0.12 + 0.21 + 0.2 + 0.16 + 0.14 + k = 1$	M1	
		$k = 0.17$	W1	
	(ii)	$P(2 < X \leqslant 5) = P(X = 3) + P(X = 4) + P(X = 5)$	M1	
		$= 0.2 + 0.16 + 0.14$		
		$= 0.5$	W1	
	(iii)	$E(X) = (1 \times 0.12) + (2 \times 0.21) + (3 \times 0.2) + (4 \times 0.16) + (5 \times 0.14) + (6 \times 0.17)$	M1	
		$= 3.5$	W1	
		$E(X^2) = (1^2 \times 0.12) + (2^2 \times 0.21) + (3^2 \times 0.2) + (4^2 \times 0.16) + (5^2 \times 0.14) + (6^2 \times 0.17)$	M1	
		$= 14.94$	W1	
		$Var(X) = E(X^2) - [E(X)]^2$	M1	
		$= 14.94 - 3.5^2$		
		$= 2.69$	W1	10
2	(i)	Let X be r.v. "No. of hits in one-minute period"		
		$X \sim Po(2.6)$	M1	
		$P(X = 4) = \frac{e^{-2.6} \times 2.6^4}{4!} = 0.141$ (3 s.f.)	MW1W1	
	(ii)	Let Y be r.v. "No. of hits in two-minute period"		
		$Y \sim Po(5.2)$	M1	
		$P(X = 4) = \frac{e^{-5.2} \times 5.2^4}{4!} = 0.168$ (3 s.f.)	MW1W1	
	(iii)	$X \sim Po(2.6)$		
		$P(X \geqslant 2) = 1 - [P(X = 0) + P(X = 1)]$	M1W1	
		$= 1 - [e^{-2.6}(2.6^0 + 2.6^1)]$	W1	
		$= 1 - 3.6e^{-2.6}$		
		$= 0.732615 = 0.733$ (3 s.f.)	W1	10

			AVAILABLE MARKS
3 Let X be r.v. "No of correct answers"			
(i) $X \sim \text{Bin}(10, 0.2)$	M1		
$P(X = 4) = \binom{10}{4}(0.2)^4(0.8)^6$	MW1W1		
$= 0.0881$ (3 s.f.)	W1		
(ii) $P(X \geqslant 1) = 1 - P(X = 0)$	M1		
$= 1 - \binom{10}{0}(0.2)^0(0.8)^{10}$	MW1		
$= 1 - 0.107$			
$= 0.893$ (3 s.f.)	W1		
(iii) 2 answers: $E(X) = np = 10 \times 0.2 = 2$	M2		9
4 **(i)** **(a)** 15, 25, 35, 45	MW1		
(b) 14.5, 24.5, 34.5, 45	MW1MW1		
(c) 15, 25, 35, 45.5 (3 s.f.)	MW1MW1		
(ii) for **(b)** mean = 25.5, SD = 7	MW2		
for **(c)** mean = 26, SD = 7	MW1		8

		AVAILABLE MARKS
5 Let X be r.v. "time, in minutes, spent at Cyber Zone"		
$X \sim N(72, 15^2)$		
(i) $P(X < 60) = P\left(Z < \frac{60-72}{15}\right)$	M1	
$= P(Z < -0.8)$	W1	
$= 1 - \Phi(0.8)$	M1	
$= 1 - 0.7881$	W1	
$= 0.2119 = 0.212$ (3 s.f.)	W1	
(ii) $P(60 < \times < 90) = \left(\frac{60-72}{15} < Z < \frac{90-72}{15}\right)$	M1	
$= P(-0.8 < Z < 1.2)$	W1	
$= \Phi(1.2) - \Phi(-0.8)$	M1	
$= \Phi(1.2) - (1 - \Phi(0.8))$		
$= 0.8849 - 0.2119$	W1	
$= 0.673$ (3 s.f.)	W1	
(iii) $P(X > 90) = 1 - 0.8849 = 0.1151$	M1W1	
$E(X) = 1.5 \times 0.2119 + 2.5 \times 0.673 + 3.5 \times 0.1151$	M1	
$= 2.4032$	W1	
$E(X) = £2.40$ (to nearest penny)	W1	15

			Marks	AVAILABLE MARKS
6	**(i)**	$P(2 \leqslant X \leqslant 3) = \int_2^3 \frac{3}{125} x^2 \, dx$	M1	
		$= \left[\frac{x^3}{125} \right]_2^3$	W1	
		$= \frac{27-8}{125} = \frac{19}{125}$	W1	
	(ii)	$E(X) = \int_0^5 x \frac{3}{125} x^2 dx = \int_0^5 \frac{3x^3}{125} \, dx$	M1	
		$= \left[\frac{3x^4}{500} \right]_0^5$	W1	
		$= \left(\frac{3 \times 625}{500} \right) = \frac{15}{4} = 3\frac{3}{4}$	W1	
	(iii)	$E(X^2) = \int_0^5 x^2 \frac{3}{125} x^2 dx = \int_0^5 \frac{3x^4}{125} \, dx$	M1	
		$= \left[\frac{3x^5}{625} \right] = \frac{3 \times 5^5}{5^4} = 15$	W1 W1	
		$\text{Var}(X) = E(X^2) - [E(X)]^2$	M1	
		$= 15 - 3.75^2 = 0.9375 = 0.938$(3s.f.)	W1	11
7	**(i)**	$(1 - p) \times 1.1p$	M1W2	
	(ii)	$1.1p - 1.1p^2 = 0.176$	M1	
		$1.1p^2 - 1.1p + 0.176 = 0$		
		$p^2 - p + 0.16 = 0$		
		$(p - 0.2)(p - 0.8) = 0$		
		$p = 0.2$ or 0.8	W1	
		but $p < 0.5$ so $p = 0.2$	W1	
	(iii)	P(passes at 3rd attempt)		
		$= (1 - 0.2) \times (1 - 1.1 \times 0.2) \times (1.1 \times 1.1 \times 0.2)$	M1MW4	
		$= 0.151008$		
		$= 0.151$ (3 s.f.)		
			W1	12
			Total	**75**