1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Theorem

\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)!r!} \)

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1 \\
\sec^2 A = 1 + \tan^2 A \\
\cosec^2 A = 1 + \cot^2 A
\]

Formulae for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A \\
\Delta = \frac{1}{2} bc \sin A
\]
1 (a) Write \( \log_{27} x \) as a logarithm to base 3. [2]

(b) Given that \( \log_a y = 3(\log_a 15 - \log_a 3) + 1 \), express \( y \) in terms of \( a \). [3]
The diagram shows the graph of $y = |f(x)|$ passing through $(0, 4)$ and touching the $x$-axis at $(2, 0)$. Given that the graph of $y = f(x)$ is a straight line, write down the two possible expressions for $f(x)$. [2]

(b) On the axes below, sketch the graph of $y = e^{-x} + 3$, stating the coordinates of any point of intersection with the coordinate axes. [3]
3 (a) Find the matrix $A$ if $4A + 5\begin{pmatrix} 4 & 0 & -1 \\ 3 & -2 & 5 \end{pmatrix} = \begin{pmatrix} 52 & -8 & 19 \\ 31 & 2 & 65 \end{pmatrix}$. [2]

(b) $P = \begin{pmatrix} 30 & 25 & 65 \\ 70 & 15 & 80 \\ 50 & 40 & 30 \\ 40 & 20 & 75 \end{pmatrix}$, $Q = \begin{pmatrix} 650 & 500 & 450 & 225 \end{pmatrix}$

The matrix $P$ represents the number of 4 different televisions that are on sale in each of 3 shops. The matrix $Q$ represents the value of each television in dollars.

(i) State, without evaluation, what is represented by the matrix $QP$. [1]

(ii) Given that the matrix $R = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, state, without evaluation, what is represented by the matrix $QPR$. [1]
The diagram shows a circle, centre $O$, radius 8 cm. The points $P$ and $Q$ lie on the circle. The lines $PT$ and $QT$ are tangents to the circle and angle $POQ = \frac{3\pi}{4}$ radians.

(i) Find the length of $PT$. [2]

(ii) Find the area of the shaded region. [3]

(iii) Find the perimeter of the shaded region. [2]
5 (a) A lock can be opened using only the number 4351. State whether this is a permutation or a combination of digits, giving a reason for your answer. [1]

(b) There are twenty numbered balls in a bag. Two of the balls are numbered 0, six are numbered 1, five are numbered 2 and seven are numbered 3, as shown in the table below.

<table>
<thead>
<tr>
<th>Number on ball</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>2</td>
<td>6</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

Four of these balls are chosen at random, without replacement. Calculate the number of ways this can be done so that

(i) the four balls all have the same number, [2]

(ii) the four balls all have different numbers, [2]

(iii) the four balls have numbers that total 3. [3]
A particle $P$ is projected from the origin $O$ so that it moves in a straight line. At time $t$ seconds after projection, the velocity of the particle, $v$ ms$^{-1}$, is given by $v = 2t^2 - 14t + 12$.

(i) Find the time at which $P$ first comes to instantaneous rest. [2]

(ii) Find an expression for the displacement of $P$ from $O$ at time $t$ seconds. [3]

(iii) Find the acceleration of $P$ when $t = 3$. [2]
7 (a) The four points $O, A, B$ and $C$ are such that

\[
\overrightarrow{OA} = 5\mathbf{a}, \quad \overrightarrow{OB} = 15\mathbf{b}, \quad \overrightarrow{OC} = 24\mathbf{b} - 3\mathbf{a}.
\]

Show that $B$ lies on the line $AC$. [3]

(b) Relative to an origin $O$, the position vector of the point $P$ is $\mathbf{i} - 4\mathbf{j}$ and the position vector of the point $Q$ is $3\mathbf{i} + 7\mathbf{j}$. Find

(i) $|\overrightarrow{PQ}|$.

(ii) the unit vector in the direction $\overrightarrow{PQ}$.

(iii) the position vector of $M$, the mid-point of $PQ$. [2]
8  (a)  (i)  Find \( \int e^{4x+3} \, dx \).  \[2\]

(ii) Hence evaluate \( \int_{2.5}^{3} e^{4x+3} \, dx \).  \[2\]

(b)  (i)  Find \( \int \cos \left( \frac{x}{3} \right) \, dx \).  \[2\]

(ii) Hence evaluate \( \int_{0}^{\frac{\pi}{6}} \cos \left( \frac{x}{3} \right) \, dx \).  \[2\]
(c) Find \( \int (x^{-1} + x)^2 \, dx \). \[4\]
9  (a) Find the set of values of $x$ for which $4x^2 + 19x - 5 \leq 0$.  

(b) (i) Express $x^2 + 8x - 9$ in the form $(x + a)^2 + b$, where $a$ and $b$ are integers.  

(ii) Use your answer to part (i) to find the greatest value of $9 - 8x - x^2$ and the value of $x$ at which this occurs.
(iii) Sketch the graph of \( y = 9 - 8x - x^2 \), indicating the coordinates of any points of intersection with the coordinate axes. [2]
The relationship between experimental values of two variables, $x$ and $y$, is given by $y = Ab^x$, where $A$ and $b$ are constants.

(i) By transforming the relationship $y = Ab^x$, show that plotting $\ln y$ against $x$ should produce a straight line graph. [2]

(ii) The diagram below shows the results of plotting $\ln y$ against $x$ for 7 different pairs of values of variables, $x$ and $y$. A line of best fit has been drawn.

By taking readings from the diagram, find the value of $A$ and of $b$, giving each value correct to 1 significant figure. [4]

(iii) Estimate the value of $y$ when $x = 2.5$. [2]
The Venn diagram above shows the sets $A$, $B$ and $C$. It is given that
$n(A \cup B \cup C) = 48,$
$n(A) = 30,$  $n(B) = 25,$  $n(C) = 15,$
$n(A \cap B) = 7,$  $n(B \cap C) = 6,$  $n(A' \cap B \cap C') = 16.$

(i) Find the value of $x$, where $x = n(A \cap B \cap C)$. [3]

(ii) Find the value of $y$, where $y = n(A \cap B' \cap C)$. [3]

(iii) Hence show that $A' \cap B' \cap C = \varnothing$. [1]