Rewarding Learning

ADVANCED
General Certificate of Education
January 2009

Mathematics

Assessment Unit M2

assessing

Module M2: Mechanics 2

AMM21

[AMM21]

TUESDAY 27 JANUARY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answer should be given to three significant figures unless otherwise stated.

1 A truck of mass 1500 kg is travelling along a straight horizontal track at a constant speed of $5\,\text{m}\,\text{s}^{-1}$

(i) Find the kinetic energy of the truck. [2]

The driver applies the brakes and the truck is brought to rest.
The constant retarding force produced by the brakes is F newtons.

(ii) Find the work done by F. [3]

The truck travels 30 m after the brakes are applied.

(iii) Find the magnitude of F. [3]

2 A particle, P, of mass 0.5 kg, moves under the action of two forces $(4\mathbf{i} + \mathbf{j})$ N and $(-2\mathbf{i} + \mathbf{j})$ N.

(i) Find the acceleration of P. [4]

At time $t = 0$ seconds P is at the origin O and is moving with velocity $(\mathbf{i} - \mathbf{j})\,\text{m}\,\text{s}^{-1}$

(ii) Find the **speed** of P when $t = 1$ [4]

(iii) Find the position vector of P when $t = 4$ [2]

3 The displacement of a particle, P, from a fixed origin O, at any time t seconds, is

$$\mathbf{r} = 3t^2\mathbf{i} + (2t^3 - t)\mathbf{j} + 2t\mathbf{k}$$

(i) Find an expression for the velocity of P at any time t. [2]

(ii) Find the initial velocity of P. [2]

(iii) Find the acceleration of P when $t = 3$ [3]

(iv) Briefly explain why the particle will not pass through O for a second time. [1]

4 A pump in a water feature raises 100 litres of water through a height of 8 m vertically upwards every second.
The water issues as a jet with a speed of $20\,\text{m}\,\text{s}^{-1}$
1 litre of water has a mass of 1 kg.

(i) Find the power developed by the pump. [5]

(ii) Given that the jet of water is directed vertically upwards, find how much **further** the water will rise. [3]

(iii) State one assumption you have made when answering **(ii)**. [1]

5 **Fig. 1** below shows a box of mass M kg sliding down a smooth slope from a point A to a point B.
The slope is inclined at 30° to the horizontal.
The speed of the box at A is u m s^{-1} and at B is $2u$ m s^{-1}
The distance AB is d metres.

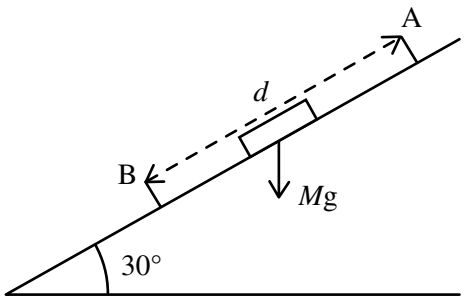

Fig. 1

Take the potential energy at A to be zero.

(i) Find the potential energy of the box at B. [3]

(ii) Hence show that

$$u = \sqrt{\frac{gd}{3}}$$ [4]

6 A stone is projected horizontally with a speed of 20 m s^{-1} from a point 19.6 m vertically above horizontal ground as shown in **Fig. 2** below.

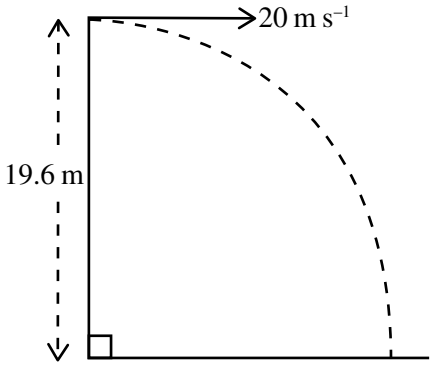

Fig. 2

(i) Show that the time taken for the stone to hit the ground is 2 s. [2]

(ii) Find the horizontal distance travelled by the stone before it hits the ground. [2]

(iii) Find the direction of motion of the stone when it has been travelling for 1 s. [5]

7 A racing car of mass 1000 kg travels around a bend in a road.
The bend has radius 50 m and is banked at $\theta°$ to the horizontal as shown in **Fig. 3** below.

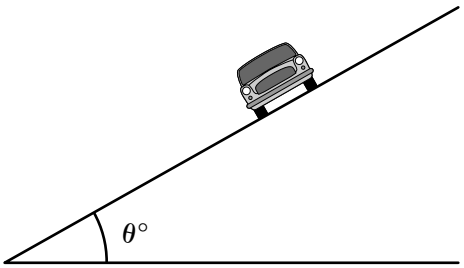

Fig. 3

$\sin\theta° = \frac{5}{13}$

The coefficient of friction between the car's tyres and the road is 0.4
The car is travelling at the maximum speed, v m s^{-1}, at which it can safely negotiate the bend.
Model the car as a particle.

(i) Draw a diagram showing all the external forces acting on the car. [2]

(ii) By resolving vertically, find the normal reaction between the car and the road. [5]

(iii) Find v. [5]

8 **Take g = 10 m s^{-2} in this question.**
A ball of mass 0.2 kg is thrown vertically upwards from ground level.
It is known that the air resistance on the ball has magnitude $0.004v^2$ newtons.
At time t seconds the ball is moving with velocity v m s^{-1}, at a displacement x metres above ground level.

(i) Show that the motion of the ball can be modelled by the differential equation

$$v\frac{\mathrm{d}v}{\mathrm{d}x} = -0.02(500 + v^2)$$ [4]

At $t = 0$, $v = 15$

(ii) Find the maximum height reached by the ball. [8]

THIS IS THE END OF THE QUESTION PAPER