ADDITIONAL MATHEMATICS

Paper 0606/11
Paper 11

Key messages

This paper required candidates to recall and use a range of mathematical techniques, to devise mathematical arguments and present those arguments precisely and logically. Good responses were set out clearly and demonstrated a good understanding of fundamental techniques. They also showed a good understanding of mathematical language.

General comments

A good range of responses were provided, showing that many candidates had worked hard and understood the syllabus objectives, being able to apply them appropriately. Candidates appeared to have no timing issues and most candidates attempted all the questions.

There were some topics where candidates appeared to be less familiar with the techniques required. These included sketching and interpretation of graphs of cubic functions, integration of algebraic fractions, solution of equations involving exponential functions and interpretation of displacement–time and velocity–time graphs. They would benefit from practice in answering questions from these areas of the syllabus.

Candidates should be aware that if a method is specified by the question, then they must use that method for their solution. The use of the words 'Hence' or 'use your …' in the second part of a question is an indication that the method employed should use the result from the previous part. Care should be taken to read the wording of such questions. Candidates should also be aware that if they are requested not to use a calculator it is particularly important to show all steps in their working.

Candidates should read questions carefully and check that they have fully answered the question and have given their answer in the required form.

Comments on specific questions

Question 1

(a) Most candidates knew that a cubic graph was required but marks were lost by curves that did not extend beyond $x = -1$ and $x = 2$ and by miscalculation of the y-intercept. The x-intercepts were usually correct.

(b) Candidates should be aware that 'hence' indicated that their graph should be used to obtain the inequalities, with no calculations necessary. Most candidates whose graphs extended beyond 2 obtained $x > 2$ but few obtained the other inequality. Candidates should be aware of the > sign in the question and use strict inequalities in their answer.

Question 2

Candidates often obtained the integral of $\frac{1}{x-1}$ but were less successful with the integral of $\frac{1}{(x-1)^2}$ which many also tried to express as a logarithm. Care was required with the bracket in $\ln(x-1)$, but many candidates clearly knew how to apply limits correctly. There was, however, some confusion with signs when evaluating the final answer.

Question 3

(a) Most candidates showed a good understanding of the factor and remainder theorems and this question was very well done.

(b) Most candidates used algebraic long division and many were correct.

(c) Most candidates evaluated the discriminant for the quadratic equation obtained in the preceding part and found that it was less than zero but few candidates then stated a full conclusion relating to both q(x) and p(x). Candidates should be aware that, as in the question, the comment should relate to real solutions not just solutions.

Question 4

Many candidates showed a good knowledge of the application of the binomial theorem and most expanded $(a+x)^3$ correctly. The expansion of $(1-\frac{x}{3})^5$ was often marred by slips such as loss of the negative sign in $\frac{-5}{3}x$ and mistakes in expanding $(-\frac{x}{3})^2$. Although some candidates correctly obtained *b* and *c*, others did not realise that *b* was obtained by identifying and adding two products and *c* obtained by identifying and adding three products. The value of *a* was nearly always obtained correctly.

Question 5

(a) Successful candidates used differentiation or completing the square to find a minimum value. In some responses there seemed to be some confusion between domain and range. Care had to be taken to use $\geqslant$ and not > as the inequality.

(b) Candidates should be familiar with the shape of the graph of the exponential function. Those who knew that $e^x > 0$ found it straightforward to add 1 to that. Care had to be taken to use > and not $\geqslant$ as the inequality.

(c) Some responses showed a good understanding of composite functions and obtained $\left(1+e^{2x}\right)^2+4\left(1+e^{2x}\right)$. The majority of these did not realise that they had obtained a quadratic equation in e^{2x} and tried to find an expression for ln *x* before finding e^{2x}. Candidates who successfully found $x=\frac{1}{2}\ln 2$ often neglected to express their answer as a single logarithm as required in the question and lost the final mark. Some candidates misunderstood composite functions and obtained $\left(1+e^{2x}\right)^2+4x$ and could not proceed.

Question 6

(a) (i) Most responses were correct.

(ii) This was not as well done as the first part but there were many correct responses.

(iii) Successful candidates had a clear plan to consider cases taking into account that 9 was both odd and could be the first digit of a number greater than 60 000. Other responses tended to be unstructured and unsuccessful.

(b) Most candidates attempted to use the formula given on the formula list. The left-hand side was usually correct but a common error was to use (n – 5)! rather than (n + 1 – 5)! in the denominator of the right-hand side. Candidates who had $n(n-1)(n-2)(n-3)$ correctly on the right-hand side and tried to obtain $(n+1)(n+1)n(n-1)(n-2)(n-3)$ on the left-hand side often omitted the n or one of the (n + 1)s. Candidates would benefit from practice in the manipulation of algebraic factorials.

Question 7

(a) (i) There was some confusion between displacement and distance and –10 was a common answer. Some responses clearly related to a velocity–time graph as calculations were made to find the area under the graph.

(ii) Few responses showed an attempt to calculate gradients from the displacement–time graph. In responses where constant speed was shown by horizontal lines it was usual to obtain the line at velocity = 5 but not the one at velocity = –2.

(b) (i) Most candidates knew that they had to integrate and many did so correctly with some a factor of 2 out. Some responses did not include a constant of integration so could not use the given information to find an expression for *v*. However, many good solutions were seen.

(ii) A good number of candidates proceeded from a correct previous part to obtain a correct integral. An absence of a constant of integration in this part and/or the previous part meant some candidates could go no further. Some candidates integrated their 5 from the previous part as 5*x* rather than 5*t*.

Question 8

Candidates should be aware that the instruction not to use a calculator in either part means that they have to be particularly careful to show all relevant working.

(a) Most candidates found *x* correctly showing full working for the rationalisation. Candidates who calculated *x* first were usually more successful when calculating *y* as no further rationalisation was required. Those who found *y* first sometimes found a difficult route, not realising that $2-\sqrt{3}$ could be cancelled. Repeated attempts at expansion and rationalisation then led to arithmetic errors and miscopying from one line to the next.

(b) Most candidates knew what to do to find a stationary point. Sign and arithmetic errors were evident in the subsequent manipulation and some candidates omitted the relevant rationalisation steps that were required in a question where calculators could not be used. Care should be taken to express the final answer in the form requested.

Question 9

(a)(i) Most candidates obtained the correct factors.

(ii) Candidates who connected this question to the previous part usually went on to solve at least one trigonometrical equation correctly. Very few, however, obtained a complete set of answers and candidates should be advised on techniques for obtaining all solutions in range.

(b) Candidates should be aware of and learn fundamental trigonometric relationships that are not included in the formula list. Lack of awareness that $\sec A = \dfrac{1}{\cos A}$ lost all the marks in this question. Candidates who knew this usually went on to find the value of $\cos\left(2\phi+\dfrac{\pi}{4}\right)$. Most responses employed the correct order of operations to find at least one value of ϕ, but few candidates realised how many solutions there were in the given range and should be advised on techniques for obtaining all solutions in range.

Question 10

(a) Most candidates either used the sine of the half angle or the cosine of the angle correctly. However, it was necessary to show an answer of 1.696 rounded to 1.70 to be clear where the rounded answer had come from. Candidates should be advised that finding an angle in degrees first was rarely successful.

(b) The length of the major arc was often calculated correctly but finding *BC* or *AC* was more problematic. Many candidates did not recognise that the easiest method was to use the cosine rule for triangle *OBC*. Those who calculated the height of triangle *ABC* were less successful with the two steps involved. A common misunderstanding was to believe that the angle *OBC* was a right angle.

(c) Successful candidates had a clear plan and a clear idea of the calculations required. The area of the major sector was often found correctly. It was often unclear what candidates intended to add to the sector area with many adding both the area of triangle *AOB* and the area of the kite *AOBC*. Others used an incorrect value of *AC* from the previous part. The wording of the question and the diagram required careful reading.

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

Candidates should be aware that they may need to refer back to the previous part of a question to help them with a solution. They should not rely on seeing the use of the word 'Hence' to indicate this. A check should also be made to ensure that the demands of the question have been met fully and that solutions are in the required form and to the required level of accuracy. Candidate should also know the formulae involving permutations and combinations in terms of factorials.

General comments

It was pleasing to see that many candidates had been well prepared for this examination, showing a good understanding of the syllabus. Most candidates were able to attempt all the questions, setting their solutions out clearly. There appeared to be no timing issues or issues with insufficient working space. There were very few scripts involving overwriting so the process of marking was not hindered by not being able to see a candidate's work clearly.

Comments on specific questions

Question 1

Most candidates were able to manipulate the indices correctly for the majority of the terms. Many correct solutions were seen.

