Rewarding Learning

ADVANCED
General Certificate of Education
January 2009

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

AMF21

THURSDAY 29 JANUARY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that
$\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find the general solution of the equation

$$\tan\left(2\theta + \frac{\pi}{4}\right)\cot\left(\frac{\pi}{3} - 3\theta\right) = 1$$ [6]

2 **(i)** Prove, by the method of partial fractions, that

$$\frac{x^3 - 4x^2 + 9x + 10}{(x^2 + 5)(x - 3)^2} \equiv \frac{x}{x^2 + 5} + \frac{2}{(x - 3)^2}$$ [8]

(ii) Hence solve the differential equation

$$(x^2 + 5)\left[(x - 3)\frac{\mathrm{d}y}{\mathrm{d}x} - y\right] = x^3 - 4x^2 + 9x + 10$$

given that $y = -2$ when $x = 4$ [10]

3 **(i)** Use Maclaurin's theorem to write out the series expansion for $\ln(1 + x)$ up to the term in x^5 [5]

(ii) Hence write out the series expansion for

$$\ln\left(\frac{1 + x}{1 - x}\right)$$ [3]

(iii) Hence find an approximation for $\ln 2$ in the form $\frac{n}{1215}$, where n is a natural number. [3]

4 **(a)** Find the exact integer value of

$$\frac{(\cos\frac{\pi}{7} + \mathrm{i}\sin\frac{\pi}{7})^3}{(\cos\frac{\pi}{7} - \mathrm{i}\sin\frac{\pi}{7})^4}$$ [4]

(b) Find the roots of the equation

$$z^4 + 4 = 0$$

and plot them on an Argand diagram. [8]

5 **(i)** If $\mathbf{A} = \begin{pmatrix} x & 1 \\ 0 & 1 \end{pmatrix}$ prove by the method of mathematical induction that

$$\mathbf{A}^n = \begin{pmatrix} x^n & \dfrac{x^n - 1}{x - 1} \\ 0 & 1 \end{pmatrix}$$

where n is a positive integer and $x \neq 1$ [7]

(ii) Hence if $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$, find $\mathbf{B}^{10}$ [2]

6 The ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ is shown in **Fig. 1** below.

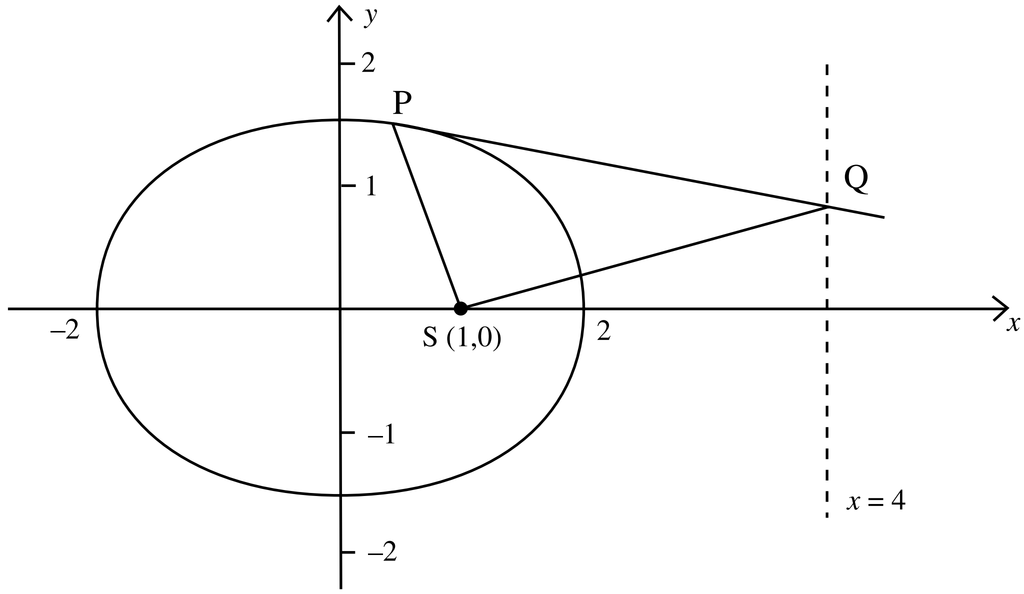

Fig. 1

(i) Prove that S (1,0) is a focus of the ellipse and that the line $x = 4$ is a directrix. [4]

(ii) Verify that the point P on the ellipse can be represented parametrically as $(2\cos\theta, \sqrt{3}\sin\theta)$ [2]

(iii) Show that the equation of the tangent to the ellipse at P can be written as

$$\frac{x}{2}\cos\theta + \frac{y}{\sqrt{3}}\sin\theta = 1$$ [6]

The point where the tangent at P meets the directrix $x = 4$ is Q.

(iv) Prove that $P\hat{S}Q$ is a right angle. [7]