Cambridge Assessment International Education

Cambridge IGCSE™

ADDITIONAL MATHEMATICS **0606/21**

Paper 2 **October/November 2023**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Partial marks
1(a)	$-3(x+2)^2+31$	**B4**	**B2** for $-3(x+2)^2$ or **B1** for $(x+2)^2$ or $a=-3$ and $b=2$ **B2** for $c=31$ or **B1** for $-4\times-3+19$ soi
1(b)	Maximum value 31 when $x=-2$	**B2**	**Strict FT** *their c* from part **(a)** and $-$*their b* from part **(a)** **B1** for either without contradiction
1(c)	$-3\left(\sqrt{u}+2\right)^2=-31$ oe	**M1**	**FT** an expression of correct form from part **(a)**
	Rearranges as far as $\sqrt{u}=-2\pm\sqrt{\frac{31}{3}}$	**A1**	
	1.48 cao or 1.475[13…] rot to 3 or more dp or $\frac{43-4\sqrt{93}}{3}$ isw	**A1**	
2	Correct method to eliminate *y*: $5x-3(1-x)=2$ oe or adds $3x+3\ln y=3$ $5x-3\ln y=2$ to obtain $3x+5x=3+2$ or better	**M1**	
	$x=\frac{5}{8}$ oe	**A1**	
	$\ln y=\frac{3}{8}$ oe	**M1**	
	$y=e^{\frac{3}{8}}$ oe or 1.45	**A1**	

Question	Answer	Marks	Partial marks
2	**Alternative method**		
	Correct method to eliminate x: $5(1 - \ln y) - 3\ln y = 2$ oe or subtracts $5x - 3\ln y = 2$ from $5x + 5\ln y = 5$ to obtain $5\ln y - (-3\ln y) = 5 - 2$ or better	**(M1)**	
	$\ln y = \frac{3}{8}$ oe	**(M1)**	
	$y = e^{\frac{3}{8}}$ oe or 1.45	**(A1)**	
	$x = \frac{5}{8}$ oe	**(A1)**	
3(a)	$2x^2 + 5x - \frac{1}{2}\ln(2x+3) + c$ oe	**B3**	**B2** for $2x^2 + 5x - \frac{1}{2}\ln(2x+3)$ or $2x^2 + 5x - \frac{1}{2}\ln 2x + 3 + c$ or $2x^2 + 5x + k\ln(2x+3) + c$ with $k \neq 0$ or **B1** for $2x^2 + 5x + \ldots + c$ or $\ldots - \frac{1}{2}\ln 2x + 3$ or $\ldots + k\ln(2x+3)$ with $k \neq 0$
3(b)	Substitutes limits and subtracts in correct order	**M1**	**FT** *their* part (a) providing it includes a term $k\ln(2x+3)$ with $k \neq 0$
	$\left[18 + 15 - \frac{1}{2}\ln 9\right] - \left[2 + 5 - \frac{1}{2}\ln 5\right]$	**A1**	
	$26 - \frac{1}{2}\ln\frac{9}{5}$ or $26 + \frac{1}{2}\ln\frac{5}{9}$ oe	**A1**	

Question	Answer	Marks	Partial marks
4	$(2+ax)^5 = 2^5 + 5\times 2^4 ax + 10\times 2^3 a^2x^2 + ...$	B1	
	$(2+ax)^5(1+bx) =$ $32 + 80ax + 80a^2x^2 + 32bx + 80abx^2...$	M1	
	$80a + 32b = 112$ oe, isw	A1	
	$80a^2 + 80ab = -240$ oe, isw	A1	
	$3a^2 - 7a - 6 = 0$ oe	M1	
	$(3a + 2)(a - 3) = 0$	M1	
	$a = 3$ and $b = -4$ and no other values	A1	
5	$[m_{\text{tangent}} =] -2px^{-3} + 5$ oe	B1	
	[When $x = 1$, m_{normal} =] $\frac{-1}{-2p+5}$ or gradient of tangent = 1 nfww	B1	**FT** $\frac{-1}{\textit{their}\left.\frac{dy}{dx}\right\|_{x=1}}$ if appropriate
	$\frac{-1}{\textit{their}(-2p+5)} = -1$ oe or *their*($-2p + 5$) = 1	M1	**FT** $\frac{-1}{\textit{their}\left.\frac{dy}{dx}\right\|_{x=1}}$ or *their* $\left.\frac{dy}{dx}\right\|_{x=1}$ and *their* evaluation of $\frac{-1}{-1}$
	$p = 2$ nfww	A1	
	[When $x = 1$] $\textit{their}\, 5 = -1 + q$ or $y = -x + 6$	M1	**FT** y = (*their p*) + 3 providing at least 2 of the first 3 marks awarded
	$q = 6$ nfww	A1	
6	$ax^2 - 5x + 2 = 2ax + x - 10$	M1	
	$ax^2 - (2a+6)x + 12[=0]$ oe	A1	
	Correct use of $b^2 - 4ac$ [*0]: $(-(2a + 6))^2 - 4(a)(12)$ [*0] oe	M1	where * is any inequality sign or =; **FT** *their* 3-term quadratic in x and a
	$4a^2 - 24a + 36$ [*0]	A1	
	Factorises or solves *their* 3-term quadratic in a	M1	**FT** *their* 3-term quadratic in a
	$a = 3$	A1	