Question 2

(a) A correctly shaped graph was produced by most candidates. Some candidates omitted to state the intercepts of their graph with the coordinate axes.

(b) Most candidates were able to obtain the correct critical values of -1 and $\frac{11}{3}$, usually by considering two linear equations obtained from the consideration of the modulus equation. This was by far the most straightforward and quickest method, although squaring each side of the modulus equation to obtain a quadratic equation was just as acceptable. It was expected that candidates make use of their sketch in **part (a)** to help them decide on the correct range of values using the critical values. It was evident that this was not done in many cases with answers such as $x \leqslant -1,\ x \leqslant \frac{11}{3}$ being all too common. Candidates should always be aware that they may need to refer to a previous part in a question to help them with a solution.

Question 3

(a) Most candidates were able to find the vector $\overrightarrow{OP}$ using a correct method. Any errors were usually due to sign errors.

(b) It was important that sufficient detail was shown in order to gain full marks. The mark allocation of two marks should have alerted candidates to the fact that this part of the question could be done within two or three lines. This was another example of where the work done in a previous part should be used to help with a solution. Candidates had found vector $\overrightarrow{OP}$ in **part (a)** and the intention was that the vector $\overrightarrow{OP}$ was found in a different form using the given ratio. The two different expressions for $\overrightarrow{OP}$ could then be equated to obtain the given result. Many candidates did use this method, whilst other candidates did obtain the given result by using less straightforward, but equally correct, methods.

Question 4

Integration of the given equation to obtain an equation for $\frac{dy}{dx}$ was done correctly by most candidates, with errors usually being in the value of the coefficient of $(3x+2)^{\frac{2}{3}}$. Many candidates then attempted to find the value of the arbitrary constant using the given conditions correctly, although some did not consider an arbitrary constant at all. A second attempt at integration was made by most candidates, again with errors being in the value of the coefficient of $(3x+2)^{\frac{5}{3}}$. Many then attempted to find the value of a second arbitrary constant. There were many correct solutions. It was important that the final answer was given in the form of an equation as required. As some candidates did not do this, it highlights the need for candidates to check that they have given their final answer in the form demanded in the question. Some candidates, having found $\frac{dy}{dx}$ went on to find the equation of a tangent rather than a curve.

Question 5

(a) Many completely correct solutions were seen, with candidates making the correct use of the addition and subtraction rules for logarithms. The incorrect statement that $\log_a p + \log_a 5 - \log_a 4 = \frac{\log_a 5p}{\log_a 4}$ was treated as an error which led to often a fortuitously 'correct' answer.

(b) It was essential that candidates recognise the given equation as a 'disguised quadratic equation' in 3^x. Many did just this and went on to find the only valid solution of $x = -1$. Some candidates introduced their own variable for 3^x which was perfectly acceptable as long as they remembered to given their final solution in terms of *x* and not leave it in terms of their variable.

(c) Most candidates realised that a change of base for either one of the two terms on the left-hand side of the equation was necessary. Most changes of base were correct, but some candidates were unsure of the next step needed in the solution. Most success was had by those candidates who again introduced their own variable and then obtained a quadratic equation in terms of this variable, which they were able to use to obtain a correct final solution.

Question 6

It was essential that sufficient detail be given, in both parts of this question, to provide evidence that a calculator had not been used.

(a) Most candidates attempted to differentiate to find $\frac{dy}{dx}$. Many correct derivatives were seen although some candidates were unable to deal with the inclusion of surds in the terms, not treating them as any other constant. The resulting equation was invariably equated to zero and a value for x found. It was essential that evidence of rationalisation of this term be seen, with most candidates doing exactly that. Many correct responses were seen. Some candidates used the fact that the given equation was a quadratic equation and were able to state the *x*-coordinate of the stationary point using the fact that '$x = -\frac{b}{2a}$'. This was equally acceptable and equally successful. Other candidates misunderstood the demands of the question and equated *y* to zero and then attempted to solve the resulting quadratic equation to find the *x*-coordinates of the intercepts on the *x*-axis. This highlights the need to ensure that question is read carefully so that incorrect assumptions of the demands are not made.

(b) All that was required in this part was a substitution of the value of *x* obtained in **part (a)** into the original quadratic equation. Many candidates with a correct answer in **part (a)** gained full marks, provided sufficient detail of the substitution and resulting expansions has been seen. It was evident that some candidates had checked their calculations when their answers did not match the required form and made appropriate and correct amendments. For those candidates with an incorrect solution to **part (a)** method marks were available. The fact that an answer in the required form could not be obtained, should have prompted further checking of work in **part (a)**.

Question 7

(a) (i) Most candidates provided a correct answer.

(ii) Fewer correct answers were seen but many candidates were able to gain credit for recognising that there were 5 ways of choosing the first position in the password and 4 ways of choosing the last position in the password (or equivalent ${}^5P_2$). It was essential that this was part of a product. Credit was also given if it was recognised that there were 360 or ${}^6P_4$ ways of choosing the remaining positions, again provided that this was part of a product.

(iii) Many correct solutions were seen with most recognising that there were 3! (or equivalent) ways of choosing the first three places and ${}^5P_3$ or 60 ways of choosing the remaining places.

(b) Many candidates were able to obtain a correct solution provided they were able initially to write down the given information in terms of combinations and then in terms using factorials. Some errors were made with the positioning of the 6, but method marks were available for correct simplification of the algebraic factorials to obtain *n*. It is essential that candidates be familiar with the formulae for combinations in terms of factorials. Little progress can be made otherwise, in questions of this type.

Question 8

(a) Provided it was recognised that $y = Ax^b$ can be written in the form $\lg y = \lg A + b \lg x$ with this form being subsequently used correctly, candidates obtained correct solutions. Some chose to make use of the gradient of the straight line as a starting point with others forming two simultaneous equations in terms of *b* and $\lg A$. Most errors occurred with incorrect substitutions, the incorrect matching of the gradient to *b* and incorrect use of $\lg A$.

(b) Most candidates were able to make correct use of their values for *A* and *b* obtained in **part (a)**.

(c) Most candidates were able to make correct use of their values for *A* and *b* obtained in **part (a)**.

Question 9

(a) Many correct solutions were seen. Errors were usually arithmetic or errors in signs. A few candidates, having obtained a correct solution of 19.1 for their quadratic equation, sometimes gave a response of 19 rather than 20. It was important that the statement $n = 20$ be seen or implied in a sentence. Quite a few candidates gave answers such as $n \approx 20$ or $n \geqslant 20$. This highlights the need for candidates to ensure that they have given their answer in the correct form.

(b) (i) A correct common ratio was given by the majority of candidates. Errors were usually of the type $r = \pm 3$.

(ii) Most candidates were able to find the first term of the progression and continue of to find the correct answer in the required form. There were other valid methods which were equally successful. Some candidates did not realise that $\frac{1}{27}$ could be written as 3^{-3} and hence gave their answer in decimal form.

(c) It was essential that the common ratio be identified as $\sin\theta$. The demand of the question requires an explanation, and this was deemed to be the first part of the explanation. It was then necessary for candidates to state that for the values of θ given $|\sin\theta| < 1$, or equivalent, with no incorrect statements, so the geometric progression had a sum to infinity. Many candidates were able to identify the correct common ratio but did not give enough further explanation with too many candidates stating incorrectly that $\sin\theta < 1$.

Question 10

(a) Most candidates dealt correctly with $\operatorname{cosec}^2\alpha$ and $\sec^2\alpha$, going on to obtain the equation $\frac{1}{\sin\alpha} + \frac{1}{\cos\alpha} = 0$. Some errors then occurred with candidates simplifying this equation incorrectly. However, many candidates did obtain the correct equation of $\tan\alpha = -1$. This equation could be obtained by making use of the appropriate trigonometric identities and subsequent simplification, a far lengthier method, more prone to errors being made, but as equally valid. Some candidates obtained quadratic equations by squaring an equation in $\sin\alpha$ and $\cos\alpha$, but then ended up with extra solutions which were not discounted. It was pleasing to see many correct solutions for this part of the question.

(b) (i) It was essential that necessary detail be shown. It was intended that candidates start with the left-hand side of the expression and simplify it, showing each relevant step, in order to end up with the term on the right-hand side of the expression. The steps that needed to be shown were, dealing with the fractions correctly together with an expansion of $(1-\sin\theta)^2$, followed by the use of the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$. A simplification of the terms in the numerator together with evidence of factorising to obtain a common factor in both the numerator and denominator was the penultimate step to obtain $\frac{2}{\cos\theta} = 2\sec\theta$. This penultimate step was not shown by many candidates. There were other equally valid and acceptable methods seen, but these were far less common.

