



Rewarding Learning

ADVANCED SUBSIDIARY (AS)  
General Certificate of Education  
January 2009

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## Mathematics

Assessment Unit F1

*assessing*

Module FP1: Further Pure Mathematics 1

[AMF11]



TUESDAY 13 JANUARY, MORNING

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### TIME

1 hour 30 minutes.

### INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all six** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or a scientific calculator in this paper.

### INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that

$\ln z \equiv \log_e z$

**Answer all six questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

**1** The matrix **A** is given by

$$\begin{pmatrix} 10 & 6 \\ 3 & 3 \end{pmatrix}$$

**(i)** Show that the eigenvalues of **A** are 1 and 12 [5]

**(ii)** Find an eigenvector corresponding to the eigenvalue 1 [3]

**2 (i)** The matrix **N** =  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

What transformation does this matrix represent? [2]

The transformation represented by the matrix **M** maps the points P (2, -3) and Q (4, 0) onto P' (-1, -5) and Q' (4, -4) respectively.

**(ii)** Find the matrix **M**. [4]

The triangle OP'Q' is the image of the triangle OPQ under the transformation represented by **M**.

**(iii)** Find the area of the triangle OP'Q'. [4]

The matrix **S** represents the combined effect of the transformation represented by **M** followed by the transformation represented by **N**.

**(iv)** Find the matrix **S**. [3]

3 Let the matrix  $\mathbf{T} = \begin{pmatrix} 2 & 4 & 2 \\ \lambda & 12 & 5 \\ 1 & 8 & \lambda \end{pmatrix}$

(i) Find, in terms of  $\lambda$ , the determinant of  $\mathbf{T}$ . [3]

(ii) Find the values of  $\lambda$  for which an inverse exists. [5]

(iii) If  $\lambda = 2$ , find the inverse of  $\mathbf{T}$ . [6]

(iv) If  $\lambda = 3$ , find the value of  $\mu$  for which the following system of equations has infinitely many solutions.

$$2x + 4y + 2z = \mu$$

$$\lambda x + 12y + 5z = 7$$

$$x + 8y + \lambda z = 6 \quad [3]$$

4  $\mathbf{S}$  is the set of matrices  $\begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}$ , where  $x$  is any real number.

(i) Prove that  $\mathbf{S}$  is closed under matrix multiplication. [4]

(ii) Find the identity element of  $\mathbf{S}$  under matrix multiplication. [2]

(iii) Find the inverse of  $\begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}$  under matrix multiplication. [2]

(iv) Assuming that matrix multiplication is associative, what can now be stated about the set  $\mathbf{S}$  under matrix multiplication? [1]

5 Two circles have equations

$$x^2 + y^2 + 6y - 16 = 0$$

$$x^2 + y^2 - 24x - 12y + 80 = 0$$

Find the point of intersection of the circles and hence show that the circles touch externally.

[11]

6 (a) The complex numbers  $z_1$  and  $z_2$  are given by

$$z_1 = 10 + 5i \text{ and } z_2 = 2 - i$$

(i) Find  $\frac{z_1}{z_2}$ , giving your answer in the form  $a + bi$ , where  $a$  and  $b$  are real numbers. [5]

(ii) Find the modulus and argument of  $\frac{z_1}{z_2}$  [4]

(b) (i) Sketch on an Argand diagram the locus of those points  $w$  which satisfy

$$|w - (10 + 5i)| = |w - (2 - i)| \quad [3]$$

(ii) On the same diagram sketch the locus of those points  $z$  which satisfy

$$|z - (10 + 5i)| = 6 \quad [3]$$

(iii) On your diagram shade the region which represents the locus of  $v$  where  $v$  satisfies both

$$|v - (10 + 5i)| \leq |v - (2 - i)| \text{ and } |v - (10 + 5i)| \leq 6 \quad [2]$$