Question	Answer	Marks	Partial marks
6	**Alternative method**		
	$a = \dfrac{3}{x-1}$ oe or $x = \dfrac{a+3}{a}$	**(2)**	**M1** for $2ax - 5$
	$\left(\dfrac{6}{x-1}+1\right)x - 10 = \left(\dfrac{3}{x-1}\right)x^2 - 5x + 2$ oe or $(2a+1)\left(\dfrac{a+3}{a}\right) - 10 = a\left(\dfrac{a+3}{a}\right)^2 - 5\left(\dfrac{a+3}{a}\right) + 2$ oe	**(M1)**	**FT** *their a* of the form $\dfrac{k}{bx+c}$ where k, b, c are non-zero constants or *their x* of the form $\dfrac{da+e}{fa}$ where d, e, f are non-zero constants
	$3x^2 - 12x + 12$ [= 0] oe or $a^2 - 6a + 9$ [= 0]	**(A1)**	
	Solves *their* 3-term quadratic in x as far as $x = ...$ or factorises or solves *their* 3-term quadratic in a	**(M1)**	
	$a = 3$	**(A1)**	
7(a)	$20\cos 2t + 12\sin 2t$	**2**	**B1** for $20\cos 2t$ or $12\sin 2t$
7(b)	12	**B1**	
7(c)	$\tan 2t = their\left(-\dfrac{20}{12}\right)$ oe	**M1**	**FT** $a\cos 2t + b\sin 2t$ where a and b are non-zero integers
	$t = 1.06$ or 1.055[60…] rot to 3 or more dp	**A2**	**A1** for $2t = -1.030$[3…] or $2t = 2.111$[2…]
7(d)	$s = -5\cos 2t - 3\sin 2t\ (+\ c)$	**B2**	**B1** for $-5\cos 2t$ or $-3\sin 2t$
	$-5\cos\pi - 3\sin\pi - (-5\cos\frac{\pi}{2} - 3\sin\frac{\pi}{2})$ or $s = -5\cos 2t - 3\sin 2t + 5$ and $s_{\frac{\pi}{2}} - s_{\frac{\pi}{4}} = 10 - 2$	**M1**	**FT** providing at least **B1** previously awarded
	8	**A1**	

Question	Answer	Marks	Partial marks
8	$\left(2-\sqrt{10}\right)\left(2+\sqrt{10}\right)=-6$	**B1**	
	$x=\dfrac{-1\pm\sqrt{1^{[2]}-4(2-\sqrt{10})(2+\sqrt{10})}}{2\left(2-\sqrt{10}\right)}$	**M1**	
	$x=\dfrac{-1\pm\sqrt{1-4(-6)}}{2\left(2-\sqrt{10}\right)}$	**A1**	
	$\dfrac{-6}{2\left(2-\sqrt{10}\right)}$ oe , $\dfrac{4}{2\left(2-\sqrt{10}\right)}$ oe	**A1**	
	$\dfrac{-6}{2\left(2-\sqrt{10}\right)}\times\dfrac{\left(2+\sqrt{10}\right)}{\left(2+\sqrt{10}\right)}$ or $\dfrac{4}{2\left(2-\sqrt{10}\right)}\times\dfrac{\left(2+\sqrt{10}\right)}{\left(2+\sqrt{10}\right)}$	**M1**	**FT** $\dfrac{k}{2\left(2-\sqrt{10}\right)}$ where k is a non-zero constant
	$\dfrac{-6(2+\sqrt{10})}{2(-6)}$ oe = $1+\dfrac{1}{2}\sqrt{10}$	**A1**	Must have sufficient detail shown
	$\dfrac{4(2+\sqrt{10})}{2(-6)}$ oe = $-\dfrac{2}{3}-\dfrac{1}{3}\sqrt{10}$	**A1**	Must have sufficient detail shown
9(a)	$2(e^x+1)^2-1$ [= 8]	**M1**	
	$e^x=-1+\sqrt{\dfrac{9}{2}}$ oe	**A1**	
	$x=\ln\left(\dfrac{3}{\sqrt{2}}-1\right)$ isw or 0.115 or 0.1145[06…] rot to 4 or more dp	**A1**	
9(b)	f is not one-one, hence f^{-1} does not exist oe	**B1**	
	$g^{-1}(x)=\ln(x-1)$	**2**	**M1** for $x=\ln(y-1)$ and a swop of variables at some point or $y=\ln(x+1)$ or $e^x=y-1$ and $y=\ln x-1$
	$x>1$	**B1**	