(ii) Most candidates made the link with the work done in **part (i)**, obtaining the equation $\cos 3\phi = \frac{1}{2}$. Most of these candidates were then able to solve this equation to obtain at least one correct solution and often all three correct solutions. Errors occurred usually with candidates solving the equation $\cos\phi = \frac{1}{2}$, not taking into account the multiple angle.

Question 11

This was an unstructured question which was intended to test the problem-solving skills of the candidates.

Most candidates realised that they needed to differentiate the given equation using the quotient rule. Most errors occurred when the candidates were unable to differentiate $\ln(x^2+2)$ correctly. This did not stop further method marks being available provided a correct process was seen. Many correct methods were seen, with any errors usually being arithmetic or due to incorrect signs. Some candidates found the equation of the tangent rather than the normal and other candidates found the correct equation for the normal but found the coordinates of the point where the curve meets the *x*-axis.

ADDITIONAL MATHEMATICS

Paper 0606/13
Paper 13

Key messages

Candidates should ensure that they have read each question carefully, made sure that the demand of the question is met and then given their answers in the required form and to the appropriate level of accuracy. When a diagram is given, it should be used and referred to in all parts of the question, not just the first part of a question. Candidates should also be aware that their answers to previous parts may sometimes be needed to help with a solution, even if the word 'Hence' is not used. The word 'hence' usually means that the answer to the previous question part be used so that a specific method of solution is adopted.

General comments

A good understanding of the syllabus was shown by many candidates who had clearly worked hard and revised well in preparation for the examination. It was, however, also evident that some candidates were clearly unprepared with many of these candidates leaving many questions unattempted.

Comments on specific questions

Question 1

Most candidates realised that they needed to make use of the discriminant of the quadratic equation. Some errors occurred when dealing with the constant. Whilst correct critical values were obtained by many, few candidates were able to form the correct inequalities.

Question 2

(a) The majority of candidates attempted to differentiate a product, usually obtaining a correct solution. Errors usually concerned the power of the derived exponential term.

(b) (i) Many correct responses were seen, showing a good understanding and application of the chain rule.

(ii) Use of the answer to **part (i)** was expected. Few candidates successfully made the connection with **part (i)** in spite of the word 'Hence'. Candidates are expected to know that integration is the reverse process of differentiation. Of those candidates that integrated correctly, most applied the limits correctly. Some answers were given to 2 significant figures only. Answers to 3 significant figures are expected unless otherwise stated.

Question 3

Use of the correct trigonometric identity was made by most candidates. Many then continued to obtain a quadratic equation in terms of $\cot\theta$. Some candidates made errors in the solution of this quadratic equation, usually incorrect signs in the linear factors obtained. It was intended that the answers be given in radians as implied by the given range and many candidates identified both a positive and negative angle. Some candidates still chose to give their responses in degrees. Again, the correct level of accuracy was not applied by some, with 2 significant figure solutions being given.

Question 4

(a) Most candidates obtained a correct term of 64. There were varying degrees of success with the other two terms required. Errors were usually either sign errors or errors in simplification. Some candidates did not attempt to simplify the terms in their expansion, highlighting the need for a check that the final answer is given in the form specified in the question.

(b) Few correct solutions were seen and many candidates did not attempt this part of the question. Although use of the answer to **part (i)** was required, candidates also obtained credit for the correct expansion of $\left(3-\frac{1}{x^2}\right)^2$.

Question 5

Many correct and partially correct solutions were seen, with most making use of the relationship $e^y = mx^2 + c$. Most candidates chose to find the gradient of the straight line, equate it to *m* and then use it correctly to find the value of *c*. Misuse of the laws of logarithms prevented some candidates obtaining full marks.

Question 6

(a) Most candidates recognised that an equilateral triangle was involved and were able to write down the correct angle straightaway. Others made use of the cosine rule to obtain the correct angle. It was pleasing to see that answers were given in radians.

(b) Many correct responses were seen with candidates recognising the need to use an arc length.

(c) Although there were quite a few completely correct solutions, some candidates made errors with the lengths used when calculating the area of the triangle *OAB* and the area of the sector *ODC*. Careful reference to the diagram may have prevented these errors.

Question 7

(a) (i) Recognition that there are 4 places left to fill and 8 people available to fill them meant that a correct response was obtained by most candidates.

(ii) Fewer correct responses were given. Most candidates chose to attempt to find the number of ways there were at least 2 teachers on the committee by considering the cases when there were 2 teachers, 3 teachers, 4 teachers and 5 teachers separately. There were varying levels of success using this method. Very few candidates chose to consider the alternative method. This was the recognition that there had to be at least 1 teacher on the committee anyway due to the total number of teachers involved. It was then just a case of subtracting this number (5) from the total number of ways the committee could be formed.

(b) It was necessary that candidates know the correct formula involving permutations. Of those that did, many obtained the correct solution by simplifying the factorial terms correctly.

Question 8

This was one of the more demanding questions on the paper, and few correct solutions were seen.

(a) It was essential that the value of *b* be calculated first, making use of the given period of the function. Two simultaneous equations involving *a* and *c* could then be formed making use of the intercept on the *y*-axis and the given coordinates of the point *R*.

(b) Few correct responses were seen. It was expected that either symmetry or consideration of the equation of the curve equated to zero and solved, be used.

(c) Very few correct responses were seen. It was expected that use of symmetry and reference to the given diagram be made to find the coordinates of the minimum point.

Question 9

(a) Most candidates were able to give the equation of the curve in the correct expanded out form. This demand was made in order to guide candidates as to what was required next, that is, the use of differentiation to obtain the coordinates of the stationary points. Many candidates did just this, but there were quite a few who did not recognise that differentiation was needed and attempted to find the points where the curve met the *x*-axis. Many correct responses were seen although there were some errors in the calculation of the *y*- coordinate at the point where $x = -\frac{2}{3}$.

(b) Although very few completely correct curves were sketched, most candidates were able to obtain marks for correct intercepts, basic shape and a maximum point in the second quadrant of the graph. Few were able to distinguish between the necessity of having a cusp at the point where $x = -2$ and a minimum stationary point at the point where $x = 2$.

(c) Use of the graph in **part (b)** and consideration of the *y*-coordinates obtained in **part (a)** were required. Candidates should be prepared to make use of work covered in previous parts even if the word 'Hence' is not used in the question part demand. Very few correct solutions were seen.

Question 10

(a) The mark allocation of this question part should have alerted candidates to the fact that the answer could not be written down straightaway. Candidates were intended to find the vector $\overrightarrow{OD}$ in terms of **a**, **c** and *h*, as a starting point. Use of the fact that $\overrightarrow{OE} = h\overrightarrow{OD}$ together with the fact that $\overrightarrow{OD} + \overrightarrow{DE} = \overrightarrow{OE}$ would lead to the required response. There were quite a few correct responses from using this approach.

(b) Many correct responses were obtained using the triangle *DBE* and the given ratio.

(c) It was intended that the vector obtained in **parts (a)** and **(b)** be equated and then like vectors considered. Few correct solutions were seen but many candidates obtained credit for a correct approach.

Question 11

(a) Most candidates were able to obtain the correct coordinates of one point of intersection using $x + 2y = 10$ and $x + y = 2$. Fewer candidates were able to find the correct coordinates for the second point of intersection. The error made by most candidates was in the interpretation of $|x + y| = 2$. The two equations to be used were $x + y = 2$ and $x + y = -2$. Although few correct points of intersection were seen most candidates were able to gain credit for using a correct approach with their sets of coordinates. However, some candidates made incorrect use of the coordinates of the point *C*, highlighting the need to read the information given in the question carefully.

(b) Very few correct solutions were seen. It was expected that candidates would draw a diagram of the situation to help visualise the positions of all the lines and points together with the mid-point of the line *AB*. The possible positions of *D* could then be calculated using displacement vectors or a similar approach.

ADDITIONAL MATHEMATICS

Paper 0606/21
Paper 21

Key messages

In order to do well in this paper, candidates need to show full and clear methods in order that marks can be awarded. In questions where the final answer is required in a given form candidates should be aware that full credit cannot be awarded for otherwise fully correct work unless this is done. Candidates should be aware of the general guidance on the cover sheet and ensure that all answers are given to the accuracy indicated. In questions where the answer is given, candidates are required to show that it is correct and fully explained solutions with all method steps shown are needed. Repeating information given in the question cannot be credited. In such questions candidates are encouraged to use consistent notation such as using the same variable throughout a solution and should avoid replacing a function of a variable with the variable itself. In questions that state that a calculator should not be used, omitting method steps often results in full credit not being given for a solution. Combining the steps directly into a simplified form, such as that produced by a calculator, cannot be credited. In questions that require a solution of several steps, clearly structured and logical solutions are more likely to gain credit. Candidates should be encouraged to write down any general formula they are using as this reduces errors and is likely to improve the accuracy of their solutions. In particular, method marks cannot be given for solving an incorrect equation when the solutions are taken directly from a calculator without showing any working. When a graph is required it should be completed in full and as accurately as possible with some labelling to support the sketch.

General comments

Some candidates produced high quality work displaying wide-ranging mathematical skills, with well-presented, clearly organised answers. This meant that solutions were generally clear to follow. Other candidates produced solutions with a lot of unlinked working, often resulting in little or no credit being given. More credit was likely to be given when a clear sequence of steps was evident.

Several questions were unstructured and candidates needed to plan their method carefully. There were many good solutions to these questions. Some candidates wrote down a few relevant steps but did not link them together.

Questions which required the knowledge of standard methods were done well. Candidates had the opportunity to demonstrate their ability with these methods in many questions. Most candidates showed some knowledge and application of technique. The majority of candidates attempted most questions, demonstrating a full range of abilities.

Some candidates needed to take more care when reading questions and keep their working relevant in order to improve their solutions. Candidates should also read the question carefully to ensure that, when a question requests the answer in a particular form, they give the answer in that form. This is particularly the case when the question states that an exact answer is required. Candidates should ensure also that each part of a question is answered and the answer clearly identified. When a candidate uses the blank page or an additional booklet they should make it clear which question their work relates to. It is not possible in most cases to connect work otherwise to a specific question which can lead to the loss of potential credit. When a question demands that a specific method is used, candidates must realise that little or no credit will be given for the use of a different method. They should also be aware of the need to use the appropriate form of angle measure within a question. When a question indicates that a calculator should not be used, candidates must realise that clear and complete method steps should be shown and that the sight of values clearly found from a calculator will result in the loss of marks.

Candidates should take care with the accuracy of their answers. Centres are advised to remind candidates of the rubric printed on the front page of the examination paper, which clearly states the requirements for this paper. Candidates need to ensure that their working values are of a greater accuracy than is required in their final answer.

Candidates should be advised that any work they wish to delete should be crossed through with a single line so that it can still be read. There are occasions when such work may be marked and it can only be marked when it is readable. Where a candidate feels they have made an error but is unable to offer any alternative work they are advised not to cross out their work to aid legibility. Rubbing work out then writing over it can sometimes lead to examiners being unable to read clearly the intended work.

Comments on specific questions

Question 1

(a) This was well done by the vast majority of candidates and most answers were given in the correct form. This form was specified in the question and writing the values of a and b alone was insufficient to gain full credit. Many incorrect solutions gained some credit by having the answer partially correct.

(b) This question indicated that the previous answer should be used and so credit was only applied where the coordinates followed on from that. Some candidates were confused and simply gave the minimum value rather than the coordinates of the minimum point.

Question 2

Most candidates were able to calculate the gradient of the required line correctly and in many cases the intercept also. Some candidates treated the coordinates as values of x and y rather than $\ln x$ and $\ln y$. Having found these values correctly many candidates were able to continue to a correct relationship between $\ln y$ and $\ln x$ and rearrange this to the required form. Some final answers included + rather than $\times$ despite the form stated in the question. A popular alternative involved rewriting the given equation into log form and then equating the gradient and intercept accordingly. Candidates who followed this method were most likely not to state their final answer in the required form and thus did not complete their solution.

Question 3

(a) Success was most commonly achieved by candidates who squared both sides and worked on the resulting quadratic. These candidates tended to solve the equation and then consider the inequality. Those writing down the two possible linear equations indicated by the modulus statement often made an inequality error somewhere which they occasionally attempted to correct without justification. A not uncommon error was to attempt to combine two correct inequalities into a single one.

(b) Those candidates who replaced $\sqrt{x}$ with another variable were almost always successful, most commonly by factorisation. A few treated the equation as a quadratic in $\sqrt{x}$ with similar success provided they realised that their solutions needed to be squared. Many false attempts were seen involving manipulation of the equation in its given form, or by squaring each term individually. It was possible to rearrange the equation by taking $11\sqrt{x}$ across to the other side and then squaring correctly but this was often incorrect at the next step by squaring $2x$ and 12 separately.

Question 4

(a) While many candidates were able to write down the value of a correctly there was relatively little success with finding b. The common error was to take the period of the tangent function as 360° instead of 180°.

(b) It was very rare to see a completely correct sketch. The vast majority of candidates did not seem to be aware of the shape of a tangent curve with a mixture of quadratic, sine and cosine curves most common, or no attempt at all. Of those who drew something resembling a tangent curve the number of branches was frequently too many or at best a two branch curve displaced. For those who had an approximate correct shape a little more attention to detail would have helped when considering the positions of key points. No points were requested by the question but labelling could help in those cases where accuracy was in doubt. Those candidates who attempted to plot points often produced a curve whose shape was not good enough.

Question 5

The method of finding the perpendicular bisector of the line joining two points was well understood, and full marks were often obtained. What seemed to present a difficulty for some candidates was finding the point *A* before this could be done. Whilst some accomplished this in a few brief steps others filled much of the answer space with inaccurate algebra. Some candidates wrote down the coordinates correctly but without justification which led to full marks being unavailable. In all cases where the correct coordinates were found it was still possible to proceed correctly to the equation required. In a few cases the equation of the perpendicular through *A* was found instead of that through the mid-point of *OA* and a few other candidates found the equation of the line rather than the perpendicular. Candidates should also note that finding the equation of the line itself was unnecessary and time consuming.

Question 6

The general method for this question was to find the derivative of y and then use the approximation $\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$. This was well understood by the majority of candidates. Execution of the method proved much more challenging, especially with regards to accurate differentiation. Most candidates correctly differentiated $e^{\frac{x}{2}}$ to get $\frac{1}{2}e^{\frac{x}{2}}$ or at least a multiple of this. Similarly many candidates knew that the derivative of $\cos 2x$ was a multiple of $\sin 2x$. Application of the product rule was usually accurate if attempted. This was only the case for a minority of candidates. The final step required the substitution of $x = 1$ into the derivative and multiplication of this by h. This was an example of a question where the substitution had to be clearly seen. Correct method was not implied by follow through values that were unsupported by method. Candidates should also have noted that x was given in radians, as degrees were often erroneously applied.

Question 7

This question was well done by the majority of candidates. The most successful method was to first eliminate y from the two given equations and then to set the discriminant of this quadratic in x to zero to lead to a quadratic in k. Eliminating x was also seen but with much less success due to the added complexity of the algebra required. It was also possible to equate gradients to form a connection between x and k and then substitute back into the original equation(s). This was seen occasionally but with mixed success, often stopping at the stage of equating gradients. Some candidates lost the final mark here by giving their answer immediately in decimal rather than exact form. On this occasion converting the exact form to decimals was not penalised but candidates should be careful to follow the requirements of the question. As has been reported in previous years, it is advisable to quote the quadratic formula before using it to reduce the risk of incorrect substitution.

Question 8

(a) (i) There were some good explanations here involving the validity of logs of negative values and zero. Many candidates had some idea of this but frequently only considered negative values or zero but not both. Others offered only manipulation of logs with no conclusion.

(ii) Candidates were fairly evenly split between those who applied the laws of logarithms correctly to arrive at a quadratic equation in y and those who erred immediately by misapplying the division rule. Of those who formed the correct quadratic a significant number either did not give an exact solution or gave two solutions. Careful reading of the question should have indicated that a single exact answer was expected.

(b) This question required at least four logarithm rules to be applied. These could be done in a variety of orders but the key was the answer needed to be in terms of $\log_a 9$ and therefore work in other bases only became worthy of credit when converted. Similarly, work which dealt with only part of the expression needed to be included in the full expression to be meaningful. There were several completely correct solutions. Some of these were concise while others took a little longer often combining log terms only to break them back into separate logs later. Many candidates managed to use the change of base rule appropriately then made the error of adding logs within the second term. Others dealt with $\log_a \sqrt{b}$ correctly and made no further progress. Candidates should be careful with their general arithmetic as this could prevent good log work being credited – in particular when $\dfrac{\log_a 9}{\frac{1}{2}}$ became $\dfrac{\log_a 9}{2}$.