Question	Answer	Marks	Partial marks
10(a)	$OA = \sqrt{6^2 + 8^2}$ oe or 10	**B1**	
	[Angle *AOB* =] 0.9272[95…] rot to 4 or more dp or [Angle *OAB* =] 0.6435[01…] rot to 4 or more dp	**M1**	
	[Angle *COB* =] 2.214[297…] rot to 3 or more dp	**A1**	
	[Arc *CB* =] 6(*their* 2.214)	**M1**	**FT** *their COB*
	[Perimeter =] 8 + (*their* 10 + 6) + 6(*their* 2.214)	**M1**	**FT** *their* arc *CB* and *OA*
	37.3 or 37.28[578…] rot to 2 or more dp	**A1**	
10(b)	$\frac{1}{2}\times 8\times 6+\frac{1}{2}\times 6^2\times 2.214$ oe, soi	**M2**	**FT** *their* 2.21 **M1** for $\frac{1}{2}\times 6^2\times 2.214$ soi
	63.9 or 63.85[735…] rot to 2 or more dp	**A1**	
11(a)	$\dfrac{1}{\frac{1}{\cos x}-\frac{1}{\sin x}}+\dfrac{1}{\frac{1}{\cos x}+\frac{1}{\sin x}}$	**M1**	
	Simplifies denominator $\dfrac{1}{\frac{\sin x-\cos x}{\sin x\cos x}}+\dfrac{1}{\frac{\sin x+\cos x}{\sin x\cos x}}$	**A1**	
	Writes as two simple algebraic fractions: $\dfrac{\sin x\cos x}{\sin x-\cos x}+\dfrac{\sin x\cos x}{\sin x+\cos x}$	**A1**	OR writes as a single simple algebraic fraction: $\dfrac{\sin x\cos x(\sin x+\cos x)+\sin x\cos x(\sin x-\cos x)}{(\sin x-\cos x)(\sin x+\cos x)}$
	Combines and simplifies: $\dfrac{2\sin^2 x\cos x}{\sin^2 x-\cos^2 x}$	**A1**	
	Correct simplification to given answer e.g. $\dfrac{2\cancel{\sin^2} x\cos x}{\cancel{\sin^2} x-\frac{\cos^2 x}{\sin^2 x}}=\dfrac{2\cos x}{1-\cot^2 x}$ or $\dfrac{\cancel{\sin^2} x(2\cos x)}{\cancel{\sin^2} x(1-\cot^2 x)}$ $\left[=\dfrac{2\cos x}{1-\cot^2 x}\right]$	**A1**	All steps correct nd fully justified

Question	Answer	Marks	Partial marks
11(a)	**Alternative method**		
	Common denominator: $\dfrac{\sec x + \operatorname{cosec} x + \sec x - \operatorname{cosec} x}{(\sec x - \operatorname{cosec} x)(\sec x + \operatorname{cosec} x)}$	**(M1)**	
	Simplifies: $\dfrac{2\sec x}{\sec^2 x - \operatorname{cosec}^2 x}$	**(A1)**	
	Rewrites in terms of sin*x* and cos*x*: $\dfrac{\frac{2}{\cos x}}{\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}}$	**(A1)**	OR multiplies numerator and denominator by $\cos^2 x$: $\dfrac{2\sec x}{\sec^2 x - \operatorname{cosec}^2 x} \times \dfrac{\cos^2 x}{\cos^2 x}$
	$\dfrac{\frac{2}{\cos x}}{\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}} \times \dfrac{\cos^2 x}{\cos^2 x}$	**(A1)**	
	Correct simplification to given answer e.g. $\dfrac{\frac{2\cos^{\not 2} x}{\not{\cos x}}}{\frac{\cos^2 x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x}} = \dfrac{2\cos x}{1 - \cot^2 x}$	**(A1)**	
11(b)	$\tan\left(y + \frac{\pi}{4}\right) = [\pm]\frac{1}{\sqrt{3}}$	**B1**	
	$\left[y + \frac{\pi}{4} =\right]\ \frac{\pi}{6}$, or $-\frac{\pi}{6}$, or $-\frac{5\pi}{6}$, or $-\frac{7\pi}{6}$ oe	**M1**	
	$[y =] -\frac{\pi}{12},\ -\frac{5\pi}{12},\ -\frac{13\pi}{12},\ -\frac{17\pi}{12}$ oe	**A2**	No extras within range **A1** for two correct, ignoring extras