Question 9

There were a significant number of completely correct and clear solutions here. This was another question where candidates needed to carry out multiple steps without guidance. Only a relatively small number of candidates made no attempt and a similar number ignored the calculus implied in the question and treated it as a coordinate geometry question in error. Most candidates realised the need to differentiate twice and to use the values given in order to find the constant of integration at each stage. Most of these were aware that the integral of sin involved cos and vice versa. The difficulty many had was in accurately applying this and sign and coefficient errors or both were not uncommon. Not including the constant of integration at either stage had the effect of rendering further work meaningless and candidates should have considered why the coordinates and value of $\dfrac{dy}{dx}$ had been given.

Question 10

(a) Most candidates had some idea of where to start here and there were many fully correct solutions. Some candidates could correctly find the vector $\overrightarrow{AB}$ and many could also find the modulus correctly. There were those who stopped at this point without combining to find the unit vector. It was not uncommon for candidates to add rather than subtract vectors but there was still scope for these candidates to demonstrate their other knowledge.

(b) Many answers involved very circuitous routes to finding *A*. The most successful method was to find the mid-point as if this was a coordinate geometry question. Candidates might also be reminded of the usefulness of a clear diagram in such a situation as this.

(c) This proved to be the most challenging part of the question. There were very few completely correct solutions and many candidates did not score or left the answer space blank. Perhaps the most straightforward method was to write $\overrightarrow{OE} = \dfrac{1}{1+\lambda}\overrightarrow{OD}$ and set the *y* component to –3. There were many other possible methods involving similar triangles or coordinate geometry. Common errors were to use $2 + \lambda$ or to multiply by $1 + \lambda$. Other common errors were to consider the *x* component or to set the *y* component to zero.

Question 11

(a) (i) This part proved challenging to many candidates. Some managed to gain partial credit and there were a number of completely correct solutions using a variety of methods. The three terms of the arithmetic progression were fairly readily identified, and their connection to the terms of the geometric progression established, although occasionally the same symbol, *a*, was wrongly used in both. Forming and solving appropriate equations seemed to be the most challenging task. Perhaps the more standard method of equating common ratios was the most successful. Some used the square of the ratio also which could lead to a cubic equation which was not always successfully solved. An unusual method used by a few candidates was to realise that the ratio of the differences between successive terms was also equal to *r* which led to a concise and simpler solution if carried through correctly. Attempts sometimes filled the answer space and overflowed elsewhere with much algebra which was difficult to follow. Some circular arguments were seen involving an initial assumption of the relationship to be shown.

(ii) Nearly all candidates recognised that they were given the first term and common difference and substituted them into the formula for the sum of an arithmetic progression with few making errors in calculation.

(b) (i) It was relatively straightforward for candidates to realise from the data given that both the first term and common ratio of the geometric progression were 6 and they may have had these values as part of their solution to **part (a)(i)**. Consequently there were many correct answers. A common error was to use the first term as 1 confusing it with the arithmetic progression.

(ii) Most candidates knew that the key was something to do with the value of the common ratio and many realised that as *r* was greater than 1 the sum to infinity did not exist, which was sufficient. Some answered by referring to the general condition instead. Occasionally candidates made the correct comparison then said that the sum did exist. Some also appeared to think that the sum to infinity of a geometric progression could not be negative and based their response on this.

Question 12

Many candidates realised the need to find the coordinates of *A*, *B* and *C* and that integrating the given curve might be required in order to find an area. Consequently many candidates gained partial marks and provided these points were found correctly some candidates managed to follow a method which led to the correct answer. This was a small proportion as this question proved very challenging. It is advisable in questions like this to partition the diagram in order to see which sections might need to be added or subtracted before attempting manipulation.

Often candidates used most of the answer space in long methods to find the required coordinates when *B* and *A* could actually be written down using the given factorised form for the curve and symmetry respectively. Instead the brackets were frequently expanded, rearranged, then factorised again. *A* was often found via the maximum point involving calculus or completing the square. There were more errors in finding *C*. The frequently seen method was to combine the two equations, one in terms of *x* and the other in terms of *k*. This usually led to the candidate using the discriminant, with $k = 12$ a common solution. Many used this value as a limit for integration disregarding the fact that this was greater than 9, the *x*-coordinate for *B*.

When integrating, a number of candidates decided that the coefficient of x^2 should be positive and therefore integrated a different curve to the one given, which was not condoned. There were many variations on how to find the required area, some of which were very elaborate and frequently incorrect. The most concise method was to find the area of the rectangle with opposite corners at *A* and *C* and add this to the area under the curve to the right of this.

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

To do well in this paper, candidates should read each question carefully and identify any key words or phrases, making sure they answer each question fully. Candidates need to be aware of instructions in questions, such as 'Show that...'. Such instructions mean that when a solution is incomplete, often through calculator use, a significant loss of marks will result. In questions directly assessing the solving of simultaneous equations, candidates should show the necessary method step of eliminating an unknown. When the simultaneous equations are both linear, candidates should not rely on calculators for this step. Candidates should be encouraged to use their calculator to check solutions for such questions. Sufficient method needs to be shown so that marks can be awarded. When values are incorrect and the method from which they arise is not seen, marks cannot be given. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with trigonometric expressions. In questions assessing circular measure, converting angles to degrees is unnecessary and introduces more opportunity for error.

General comments

A good proportion of candidates demonstrated knowledge and understanding of mathematical techniques and were able to interpret and communicate well mathematically. This was particularly the case in **Questions 1**, **3**, **4**, **8** and **9**. Some candidates may have improved if they had a better understanding of the necessity to use bracketing, or correct ordering of terms in a product, to ensure correct, unambiguous mathematical form. For example, in **Question 12(b)** brackets were needed around the argument of the logarithm as it was a binomial expression. Candidates found questions where they needed to apply problem-solving skills more challenging, for example **Questions 6(b)(iii)** and **13(b)(ii)**.

Candidates who wrote answers in pencil and then overwrote them in pen should be aware that this made their work difficult to interpret. Candidates who wrote answers elsewhere usually added a note in their script to indicate that their answer was written, or continued, on another page. This was very helpful. The presentation of work was often clear and good.

Showing clear and complete method for every step in a solution was essential for questions where candidates were asked to 'Show that…' a result was of a particular form. This instruction indicated that the marks would be awarded for the method as the end result had been given. Candidates needed to understand that, when showing these results to be true, they need to generate the mathematics to arrive at each result and not use the information given as an assumed part of their solution. The need for this was highlighted in **Questions 10(b)(ii)** and **11(a)** in this examination. However, the key word 'Verify…' in **Question 12(a)(ii)** indicated that they could and should use the information they were given to demonstrate that the result under consideration was correct.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

Many candidates found this to be an accessible start to the paper. Most candidates applied the binomial theorem, as required, and were also able to simplify the terms correctly. A few candidates made errors when dealing with the powers of e^{2x} and a few others omitted to sum the terms, simply stating a list. A small number of candidates omitted to include the term $(e^{2x})^4$.

Question 2

It was expected that candidates would draw a graph showing all the key features of the cubic function given, including the end behaviour. A good proportion of candidates drew sufficiently accurate curves and also marked the *y*-intercept and the three roots correctly. A few candidates drew only the section of the curve for $-2 \leqslant x \leqslant 3$, which was not condoned as the end behaviour was not indicated. A small number of candidates omitted or miscalculated the *y*-intercept, with 6 and 3 commonly seen in these cases. A few others omitted to show at least one of the roots, commonly $x = 1$. Weaker responses had graphs that were ruled in sections or that had incorrect orientation. Very few candidates offered a graph that was not accepted as an attempt at a sketch of a cubic function.

Question 3

Most candidates identified the consideration of the discriminant as the appropriate technique to solve this problem. Many candidates were sufficiently careful with the algebraic manipulation needed and were able to find the correct critical values. A few candidates went on to compose a correct answer. The incorrect answer $-2.5 < k < 2$ was quite common. It seems that these candidates misinterpreted the case where roots are real and equal as being a single root rather than a repeated root. Other candidates either stated an incorrect pair of inequalities or simply ended their solution with $k = 2$ and $k = -2.5$. Some candidates wrote a correct answer but spoiled it by then stating $k = -2, -1, 0, 1, 2$. A small number of candidates gave a final answer in *x* rather than *k*, which was not condoned. A few candidates would have improved if they had taken a little more care in dealing with the initial expression, as sign errors and arithmetic errors were common in weaker responses.

Question 4

Many candidates correctly applied the factor and remainder theorems and quickly formed a correct pair of linear simultaneous equations. A good proportion of these went on to solve these to find *m* and *n* correctly and then show that $p(2) = 0$. A few candidates would have improved if they had taken care to include all the information in their initial steps as, sometimes, they wrote $p\left(\frac{1}{3}\right)$ but omitted '= 0' and occasionally this was not recovered by sight of a correct, simplified version of the equation. Whilst it is not recommended, the elimination of the unknown could be implied in this question as it was not the main technique being assessed. However, candidates who had incorrect equations or who gave incorrect values from correct equations, with no method shown in each case, lost more marks than those who had shown their method. Weaker responses often contained sign or arithmetic slips and presentation was sometimes poor. A few candidates showed that $x - 2$ was a factor using division or by factorising $p(x)$. Whilst these were acceptable approaches, they were more prone to error. A small number of candidates formed an equation using $p(2) = 0$ and used this as one of their linear equations to find *m* and *n*. This was not permitted as it was circular reasoning.

Question 5

(a) A good number of correct responses were seen. Common incorrect answers were -2, -1, $\frac{2}{3}$ or 2.

(b) Again, a good proportion of responses were fully correct. A few candidates gave the answer 3π only, which earned partial credit, as the function was clearly defined for a domain in degrees. A small number of candidates calculated $360 \times \frac{2}{3}$.

(c) Many excellent and very neat graphs were drawn over the whole domain. Many candidates marked key points to assist them with their sketches. Mostly this seemed to be helpful, although on occasion, points were mis-plotted and no credit could be given for the resulting shape. Some graphs were very parabolic and others very straight. A few candidates earned a mark for the correct period after sketching a graph that was not sufficiently cosinusoidal or that had an incorrect midline or amplitude, for example. A small number of candidates earned partial credit for a correct graph for $0° \leqslant x \leqslant 540°$.

Question 6

(a) This part of the question was well answered. Most candidates applied the correct distance formula for two points in coordinate form. Most of these went on to state an acceptable form of the answer. Occasionally, candidates mis-recalled the distance formula, not understanding its basis in Pythagoras' theorem. These candidates may have improved if they had made a simple sketch and used Pythagoras' theorem directly with the *x* and *y* distances 6 and 10, respectively. A few candidates left their answer as $\sqrt{136}$, which was not condoned for the accuracy mark, or incorrectly rounded to 11.6.

(b) (i) A very good number of correct answers were seen for this part. It was expected that candidates would use the simplest method which was to find $\frac{-4+6}{2}$ or similar. However, some candidates found the equation of the line *AC* and then solved for *y* when *x* was 8. Whilst a correct method, this more roundabout approach was more likely to result in an error.

(ii) Again, a good number of fully correct answers were seen. Many candidates were able to deduce that the correct procedure was to find the gradient of *AC* and then use the fact that the diagonals of a square are perpendicular to each other. Many candidates gave the answer in point/gradient form whilst many others used gradient/intercept form. Any correct form was acceptable but *x* and *y* both had to be present. Weaker responses sometimes omitted the *y* with *BD* = … being common in these cases. Some candidates assumed that the sides of the square were parallel to the coordinate axes. These candidates usually stated that *B*(11, –4) and *D*(5, 6) or vice versa and used these incorrect points to find the gradient of *BD*. A simple sketch may have helped these candidates notice that the shape drawn was a rectangle and not a square.

(iii) Candidates found this part to be quite challenging. The simplest solutions involved using the perpendicular gradient found in the previous part of the question. Candidates who had assumed the sides of the square to be vertical in the previous part continued to do so here. In fact, many who had not made this assumption earlier did so in this part and stated the incorrect vectors $\begin{pmatrix}-3\\5\end{pmatrix}$ and $\begin{pmatrix}3\\-5\end{pmatrix}$. A few candidates gave the answers –5**i** + 3**j** and 5**i** – 3**j**, which gained only partial credit as the question demanded that the answers be given as column vectors.

Question 7

Candidates are advised to keep their working values as accurate as possible in questions assessing circular measure. It is better to keep calculations in exact form for method and only round to 3 significant figures for the final value. This ensures accurate final answers and reduces the likelihood of making a premature approximation error, as was seen on occasion in both parts of this question. Whilst it is not recommended that candidates work in degrees in questions assessing circular measure, it is essential that candidates make sure that they have their calculator set to the correct mode for their choice of angle. Occasionally it was clear that candidates had used the wrong mode.

(a) The majority of candidates used a correct strategy and attempted to sum the arc length and the length of the two tangents. Most of these showed full method and earned both the method marks. Some candidates prematurely rounded their working values and lost the accuracy mark. Most candidates were able to find the arc length correctly, although some candidates who converted to degrees did so incorrectly. A few candidates were not taking advantage of the simple form for the arc length when the angle given was in radians. Calculations such as $\frac{\frac{7\pi}{9}}{2\pi}\times 2\times\pi\times 18$ were not uncommon and more prone to error. A few candidates struggled to find the length of the tangent. Those who were using basic trigonometry were most successful, although some thought the tangent was the hypotenuse and used an incorrect ratio. A few candidates used the sine rule but some were unable to find angle *BCA* correctly. Weaker responses often showed the sum of the arc and one tangent.

(b) Again, many candidates used a correct strategy and attempted to subtract the area of the sector from the area of the kite. Many did this correctly. Those who used rounded working values or an unsimplified formula for the arc length in the previous part usually continued in a similar way in this part. The use of rounded values more commonly resulted in an inaccurate final answer in this part of the question. Some candidates needed to take a little more care with their strategy as, on occasion, candidates doubled the area of the kite, thinking it was the area of one of the right-angled triangles. Candidates who used $\frac{1}{2}bc\sin A$ sometimes needed to take more care over which sides and angles were required for this formula.

Question 8

(a) A good proportion of fully correct answers were seen to this kinematics problem. Most candidates were able to solve $v = 0$ to find the two times at which the particle was at instantaneous rest. Many candidates were also able to deduce the need to integrate to find an expression for the displacement and many of these then used it correctly. A very good proportion of these understood that, whilst displacement can be signed, distance is not a vector quantity and is positive. These candidates were generally fully correct. Other candidates may have improved if they had understood the relationship between velocity and displacement, as some differentiated in this part. A few candidates found a non-zero constant of integration and this was not condoned. Some candidates omitted to show sufficient method to earn full credit. These candidates may have improved if they had realised that the substitution of the limits into the integral is a necessary method step and should be shown. A few candidates found the displacements 112 and 108 and offered those as their answer, misinterpreting what was needed.

(b) Again, this was generally well answered. A few candidates spoiled an otherwise correct solution by stating a positive final answer. This was not condoned. Those candidates who differentiated in **part (a)** usually integrated in this part.

Question 9

In this question, candidates needed to solve a pair of simultaneous equations. The key step in such a solution is the elimination of one unknown. This is a necessary step in the method and should always be shown clearly in order for credit to be given. Many candidates were successful and gave neat and logical solutions. The majority of candidates rearranged the equation $xy + 4 = 0$ to make either x or y the subject and then substituted into the equation $4x^2 + 3xy + y^2 = 8$. This needed to be done with care as brackets were needed for correct mathematical form. The equation that resulted needed to be manipulated correctly into a form that was either a quadratic in x^2 or a quadratic in y^2, ready to be solved. Many candidates were able to carry out the algebra needed, often replacing x^2 or y^2 with u or t, for example, which simplified the work. Those candidates who used substitutions such as $y = x^2$ or $y = y^2$ often confused themselves and did not recover. Some candidates would have improved if they had taken a little more care with signs or with ensuring that all terms in their equation were included in their manipulation. For example, candidates who multiplied all terms by x^2 or y^2 sometimes omitted to include the 8 on the right-hand side of the equation in this step. A good number of candidates who managed to find a correct form of the equation went on to find all the solutions, although a few only gave the positive values, or incorrectly rejected the negative values, at this point. A few candidates made spurious attempts to solve. These commonly had not rearranged to the standard form $ax^2 + bx + c = 0$, but had, for example, $x^2(x^2 - 5) = -4$ which then, incorrectly, became $x^2 = -4$ and $x^2 - 5 = -4$. It was important that the values of x and y that had been found were correctly paired and candidates who did not do this were penalised.

A small proportion of candidates used approaches that relied on a great deal of correct algebraic manipulation being carried out prior to any elimination being seen. These methods increased the chances of an error being made and so are not generally recommended, even though the algebra used was often skilful. Mostly, these methods involved substituting $xy = -4$ into the first equation in some way, in order to create expressions in x and y for which the candidate could complete the square or an expression in x and y that could be factorised. Sometimes the equations that resulted were quadratic and sometimes they were linear. In all cases, including the linear equations, the key step of eliminating an unknown had to be seen for credit to be given. Candidates who relied on their calculator to solve linear equations at this stage may have improved if they had understood the instructions on the front of the examination paper indicating that all necessary method must be shown.

Question 10

(a) Some concise, correct responses were seen to this part. The simplest approach was to rewrite the integrand as e^{3x+3}, or e^3e^{3x}, and then integrate. A few candidates multiplied by 3 rather than $\frac{1}{3}$. This was condoned for partial credit on this occasion as the initial manipulation had been carried out correctly and the power had not been changed. Many candidates omitted a constant of integration. This was also condoned on this occasion. However, candidates should be aware of the appropriate use of a constant of integration as the omission of such will not always be condoned for full credit. The response $\frac{1}{4}\left(e^{x+1}\right)^4$ was a commonly seen incorrect answer.

(b) (i) A very good proportion of solutions were fully correct. A few candidates were unable to differentiate sin4*x* correctly but were able to apply the correct form of the product rule. A few other candidates were able to differentiate sin4*x* correctly but did not realise that the product rule was required. A few candidates included a spurious '+*c*' in their answer. This was inappropriate and not condoned.

(ii) Candidates found this part of the question quite challenging. As the answer had been given it was essential that candidates showed full and clear working to justify having found the answer correctly. Better responses clearly linked the integral in this part to the previous part of the question, as required. These candidates then, step by step, rearranged the equation, integrated as required and showed the clear substitution of the limits into the integral, being careful with the order of the terms, any bracketing and signs. Candidates then needed to show the evaluation of each term to fully justify the values in the given expression. Candidates who omitted to show the substitution of the limits into the integral often did not show sufficient evidence to earn the final two marks. Some candidates may have improved if they had rechecked their differentiation in the previous part. Occasionally, sign errors that had been made in the previous part of the question resulted in candidates incorrectly adjusting their work in this part.

Question 11

(a) This part of the question relied in part on prerequisite geometrical knowledge and some excellent responses were seen. Candidates needed to form an expression for the volume of the given solid and write an expression for *y* in terms of *x*. Some candidates omitted to read the question carefully and gave an expression for *h* in terms of *r* or *x*, for example. Other candidates stated $x = r$ without actually using it as such. It would have been much simpler if they had just written expressions using *x* instead of making this statement. These candidates were all penalised. A few candidates used the volume of a sphere, rather than a hemisphere in their calculations. It was evident from the inclusion of several 'halves' in initial calculations, or from changes in coefficients, that some candidates had found their answer for the surface area to be incorrect and went back and corrected their error. This was excellent strategy. Other candidates may have done better if they too had adopted this strategy, rather than trying to make their working fit the given form. Weaker responses seen included using the given surface area and equating to $3\pi x^2 + 2\pi xy$ and then solving for *y*. This was not credited.

(b) Most candidates attempted to differentiate the expression for *S* given in **part (a)**. Many candidates did this fully correctly and went on to find the correct value for *x*. Most candidates were able to differentiate $\frac{5}{3}\pi x^2$ correctly but fewer were successful in differentiating $\frac{1000}{x}$, with sign errors or 1000 commonly seen. A good proportion of candidates knew the appropriate method was to equate the derivative to 0 and solve for *x*. Many earned the method mark available for this strategy. A few candidates found the second derivative and equated that to 0. Some candidates went on to find the minimum value of *S* and/or demonstrate that it was a minimum value, neither of which were requirements of the question.

Question 12

(a) (i) This part of the question was very well answered with almost all candidates able to give the correct coordinates in an acceptable form.

(ii) Candidates were able to use the value $x = 2$ to demonstrate that the point *C* was both a point on the curve and the line. Most candidates did not appreciate that the use of the command word 'verify' allowed them to use the given information in this way. To earn both marks, full correct method had to be shown. Most candidates equated the expressions and formed a quadratic equation in *x*. Providing the equation was correct, the method of solution was shown and the negative solution which arose was discarded, this was given full credit. A good proportion of candidates omitted to show any method of solution for their quadratic equation. This was not accepted as the answer had been given and choosing this approach meant that sight of the correct method was necessary. Few candidates chose to use the simpler approach of substituting $x = 2$ into the equations of the line and the curve and showing that the *y*-coordinates were the same. Again, for full marks, the method shown had to be complete and no step should be assumed. The simplest way to do this was to write each equation with *y* as the subject and then substitute $x = 2$. Candidates who changed the subject first were more successful. Candidates who substituted first tended to omit some working.

(b) Some neat, clear and fully correct answers were seen to this part of the question. Candidates who integrated the equation of the curve correctly and were careful with writing down their logarithms with brackets to show correct mathematical form often went on to give a completely correct solution. Candidates who omitted brackets often made errors in the work that followed. A few candidates multiplied $\ln(2x + 1)$ by 2 rather than $\frac{1}{2}$. Some candidates had a correct strategy, but not all could carry out their plan correctly. Candidates who found the area of the triangle using $\frac{1}{2}\times\text{base}\times\text{height}$ were more successful than those who integrated the expression for the line. Attempts at using integration to find the area of the triangle often contained sign errors, errors when substituting limits or use of incorrect limits. Candidates were not allowed to use their calculators to answer this question and so use of decimals in the working was penalised. Weaker responses typically included integrating $\frac{1}{2x+1}$ as $\frac{(2x+1)^0}{0}$.

Question 13

(a) Most candidates used a correct order of composition for the expression fg(*x*). A good proportion of these candidates went on to find an acceptable, fully simplified form. However, some candidates were unable to fully simplify the expression which resulted. To be fully simplified, it was required that there were no common factors and/or that the numerator or denominator of any fraction was not itself fractional. A small number of candidates formed a product of functions, $f(x) \times g(x)$. This misunderstanding was not condoned. A few other candidates attempted to form gf(*x*), which was also not condoned.

(b) (i) Many correct answers were seen to this part of the question. Common incorrect offerings were

$x > 0$, 'the range of f^{-1} is > 0' and $y \in \mathbb{R}$.

(ii) Candidates found this final question very challenging. The best candidates understood that they could form a quadratic equation in *x* and *y* and solve it for either *y* or *x* using the quadratic formula. A few excellent, fully correct answers were seen, with the choice of the positive square root fully justified although most of these candidates assumed the positive square root without comment and were penalised. Some candidates omitted to notice that the square root was positive and gave an answer including '±'. Perhaps consideration of the one-to-one nature of inverse functions may have helped these candidates. A few candidates earned a mark for at least writing the quadratic in a useful form. Most candidates made no real progress with this part.

ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 23

Key messages

To do well in this paper, candidates should read each question carefully and identify any key words or phrases, making sure they answer each question fully. Sufficient method needs to be shown so that marks can be awarded. When values are incorrect and the method from which they arise is not seen, marks cannot be given. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with trigonometric expressions, particularly in questions involving calculus.

General comments

Some candidates seemed to be reasonably well-prepared for this examination. A good proportion of candidates were able to use algebraic manipulation when needed, particularly in **Questions 2**, **3**, **5** and **6**. Some candidates were also able to write or interpret problems using correct mathematical form, as required in **Question 9(a)(i)** and **9(a)(ii)**, for example.

Questions 6(b), **8(b)**, **9(b)(ii)**, **10(a)** and **11(a)(ii)** required candidates to 'Show that' a result was correct or an expression was of a particular form. Showing clear and complete method for every step in these solutions was essential as the answer had been given and the marks were awarded for the method.

Candidates should also understand that, when a part of a question begins with the word 'Hence...', it is expected that they should use the previous part or parts of the question to answer the current part. This will often be the most straightforward method of solution and will be assessing a specific skill. This was seen in **Questions 5(b)** and **9(a)(ii)** in this examination.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

Candidates generally found this question to be an accessible start to the paper. Many candidates performed well and all those who made progress showed sufficient terms in the numerator and denominator to demonstrate that they had not used a calculator. A few candidates made an arithmetic slip and lost the accuracy mark.

Question 2

A reasonable proportion of candidates earned all of the marks available, presenting complete and logical answers which included discarding the extraneous solution that arose. The most common error from candidates who made little, or no, progress was to form the equation $30 = 14^{2x} - 21^{x+1} + 19$.

Question 3

(a) Most candidates attempted this part of the question. Some candidates were careful with converting the roots to powers and simplified correctly. Other candidates made slips in their working. Commonly the x in the numerator was omitted or $\sqrt[4]{81}$ was stated as 3^4. A few order of operations errors were also made. A small number of candidates used a rounded decimal value for the power $\frac{8}{3}$. This was not condoned.

(b) (i) A two-step process was needed to find the value of *a*. A reasonable number of candidates were able to manipulate the equation into a solvable form either by dividing or multiplying both sides by 2 or by rewriting the argument of the logarithm as 8^2, for example. A good proportion of candidates then went on to complete the solution correctly, although occasionally slips were made in solving the exponential equation that resulted. A small number of candidates made no progress with this part of the question.

(ii) A multi-step approach was needed for this part of the question also and a few excellent responses were seen. A key piece of information, which was overlooked by some candidates, was that the answer needed to be in base *a*. This should have guided them to the first sensible step of applying the change of base rule to change the base to *a*. Once this had been achieved, the candidates then needed to manipulate the denominator so that it was written as a single constant. Final answers of $\frac{1}{2}\log_a 3a$ or $\log_a \sqrt{3a}$ were acceptable. A small proportion of candidates confused the base and the argument of the logarithm and worked as if they had $\log\left(a^2\right)^{3a}$. Some candidates made no attempt to answer.

Question 4

A small proportion of candidates identified the most efficient method of solution by rewriting $y = \frac{\sin x}{\cos x}$ as $y = \tan x$ and then differentiating. Most candidates attempted to apply the quotient rule. Some candidates were successful with this, although sign errors were more common with this approach. The next step was to use the derivative to find the approximate change in *y* and some excellent solutions were seen. Common errors seen in weaker responses were omitting to substitute $x = -\frac{\pi}{4}$, having the calculator in degree mode or using $h - \frac{\pi}{4}$ instead of *h*.

Question 5

(a) Again, some excellent responses were seen to this part. A good proportion of candidates were able to find the correct critical values for the quadratic inequality. Some candidates were then able to form the correct inequality in *x* as their final solution. Other candidates offered only the critical values or wrote incorrect inequalities, with $x \leqslant 1.5 \quad x \leqslant 7$ being very common.

(b) A reasonable proportion of candidates understood the need to integrate between the critical values found in **part (a)**. Some responses were fully correct. A few candidates made errors when integrating. Other errors seen included making rounding errors or omitting to write the final answer as a positive value, which was required. Some candidates made no attempt to answer.

Question 6

(a) Most candidates used the factor theorem and correctly showed that p(–0.25) = 0. Some candidates omitted to write ‘ = 0’ which was not condoned, or made slips when writing powers.

(b) Good responses involved forming the factor 4*x* + 1 and using this to find the quadratic factor. Most candidates used algebraic long division. Candidates should know that factors of polynomials are generally of the form *ax* + *b* where *a* and *b* are integers. Most candidates used the factor *x* + 0.25, which was condoned for this particular question. There was some confusion between factors and roots, with candidates finding repeated factors and identifying them as repeated roots. This was not condoned.

Question 7

(a) Very few reasonable graphs were drawn. Some candidates drew the part of the curve in the first quadrant only. This was not accepted as the full shape needed to be attempted, including seeing the asymptotic behaviour. Some candidates had graphs with incorrect curvature and many made no attempt to answer.

(b) Candidates were more successful in this part. A good proportion were able to find the *y*-coordinate, although many wrote decimal values, rather than giving the exact form. Some decimals were not given to at least three significant figures and this was not accepted. Good responses showed correct differentiation. A few candidates appreciated that the form of the derivative was a multiple of $\frac{1}{4x-3}$ but did not multiply by 4. A small number of candidates found the correct gradient of the tangent but then continued to find the gradient of the normal and use that in their equation. Candidates who wrote the *y*-coordinate in exact form, more consistently earned the final mark for forming the equation.

Question 8

(a) (i) Some good, fully correct responses were seen to this part of the question. A few candidates earned one mark for knowing that the argument of the cosine remained unchanged and stating a multiplier that was either negative or 3. Some candidates simply stated $\cos\left(\frac{\phi+\pi}{3}\right)$ which was not sufficient. Some candidates made no attempt to answer. Some, otherwise good, responses omitted the constant of integration. This was condoned for this question but it is expected that candidates know the appropriate use of a constant of integration.

(ii) It was rare to see a fully correct answer to this part of the question. Some candidates did observe that a correct first step was to rewrite the integrand as 5. A small number of candidates integrated this correctly with respect to θ. For this question, the constant of integration was necessary for full credit to be awarded. A few candidates were penalised for omitting it. Weaker responses included integrating $5\sin^2\theta$ as $\pm 5\cos^2\theta$ and $5\cos^2\theta$ as $5\sin^2\theta$.

(b) A few candidates expanded the brackets and simplified prior to integrating. Some of these candidates were able to integrate one or other of the two terms that resulted, but correct integration of both terms was rarely seen. A few candidates simplified but omitted to integrate. Some candidates misinterpreted $2x^{-1}$ as $\frac{1}{2x}$ whereas other candidates integrated $\frac{1}{x^2}$ as $\ln x^2$. Many candidates made no real progress as they attempted to integrate the integrand in its given form.

Question 9

(a) (i) Success was varied in this question. A small proportion of candidates gave a fully correct completed square form. A few candidates earned a mark either for $(x + 1)^2$ or for the correct constant. Many candidates made no real progress

(ii) In this part of the question it was necessary to interpret the completed-square form from **part (i)**. As few candidates had an expression of the correct form in the previous part, correct answers were rarely seen. The most common incorrect response was to state that the range was the set of real numbers. Many candidates made no attempt to answer.

(b) (i) Very few correct responses were seen. Candidates often expressed both the domain and range in terms of *x* or gave the domain using *y* and the range using *x*. This may have been an issue with misinterpreting 'the domain of g is the range of g^{-1}' and 'the range of g is the domain of g^{-1}'.

(ii) Candidates found this part of the question very challenging. Many made no real progress. A small proportion of very good solutions were seen, usually by completing the square. However, the selection of the positive square root was not justified and this was needed for full credit to be awarded.

Question 10

(a) A few fully correct responses were seen. Some candidates were able to find a correct expression for *y* in terms of *x* but made no progress beyond that. The few that did substitute into a correct expression for the surface area sometimes found the manipulation needed to arrive at the correct form of the answer too challenging. Many candidates made no attempt to answer.

(b) Again, some fully correct responses were seen, although they were rare. Some candidates were unable to successfully apply the chain rule to $\sqrt{x^6+900}$, with slips in the power of the derivative not uncommon. A few candidates attempted to apply the quotient or product rule and success was varied. Candidates who knew that the next step in the process was to equate the derivative to 0 and solve for *x* were rewarded for persevering to that point. It seems likely that some candidates had not studied this particular topic as many candidates made no attempt to answer.

Question 11

(a) (i) A reasonable proportion of candidates gave neat, fully correct solutions to this part of the question. Various successful approaches were seen, including equating expressions for the difference, equating expressions for $\frac{1}{q}$ and using the sum of the three given terms to find the difference. Some candidates made a correct initial step, equating expressions, but were unable to demonstrate what was required correctly, either because they were unsure how to complete the solution or because they had made a slip in simplification. This was more common if their first step involved both *p* and *q* terms. A few candidates earned a mark for making a correct single statement. Other candidates offered no response of any value.

(ii) Candidates were more successful in this part with a reasonable number earning full marks. Occasionally slips were made in simplifying the algebraic expression or brackets were omitted in the first step and subsequent errors were made because of this. Some candidates may have improved if they had referred to the formula page at the start of the paper, as the expression for the *n*th term of an arithmetic progression was not always stated correctly.

(b) The majority of candidates were able to write a correct pair of simultaneous equations using the sum to infinity and the second term. Occasionally candidates attempted to use the sum to *n* terms formula instead of the sum to infinity, so perhaps a little more care was needed when studying the formula page. The equations needed to be combined correctly to gain further credit and this was not always achieved. Candidates who were careful when rearranging terms were more successful. Some candidates were writing the quadratic equation with a fractional constant and solving it without simplification of the terms to integers. This was condoned, even though the factors that resulted were not of the form $ax + b$ where *a* and *b* are integers, as was expected. On this occasion, candidates who made slips in their method of solution from which they recovered, were not penalised for the slips which had been seen.

Question 12

Candidates found this final question to be very challenging with very few marks being awarded in any part. In **part (a)**, a very small number of candidates differentiated correctly. Most of these went on to find either a negative time with the calculator correctly in radian mode or a positive time with the calculator incorrectly in degree mode. It is vital that candidates understand that, for this specification, arguments for trigonometric functions in calculus questions will always be in radians and radian mode should always be used. Most candidates made no attempt to answer either of the last two parts of the question and those that did were unsuccessful.